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Control of Unicycle Robot Using Robust Estimation Techniques

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Dedication:

Praise be to Allah, first and last, outwardly and inwardly, who inspired me with patience and gave me strength and success, so praise and thanks be to Him for His countless blessings and for bringing me this achievement after a long time of endeavor and struggle.

To my dear mother, the source of tenderness and reassurance, who accompanied me with her prayers and smile despite all circumstances, and to my dear father, the first supporter, who instilled in my heart the value of knowledge and diligence...

You have my love and gratitude, your presence is the real support in my life.

To myself...

You who walked your path despite fatigue, fell and then stood up again, you who believed in the dream when many doubted it, and persevered silently in times when no one was with you...

I am proud of you because you didn't give up, because you chose to continue despite everything, and because you believed that a small step today makes a difference tomorrow. Thank you for being real, strong, and believing that the light will come, no matter how long it takes.

To my dear siblings, you are the light in my days, the silent supporters, and the joy that accompanies me in every achievement.

Finally, to my esteemed professors, whose words were a light and whose guidance was a boost on my journey, I am very grateful to you, for you are the beacons of this path.

Kawther GACEMI

Dedication:

To myself ...

To the one who worked hard and persevered until I reached this moment.

To the one who worked hard despite the challenges and faced fatigue with patience and perseverance.

Success came from Him, and this moment was the fruit of great effort and toil.

To my mom and dad, You are the light that illuminated my path, you are the real support and the first motivation. You are the partners of this success.

To my dear family,For your support, prayers and words that lifted my spirits, your presence around me has been a source of strength and warmth at all stages of this journey.

To my honored professors,To everyone who taught me a letter, and to everyone who gave me feedback or guidance, you have been beacons that illuminated the path of learning, and essential pillars of this achievement.

In conclusion ,I thank myself for every moment of patience despite fatigue, for every attempt despite fatigue, and for insisting on moving forward. Thank you to me for not giving up and succeeding, and thank you to myself. To those who tired and persevered until I reached this moment. To those who worked hard despite challenges and faced fatigue with patience and perseverance, to those who believed that God does not lose the reward of those who work well.

imtinanemaazouzi

Dedication :

We would like to dedicate this work to our esteemed supervisor, **Professor Noureddine Gazzam**, whose guidance, patience, and continuous encouragement have been essential throughout the journey of our graduation project.

We are deeply grateful for the time and effort he devoted to us, always offering his knowledge generously and motivating us to do our best. His valuable insights and constructive advice not only enriched this work but also helped us grow as students and as individuals.

We would also like to extend our sincere thanks to the President of the jury, **Dr. Oubbati Brahim Khalil** and the examiner, **Dr. Sahraoui Khaled**, for their valuable time, effort, and contribution in evaluating our work. Their presence and remarks are greatly appreciated.

It has been a privilege to work under such guidance and to be evaluated by such distinguished scholars. We will always remember their support and kindness with sincere appreciation.

ملخص:

تهدف هذه المذكرة إلى تطوير نظام تحكم يضمن تتبعاً دقيقاً لمسارات الروبوت أحادي العجلة، خاصة في الحالات التي تتأثر فيها القياسات بالضجيج أو بقيود الحساسات. لتحقيق ذلك، تم اعتماد **خوارزمية التحكم بالانزلاق (Sliding Mode Control - SMC)** لما تتميز به من قدرة عالية على مقاومة الاضطرابات وعدم اليقين في النمذجة، إلى جانب **خوارزمية التقدير Super-TwistingEstimator (STE)** التي تتيح استخراج المشتقات الزمنية لمسار المرجع، مثل السرعة، بدقة عالية ودون تأثر كبير بالضجيج.

شملت الدراسة بناء النموذج الكينماتيكي للروبوت، وتصميم قوانين تحكم من الرتبتين الأولى والثانية، ثم إجراء محاكاة عددية باستخدام برنامج **MATLAB**. تمت مقارنة الأداء لاختبار تتبع المسار في حالتين: باستخدام السرعات المرجعية الحقيقية، وباستخدام السرعات المقدَّرة عبر الخوارزمية.

أظهرت النتائج أن النظام المقترح يحقق تتبعاً جيداً للمسار المطلوب مع مقاومة واضحة للضجيج، وتأخر زمني مقبول في التقدير، مما يؤكد فعالية الجمع بين **التحكم بالانزلاق (SMC)** و**خوارزمية Super-TwistingEstimator (STE)** في تطبيقات الروبوتات المتنقلة.

Abstract:

This paper aims to develop a control system that ensures accurate tracking of single-wheeled robot trajectories, especially in cases where measurements are affected by noise or sensor limitations. To achieve this, the Sliding Mode Control (SMC) algorithm was adopted (SMC) algorithm has been adopted for its high resistance to disturbances and uncertainty in modeling, along with the Super-Twisting Estimator (STE) algorithm, which allows the extraction of time derivatives of the reference trajectory, such as speed, with high accuracy and without significant noise interference.

The study included building a kinematic model of the robot, designing first- and second-order control laws, and then performing numerical simulations using MATLAB to test performance. Trajectory tracking was compared in two cases: using actual reference velocities and using velocities estimated by the algorithm.

The results showed that the proposed system achieves good tracking of the desired path with clear noise resistance and acceptable estimation delay, confirming the effectiveness of combining sliding mode control (SMC) and the Super-Twisting Estimator (STE) algorithm in mobile robot applications.

Résumé:

L'objectif de cet article est de développer un système de contrôle qui assure un suivi précis des trajectoires d'un robot à une roue, en particulier dans les cas où les mesures sont affectées par le bruit ou les contraintes des capteurs. Pour ce faire, l'algorithme de commande par mode glissant (SMC) a été adopté en raison de sa grande résistance aux perturbations et aux incertitudes de modélisation, ainsi que l'estimateur Super-TwistingEstimator (STE), qui permet d'extraire les dérivées temporelles de la trajectoire de référence, telles que la vitesse, avec une grande précision et sans être affecté par le bruit.

L'étude a consisté à construire le modèle cinématique du robot, à concevoir des lois de contrôle du premier et du second ordre, puis à effectuer des simulations numériques à l'aide de MATLAB pour tester les performances. Le suivi des trajectoires a été comparé dans deux cas : en utilisant les vitesses de référence réelles et en utilisant les vitesses estimées par l'algorithme.

Les résultats montrent que le système proposé permet un bon suivi de la trajectoire avec une bonne résistance au bruit et un délai d'estimation acceptable, ce qui confirme l'efficacité de la combinaison du SMC et de l'algorithme Super-TwistingEstimator (STE) dans les applications de robotique mobile.

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Abbreviations:

SMC: Sliding Mode Control.
PID: Proportional-Integral-Derivative.
LQR: Linear Quadratic Regulator.
MPC: Model Predictive Control.
STE: Super-Twisting Estimator.

Notation and Basic Terms:

- **State Variables:**
  - x, y: position coordinates in 2D plane (world frame)
  - θ: orientation angle with respect to x-axis of global frame
- **Control Variables:**
  - v: linear velocity along forward direction
  - ω: angular velocity (rate of orientation change)
- **System Vectors:**
  - q(t): state vector $[x(t), y(t), \theta(t)]^T$
  - $q_r(t)$: reference trajectory or desired state
  - e(t):trackingerrorvector, $e(t) = q(t) - q_r(t)$
  - u(t): control input vector $[v(t), \omega(t)]^T$

General Introduction

Unicycle robots are important robotic systems in academic research and industrial applications due to their ability to move flexibly in irregular environments and the simplicity of their structural design compared to multi-wheeled systems. This memoir aims to present an integrated study that starts with building a theoretical foundation for modeling the robot's motion and kinematics, through state estimation and control strategies, to performance simulation and analysis of results to ensure the effectiveness of the proposed methodology.

In the first chapter I, “Unicycle Robot Concepts and Modeling - Kinematics”, we highlight differential kinematics models for a unicycle robot, defining basic concepts such as kinematic variables, nonholonomic constraints, and equations that relate the inputs (linear velocity and angular velocity) to the derivatives of spatial and angular states. These fundamentals can form the basis for the design of any subsequent control algorithm.

Chapter II, “Robust Control and Estimation for Unicycle Robot,” moves on to address how to estimate the true state of the robot in real time using techniques such as the integra, and then reviews a range of control strategies (such as adaptive model control and sliding state control) to ensure accurate and stable trajectory tracking.

Finally, in Chapter III, “Trajectory Tracking of a Unicycle Robot using SMC and STE,” we move to a digital simulation environment where we apply the models and estimation, and control algorithms to multiple scenarios. We analyze key performance indicators such as tracking accuracy, settling time, and sensitivity to noise and disturbances, resulting in a comprehensive evaluation that reflects the validity of the proposed methodology and suggests future improvements.

Together, these three chapters embody a comprehensive methodology from theoretical construction, to numerical implementation to critical performance analysis, ensuring that the research goals of developing highly efficient uniaxial robots for locomotion and control are achieved.

Chapter I: Unicycle Robot Concepts and Kinematics Model

I.1 Introduction:

Mobile robots play an important role in modern engineering and robotics research, with huge applications in industrial, service, and army environments. Among the kinematic fashions used to symbolize the movement of those robots, the unicycle robot model stands out as a simplified yet effective representation of mobile robot motion on a flat floor in dimensions. This version is capable of constituting the primary kinematic houses of maximum wheeled robots even as minimizing the mathematical complexity of greater specialized fashions.

The single-wheeled robotic version is primarily based totally at the perception that the robotic is a factor transferring with linear and angular pace approximately its vertical axis, as it should be reflecting the non-Holonomic movement constraints that represent this sort of system. Despite its simplicity, this model can simulate complex kinematic behaviorsand is extensively used withinside the improvement of algorithms for navigation, trajectory tracking, and movement control **[1].**

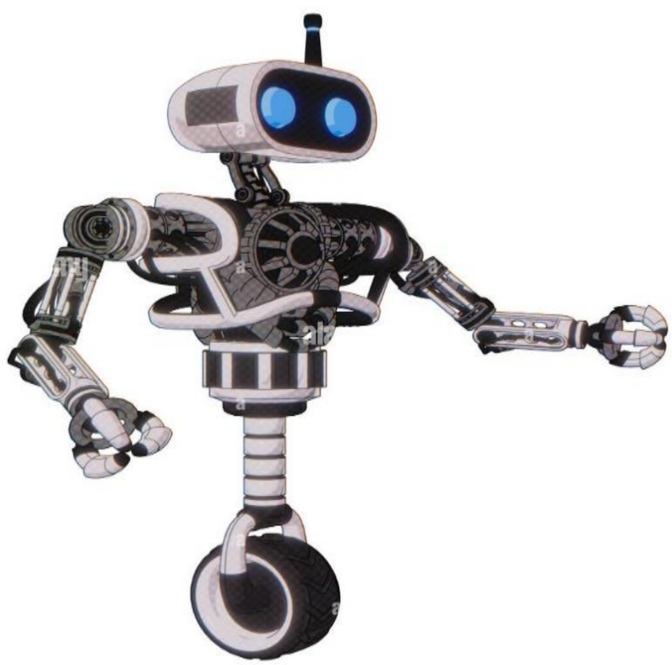


Figure I.1: Conceptual design of a unicycle robot with a single wheel and articulated arms, illustrative model **[1].**

I.2 Project Motivation:

Single-wheeled robots are fundamental to robotics research due to their mechanical simplicity and complex nonlinear dynamics. They provide an ideal environment for testing advanced control algorithms and have direct applications in automation.

The main challenge lies in achieving accurate path tracking relying solely on partial measurements of the state (position coordinates) that may be affected by sensor noise. Traditional control methods fail when direction angles must be estimated from noisy position derivatives, leading to performance degradation and potential instability. This project develops an integrated framework that combines algebraic estimation and slip control to address these limitations, aiming to:

- Robust state estimation despite measurement noise
- High-precision path tracking in realistic conditions
- Real-time feasibility
- Formal guarantees of stability

The single-wheel model was chosen due to its high mathematical analyzability, its industrial relevance in wheeled robotics, and its established role as a reference system for testing control techniques**[2].**

I.3 Control Objective:

The primary objective is to design a control system enabling precise trajectory tracking of predefined reference paths $q_r(t) = [x_r(t), y_r(t), \theta_r(t)]^T$ despite measurement noise and partial state information.

Main objectives:

- Trajectory tracking: minimize the tracking error $e(t) = q(t) - q_r(t)$ while ensuring convergence in a limited time.
- Noise resistance: Maintain performance even in the presence of noisy position measurements.

- State estimation: Estimate the unmeasured heading angle θ from the available noisy position data.

Control strategy: First-order sliding mode control (SMC) was chosen due to its natural immunity to disturbances, its ability to converge in a finite time, and its insensitivity to identical disturbances**[3].**

Performance specifications:

- Position error less than 5 cm, heading error less than 2 degrees.
- Stable operation with measurement noise up to 10% of signal amplitude.
- High real-time computational efficiency suitable for embedded systems.

I.4 The evolution of robots throughout history:

The first theoretical studies of single-wheel robots began in the late 1970s when researchers sought to understand methods of controlling robots with movement constraints. This model gained widespread popularity due to its mathematical tractability while maintaining the essential characteristics of real wheeled robots. The following decades saw significant developments in the understanding and application of this fundamental class of non-holonomic mobile robots, with modern applications evolving to include autonomous navigation systems, educational platforms, and mobile industrial robots [4].The single-wheel robot model has become a reference standard for testing control algorithms and understanding the fundamental challenges in controlling constrained mechanical systems, providing valuable insights into the fundamental challenges of controlling constrained mechanical systems[5].

I.5 Definition and description of Unicycle Robot:

I.5.1 Definition:

A unicycle robot is a simplified version of a non-holonomic mobile robot, consisting of a chassis with a single wheel or set of wheels aligned linearly to provide one linear degree of freedom (to move forward and backward) and one angular degree of freedom (to rotate about the vertical axis), without the possibility of direct lateral

Figure I.2: LEGONXT-Based Self-Balancing Unicycle Robot Prototype **[2].**

I.5.2 Description:

I.5.2.1 Kinematic Features:

o non-holonomic constraints prevent lateral sliding, forcing the robot's lateral velocity to always be zero.

o Two key variables are included in its kinematic equation: Linear velocity v and angular velocity ω/omega, making the 2D motion model easy to formulate.

I.5.2.2 Dynamic Complexity:

o Despite the simplicity of the kinematic model, the dynamic characteristics of a unicycle robot are intrinsically nonlinear and may be unstable without proper control.

o Requires continuous balancing and path deviation correction, given the system's sensitivity to sensory noise, mathematical model inaccuracies, and external interferences.

I.5.2.3 Application and relevance:

o It is a reference model in nonlinear control and observability research, and is used to test algorithms such as Sliding Mode Control, Model Predictive Control, and others.

o Its principles apply to other, more complex robots (such as parallel wheel robots and stabilizers) and help to understand the fundamental challenges in controlling vehicles with constrained motion.

I.5.2.4 Physical and Kinematic Characteristics:

Below are the physical and kinematic characteristics of the robot:

Feature	Description
Inherent Instability	Requires active control to maintain balance
Non-Holonomic Constraint	Motion is restricted in certain directions
High Maneuverability	Capable of sharp turns and efficient mobility
Sensitivity to Disturbances	Affected by wind, slope, and terrain
Sensor Dependence	Requires precise sensors
Energy Efficiency	Requires optimized control for prolonged operation

Table I.1: Physical and kinematic characteristics of the unicycle robot **[4].**

I.6 Types of single-wheeled robots

I.6.1 Traditional single-wheeled robots:

These rely on a single large wheel with a simple balancing system using a gyroscope. They are characterized by low energy consumption and ease of design. They are mainly used in educational and research applications.

Figure I.3: Traditional Single-Wheeled Robot **[3].**

I.6.2 Self-balancing single-wheel robots:

These contain advanced sensor systems and intelligent control algorithms that automatically maintain balance. They are characterized by their ability to recover from disturbances and navigate complex environments. They are used in personal transportation and surveillance applications.

Figure I.4: Self-Balancing Single-Wheel Robot **[4].**

I.6.3 Ball Robots:

Consist of a spherical shell that protects the internal components with an internal propulsion mechanism. They are characterized by movement in all directions and high shock resistance. They are used in security surveillance and exploration in hazardous environments.

Figure I.5: Ball Robot **[5].**

I.7 The Kinematic Model:

I.7.1 Definition of Kinematics:

Kinematics is concerned with describing the motion of a robot (position and orientation) without introducing forces or moments. A wheeled robot (Unicycle) is represented by a case:

$$\boldsymbol{q} = \begin{vmatrix} x \\ y \\ \theta \end{vmatrix} \tag{I.1}$$

Where:

x, y: The coordinates of the center of the robot in the fixed lens frame (global).

θ: The orientation angle of the object (robot axis) relative to the world frame.

I.7.2 Control Variables:

Longitudinal speed v (Linear Velocity): - controls the rate at which the robot advances.

Angular velocity ω (Angular Velocity): - controls the rate at which the orientation changes.

- ✓ State Vector Definition:

$$\mathrm{q(t)} = [\mathrm{x(t)}, \mathrm{y(t)}, \theta(\mathrm{t})]^{\mathrm{T}} \in \mathbb{R}^3 \tag{I.2}$$

- ✓ Control Input Victor:

$$u(t) = [v(t), \omega(t)]^{\mathrm{T}} \in \mathbb{R}^2 \tag{I.3}$$

- ✓ State Space Form:

$$\dot{q}(t) = f(q(t), u(t)) \tag{I.4}$$

where f(q,u) represents the kinematic model function.

- ✓ Nonlinear System Représentation :

$$\begin{cases} \dot{x} = Vcos(\theta) \\ \dot{y} = Vsin(\theta) \\ \dot{\theta} = \omega \end{cases} \quad (I.5)$$

- ✓ Matrix Form :

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \theta \end{pmatrix} = \begin{pmatrix} cos(\theta) & 0 \\ sin(\theta) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \quad (I.6)$$

- ✓ General State Space Equation:

$\dot{q} = G(q)u$(I.7)

where G(q) is the input matrix dependent on current state

I.8 Derivation of the equations of motion:

We start from the definition of differential motion in the Body frame and then convert it to the World frame:

I.8.1 in the body frame:

$$\begin{cases} v = B\dot{x} \\ 0 = B\dot{y} \\ \dot{\theta} = \omega \end{cases} \quad (I.8)$$

where the motion in the transverse axis (Y_B) is zero for the Unicycle robot.

Moving to the world frame, we use a rotation with a matrix:

$$R(\theta) = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} \quad (I.9)$$

This leads to:

$$\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = \begin{pmatrix}v\\ 0\end{pmatrix} R(\theta) = \begin{pmatrix}v\ cos\theta\\ v\ \ sin\theta\end{pmatrix} \quad \text{(I.10)}$$

with $\omega = \dot{\theta}$ gives us the full equations:

$$\begin{cases}\dot{x} = Vcos(\theta)\\ \dot{y} = Vsin(\theta)\\ \quad \dot{\theta} = \omega\end{cases} \quad \text{(I.11)}$$

I.8.2 Global Frame Transformation:

- ✓ Rotation Matrix from Body to Global Frame:

$$R(\theta)= \begin{pmatrix}cos(\theta) & -sin(\theta)\\ sin(\theta) & cos(\theta)]\end{pmatrix} \quad \text{(I.12)}$$

Velocity Transformation:

$$(\dot{x}) = (\cos(\theta) \ \ -\sin(\theta))(v) = (v\cos(\theta)) \quad \text{(I.13)}$$

$$(\dot{y}) \ = \ (sin(\theta) \ \ cos(\theta))\ (0) \ = \ (v\ sin(\theta)) \quad \text{(II.14)}$$

- ✓ Complete Kinematic Model :

$$\begin{cases}\dot{x} = Vcos(\theta)\\ \dot{y} = Vsin(\theta)\\ \quad \dot{\theta} = \omega\end{cases} \quad \text{(I.15)}$$

- **Geometric Interpretation:**
  - $g_1(q)$: direction of forward motion
  - $g_2(q)$: direction of rotational motion
  - Linear combination spans reachable velocity directions

I.8.3Mathematical formulation of constraints:

- The main constraint equation $\dot{x}\sin(\theta) - \dot{y}\cos(\theta) = 0$ prevents direct lateral movemen

- The robot has limited movement in the direction of its orientation only, with no speed in the vertical direction.
- Matrix: $A(q) \cdot \dot{q} = 0$ where matrix $A(q) = [sin(\theta), -cos(\theta), 0]$
- Differential: $\omega(q) \cdot dq = 0$ where $\omega(q) = [sin(\theta), -cos(\theta), 0]\ dq$
- The constraint defines the directions of prohibited velocity and creates a deviation in configuration space
- Requires sophisticated control strategies for motion planning
- Nearby configurations cannot be accessed directly and require complex motion sequences
- Path planning must respect the boundaries of the constraints
- Constraint violations occur with lateral slippage, wheel slippage, or external forces

I.8.4 Practical Applications:

The kinematic unicycle robot model finds applications in:

- Robotic navigation and path planning.
- Autonomous mobile systems.
- Educational robotics platforms.
- Industrial transportation systems.
- Stability and controllability analysis.
- Development of algorithms for nonlinear control.

I.9 Control Objectives and Applications:

I.9.1 Basic control goals:

Path tracking: The robot must follow a timed path with the required accuracy.

Point stabilization: Reach the target position (xd, yd, θd) from an initial position while ensuring zero velocities on arrival.

I.9.2 Real-world applications:

The principles of the single-wheeled robot model apply to diverse process systems including delivery robots, inspection robots, personal mobility vehicles, warehouse automation systems, and educational platforms.

I.10 Non-Holonomic Constraints:

The primary property of a unicycle robot is its non-holonomic constraint, which can be expressed as follows: $x^{\cdot}\ sin(\theta) - y^{\cdot} cos(\theta) = 0$

This limitation prevents direct lateral movement and requires sophisticated control strategies to achieve the desired movements.

I.11 Controllability and Control Analysis:

I.11.1 Controllability:

A single-wheeled robot system can be expressed as a vector field:

$$g^{\cdot} = v g_1(q) + \omega g_2(q) \quad \text{(I.16)}$$

Wherein:

$$g_1(q) = \begin{pmatrix} cos(\theta) \\ sin(\theta) \\ 0 \end{pmatrix}, \quad g_2(q) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \text{(I.17)}$$

Calculating an arc for me produces:

$$[g_1, g_2] = \begin{pmatrix} sin(\theta) \\ -cos(\theta) \\ 0 \end{pmatrix} \quad \text{(1.18)}$$

Since the vectors g_1, g_2, and $[g_1, g_2]$ are linearly independent, the system satisfies the Lee algebra rank condition (LARC), ensuring controllability

I.11.2 Observability:

The observability of a unicycle robot refers to the ability to reconstruct its full state (x, y, θ) based on measurable outputs. In most practical cases, only the position (x, y) is directly measured, while the orientation θ is not directly observable.

To ensure full observability, additional estimation methods (e.g., algebraic or Kalman filters) are used to infer θ from available data. The system is considered observable if the observability matrix, built from outputs and their derivatives, has full rank.

I.11.3 Stability Analysis:

- The system has infinite equilibrium points at any configuration $q = [x_0, y_0, \theta_0]^T$with zero control inputs.
- Traditional linear techniques are limited in effectiveness because the system does not satisfy the necessary Brocket condition.
- It is impossible to achieve point-wise stability using fixed and smooth linear control.
- Lyapunov analysis is difficult due to non-holonomic constraints and requires custom Lyapunov functions.
- Input-to-state stability (ISS) can be achieved with appropriate control design to ensure limited deviations.
- Practical solutions require time-varying controllers or discontinuous methods such as slip control.
- Regional analysis around desired trajectories are more useful than global point stability.
- The basin of attraction depends on controller parameters with a trade-off between. convergence speed and stability margins.

I.12 Practical Challenges and Solutions:

I.12.1 Slip and Friction Challenges:

Automated systems face the problem of changing friction coefficients and loss of grip, which affects control accuracy. Solutions include adaptive control, intelligent compensation algorithms, and improved mechanical design of tires.

I.12.2 Floor Irregularities:

Challenges include varied terrain, surface type changes, and weather conditions. Solutions require the use of multi-layer sensor systems, adaptive mapping, and hybrid control.

I.12.3 Sensing and control delays:

Delays result from signal processing, command transmission, and mechanical inertia. They are addressed through predictive control, fast signal processing, and low-delay design.

I.12.4 Physical limitations of motors:

Motors have inherent barriers of their performance they can't rotate at countless velocities or generate limitless torque, they require power to operate, and they warmth up with use. We clear up those troubles through wise strength control, suitable cooling systems, and the choice of green motors.

I.13 Performance Evaluation Criteria:

I.13.1 Tracking Accuracy Indicators:

- Absolute Error: Distance between target and actual location.
- RMSE: Root mean square error as a comprehensive statistical measure.
- Spatial correlation coefficient: Matching the planned path with the actual path.

I.13.2 Time response and stability:

- Initial response time: From command to start of movement.
- Time to reach steady state: to reach the desired speed.
- Damping coefficient: a measure of system stability and resistance to oscillation.

I.13.3 Energy consumption efficiency :

- Specific consumption: energy consumed per unit distance.
- Conversion efficiency: ratio of useful energy to total energy.
- Battery life: Continuous operating time on a single charge.

I.13.4 Interference resistance:

- Signal-to-noise ratio (SNR): Indicator of signal quality.
- Disturbance rejection coefficient: Ability to ignore external influences.
- Recovery time: Speed of return to normal performance after interference.

I.14 Conclusion:

in this chapter, we presented the theoretical foundations of the unicycle robot by analyzing its kinematic model and essential properties. We discussed the non-holonomic constraints that prevent direct lateral movement and highlighted the control challenges associated with such systems.

The fundamental equations relating linear and angular velocities to the robot's position and orientation were derived, providing the mathematical basis for the design of control algorithms. We also examined the system's controllability and observability, confirming that appropriate control strategies can be effectively implemented.

Finally, the study of the robot's physical and kinematic characteristics demonstrated that, despite its structural simplicity, the unicycle robot requires continuous and robust control to maintain stability and achieve its objectives. These insights serve as a foundation for the following chapter, which will address estimation and control strategies to enhance performance in real-world applications.

Chapter II: Robust Control and Estimation for Unicycle Robot

II.1 Introduction:

The control of nonlinear systems is one of the main challenges in cybernetics, due to the complexity of their dynamics and the difficulty of accurately modeling them. The unicycle robot is a suitable example to study this type of system, as it is categorized as an inherently unstable device that requires continuous balance and precise control of steering and motion along the trajectory, making it difficult to control using traditional methods. The sliding mode control (SMC) method will be introduced, explaining how to incorporate the estimated derivatives into the control law to ensure accurate tracking of the reference trajectory, despite limited measurements and changing ambient conditions.

One of the main challenges we will face while implementing this module is that not all the variables needed for direct control are available, especially the time derivatives of the robot's position, such as the angle. Additionally, measured signals are often corrupted by noise, which negatively impacts control accuracy and stability. To address these challenges, robust derivative estimation techniques are needed to accurately extract the unmeasured variables from the available signals. This stage is necessary to ensure the integrity and effectiveness of the control law.

II.2 Modeling and control challenges in a unicycle robot:

II.2.1 Structural challenges of the unicycle robot:

The unicycle robot poses several structural and dynamic challenges due to its

Non-linear and under-actuated nature. These include:

- **Lack of Passive Stability**: The robot requires continuous control input to remain upright, as it cannot balance itself naturally.
- **High Sensitivity to Disturbances:** Environmental factors such as ground friction variations or wind can easily disrupt its motion, requiring robust compensation mechanisms.
- **Non-uniform Motion Dynamics**: Due to its single-wheel configuration and off-center rotation axis, achieving smooth and symmetric motion is difficult.
- **Limited State Observability**: Key dynamic states such as velocities and

Angular rates are not always directly measurable, which affects the precision of control implementation.

These factors reveal the limitations of classical linear controllers like PID, which assume full state availability and minimal disturbances. Consequently, advanced nonlinear control methods with integrated estimation techniques are essential for reliable performance.

II.2.2 The mathematical model of a unicycle robot:

To design control and estimation systems for a unicycle robot, it is first necessary to define its mathematical model. In general, nonlinear systems can be expressed by the formula **:**

$$\dot{x} = f(x,u) \qquad \text{(II.1)}$$

In the case of a unicycle robot, the kinematic model is given by the following equations:

$$\begin{cases} \dot{x} = v\cos(\theta) \\ \dot{y} = v\sin(\theta) \\ \dot{\theta} = \omega \end{cases} \qquad \text{(II.2)}$$

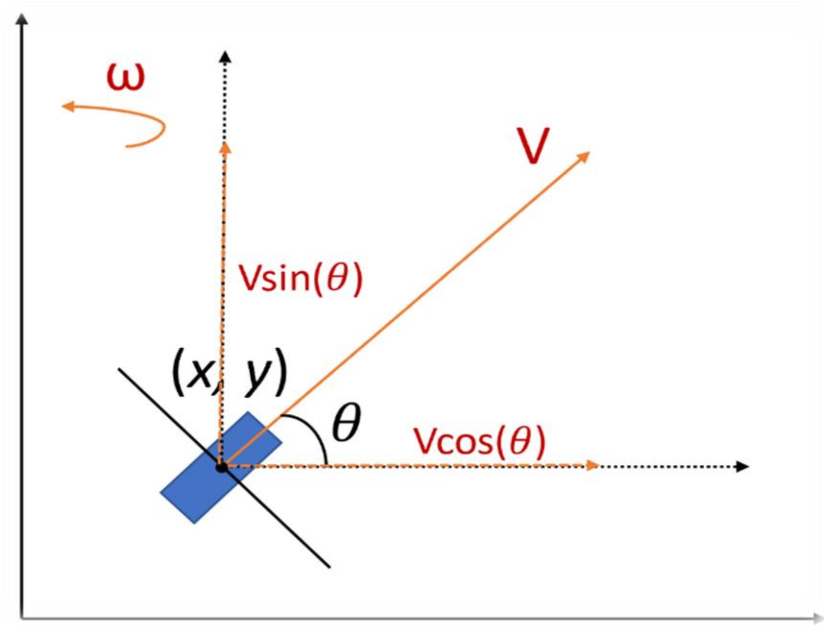
Where:

- x, y:The positional coordinates of the robot.

-θ: Steering angle.

- v: Linear speed.-ω: angular velocity.

As illustrated in Figure 2.1, the unicycle robot’s motion is defined by its position **(x, y)** and orientation $\boldsymbol{\theta}$, with velocity $\boldsymbol{v}$ and angular velocity$\boldsymbol{\omega}$**[11]**.



FigureII.1: Kinematic representation of a unicycle robot, showing the relationship between position and orientation **[6].**

This model highlights the strong coupling between positional and angular motions. In particular, any error in estimating the heading angle θ or the linear velocity v directly affects the accuracy of trajectory tracking. Consequently, precise estimation of these variables is crucial for ensuring robust path-following performance.

For this reason, advanced estimation techniques are often employed to reconstruct unmeasured states such as θ, especially when using sensors with limited accuracy or in noisy environments.

II.3 Sliding Mode Control (SMC):

II.3.1 General introduction to SMC:

II.3.1.1 About SMC:

Sliding Mode Control (SMC) is a powerful and efficient nonlinear control technique, characterized by its ability to deal with uncertainties and external disturbances in dynamic systems. The basic principle of SMC is to drive the trajectory of the system to a predefined "sliding surface" in the state space, and then keep the trajectory on that surface. Once the system reaches the glide surface, its behavior is determined by the properties of that surface itself, which can be designed to be stable and desirable.

II.3.1.2 When it is used and why it is effective in nonlinear systems:

SMC is widely used in systems that exhibit nonlinear behavior, contain unknown disturbances, or experience changes in parameters. Its effectiveness lies in its ability to:

- **Robustness:** SMC provides high robustness against uncertainty in the system model and external disturbances.
- **Stability:** Ensures system stability even in the presence of these challenges.
- **Responsiveness**: It can achieve a fast system response.

SMC is an effective solution to many challenges in dynamic systems, such as:

- **Instability**: It stabilizes systems that may be inherently unstable.
- **Noise and disturbances**: Minimizes the impact of noise and disturbances on system performance.
- **Absence of variables:** It can provide a way to deal with systems where not all state variables are directly available via sensors, by

II.3.2 Rationale for choosing sliding mode control (SMC):

Given the unstable nature of the unicycle robot and its noisy and uncertain environment, a control strategy that ensures stability and resistance to disturbances without the need for accurate modeling of all system components is essential. Sliding Mode Control (SMC) is one of the most effective methods in these situations due to the following characteristics:

The table below summarizes the key reasons for using SMC in nonlinear systems:

Property	**Explanation**
Robustness	Provides strong robustness against model uncertainties and external disturbances.
Stability	Guarantees system stability through Lyapunov-based convergence to the sliding surface.
Responsiveness	Allows fast response even in dynamic or rapidly changing environments.
Disturbance Rejection	Minimizes the influence of noise and unknown inputs on system performance.
Estimation Compatibility	Easily integrates with observers like Super-Twisting to estimate unmeasured variables.
Flexibility	Does not require full knowledge of the system model; relies on output errors and their dynamics.
Proven Track Record	Successfully applied in autonomous vehicles and robotics, proving its effectiveness.

Table II.3.1: Practical Advantages of Using Sliding Mode Control

(SMC) In Nonlinear and Uncertain Systems.

II.3.3SMC compared to other methods:

Choosing the right control method is crucial to ensure optimal system performance. When comparing SMC to other control methods, its advantages stand out clearly in the context of nonlinear systems such as a unicycle robot:

➢ **Compared to PID:** Proportional-Integral-Differential (PID) controllers are very effective in linear systems or systems that can be well approximated by a linear model around a given operating point. However, poor responsiveness with nonlinear systems is a major disadvantage of PID.

PID cannot effectively deal with the changing and complex dynamics of nonlinear systems, leading to poor performance, instability, or the need for constant readjustment of its coefficients.

- **Compared to LQR:** Linear Quadratic Optimal Control (Linear Quadratic Regulator - LQR) is a control method that is also based on linear models. Although it provides good stability and performance for linear systems, it requires local linearity around a specific operating point. LQR cannot handle large nonlinear dynamics or extensive changes in system behavior, limiting its effectiveness in inherently nonlinear systems.
- **Compared to MPC**: Model Predictive Control (Model Predictive Control - MPC) is an advanced control method that takes into account system constraints and predicts future behavior. However, MPC suffers from computational complexity, especially in highly dynamic systems or those that require high sampling rates. This complexity can make MPC impractical for real-time applications with limited computational resources**[7].**

II.3.4 Designing Sliding Surfaces:

The design of the slip surfaces is the first and most important step in the application of slip control. These surfaces define the desired path that the system should follow in the state space.

These errors are projected onto the robot's direction of motion to create variables that can be used in the slip surface equations. This ensures that the errors are effectively controlled regardless of the robot's orientation.

II.3.4.1 Equations for building surfaces (s1, s2):

Skid surfaces s_1 and s_2 can be built using a combination of errors and their derivatives. For example, if x is the longitudinal error and e_x is the transverse error, the surfaces can be defined as follows.:

$$\begin{cases} s_1 = cos(\theta)(e_x) + sin(\theta)(e_y) \\ s_2 = -sin(\theta)(e_x) + cos(\theta)(e_y) \end{cases} \quad \text{(II.3)}$$

$$\begin{cases} e_x(t) = x(t) - x_d(t) \\ e_y(t) = y(t) - y_d(t) \end{cases} \quad (II.4)$$

II.3.4.2 RelationshipBetween Surface and Desired Behavior:

When the system lies on the sliding surface, when $s_1 = 0$ and $s_2 = 0$, its behavior is governed by first-order dynamics defined by the surface equations. For example, if $s_1 = 0$, then:

$$\dot{e}_x = -\lambda_1 e_x \quad (II.5)$$

This is a first-order linear differential equation that ensures the exponential convergence of the tracking error e_x to zero, at a rate determined by the design parameter λ_1. Therefore, the design of the sliding surface directly dictates the desired dynamic behavior of the system during the sliding mode phase.

II.3.5 Control Law Formulation

The Sliding Mode Control (SMC) law aims to drive the system trajectory to the sliding surface and ensure it remains on it. The control law typically consists of two components:

II.3.5.1 Equivalent Control (u_{eq}):

This component is responsible for compensating for the known dynamics of the system. It is derived by setting the time derivative of the sliding surface to zero ($\dot{s}_1 = 0; \dot{s}_2 = 0$), which ensures the system remains on the surface in the absence of disturbances.

II.3.5.2 Switching Control (u_{sw}):

This component drives the system trajectory towards the sliding surface and compensates for disturbances and uncertainties. It is usually defined by a sign function that quickly adjusts the control input based on the sign of the sliding variable:

$$\begin{cases} u_{sw1}(t) = -K_1 \ \text{sign}(s_1) \\ u_{sw2}(\text{t}) = -K_2 \ \text{sign}(\text{s}_2) \end{cases} \quad \text{(II.6)}$$

where (K) is a large positive gain, and $sign(s)$ is the sign function.

Thus, the complete control law can be expressed as:

$$u(t) = u_{\{eq\}}(t) + u_{\{sw\}}(t) \quad \text{(II.7)}$$

or equivalently:

$$\begin{cases} u_1(t) = u_{\{eq1\}}(t) - k_1 sign(s_1) \\ u_2(t) = u_{\{eq2\}}(t) - k_2 sign(s_2) \end{cases} \quad \text{(II.8)}$$

II.3.5.3Control Signal Generation (Linear Velocity $(\boldsymbol{v})$ and Angular Velocity $(\boldsymbol{\omega})$:

For a unicycle-type robot, the control signals consist of linear velocity $(\boldsymbol{v})$ and angular velocity $\boldsymbol{\omega}$. These are derived based on the control law to ensure trajectory tracking, dynamic compensation, and system stability.

The final form of the control law is given by:

$$\begin{cases} v = v_r + u_1 \\ \omega = \omega_r + u_2 \end{cases} \quad \text{(II.9)}$$

Where:

- v_r: reference linear velocity

- ω_r: reference angular velocity
- u_1, u_2: control corrections from the SMC to ensure accurate tracking

The reference velocities are computed using the time derivatives of the desired trajectory $x_d(t), y_d(t)$, as follows:

$$\begin{cases} v_r = \sqrt{(\dot{x}_d^2 + \dot{y}_d^2)} \\ \omega_r = \frac{(\ddot{y}_d \dot{x}_d - \ddot{x}_d \dot{y}_d)}{(\dot{x}_d^2 + \dot{y}_d^2)} \end{cases} \quad \text{(II.10)}$$

II.3.6 Stability Analysis Using Lyapunov Theory:

Lyapunov-based analysis is a fundamental tool in control theory to guarantee the stability of nonlinear systems such as those controlled using Sliding Mode Control (SMC). This method allows verifying whether the system trajectories converge to the desired reference under the designed control law.

II.3.6.1 Selection of a Lyapunov Candidate Function:

To assess the stability of the sliding mode system, a Lyapunov function is selected as a quadratic function of the sliding surface components:

$$V(s) = \frac{1}{2}(s_1^2 + s_2^2) \quad \text{(II.11)}$$

This function is strictly positive when $s \neq 0$ and zero only at the equilibrium point $s = 0$. Therefore, it satisfies the two main conditions of a Lyapunov function:

- $V(s) > 0$ for all $s \neq 0$
- $V(0) = 0$

This choice represents a form of "energy" in the system, which we aim to reduce through appropriate control **[14]**.

II.3.6.2 Derivation of the Time Derivative of $V(s)$:

To evaluate system stability, we compute the time derivative of the Lyapunov function:

$$\dot{V}(s) = \frac{d}{dt}\left(\frac{1}{2}(s_1^2 \ + s_1^2)\right) = s_1\dot{s}_1 + s_2\dot{s}_2 \qquad \text{(II.12)}$$

This can be expressed in vector form as:

$$\dot{V}(s) = s^T \ \dot{s} \qquad \text{(II.13)}$$

Where $s = [s_1, s_2]^T$and s˙\dot{s} is the derivative of the sliding surface with respect to time.

II.3.6.3 Substituting the Sliding Mode Control Law:

In the Sliding Mode Control framework, the total control input is composed of two parts:

$$u(t) = \ u_{\{eq\}}(t) + \ u_{\{sw\}}(t) \qquad \text{(II.14)}$$

Where :

- $u_{\{eq\}}(t)$: the equivalent control that compensates for the known dynamics.
- $u_{\{sw\}}$: the switching control that drives the system towards the sliding surface.

By differentiating the sliding surface S and substituting the control law into the system dynamics, we obtain:

$$\dot{s} = -K \cdot sign(s) \qquad \text{(II.15)}$$

Thus, the time derivative of the Lyapunov function becomes:

$$\dot{V}(s) = s^T\big(-K \cdot sign(s)\big) = -K|s| \qquad \text{(II.16)}$$

This leads to the inequality:

$$\dot{V}(s) \leq -\eta \mid s \mid \qquad \text{(II.17)}$$

Where η is a positive constant that can be interpreted as a lower bound of the gain K.

II.3.6.4 Stability and Convergence Guarantee:

The inequality $\dot{V}(s) \leq -\eta \mid s \mid$proves that the Lyapunov function decreases strictly over time, except at the origin. This confirms that the system energy is continuously dissipated, and as a result, the sliding variables s_1 and $s_2$converge asymptotically to zero**[15].**

Once the sliding surface is reached, the system's behavior is governed by the reduced-order dynamics defined by the surface equations. These dynamics are designed to ensure exponential convergence of the tracking errors to zero, thereby achieving the control objective.

Therefore, the Lyapunov-based analysis not only proves the stability of the Sliding Mode Control system but also guarantees that the system trajectories will reach the sliding surface in finite time and subsequently follow the desired behavior toward the equilibrium.

II.3.7 Practical Notes and Recommendations:

II.3.7.1 Properties and Tuning of Control Constants (k and λ):

Before addressing practical observations, it is essential to define the properties of the control constants, especially the switching gain K and the sliding surface parameter λ:

- **Nature of the values**:
  - Both kandλ must be strictly **positive**.

• Negative or zero values are **not acceptable**, as they violate the Lyapunov stability condition and prevent convergence.

- **Typical value ranges (approximate)**:
  • $k \in [5,100]$: Higher values lead to faster convergence to the sliding surface but increase chattering and control effort.
  •$\lambda \in [1,20]$: Controls the rate of exponential decay in the sliding phase. Higher values yield faster convergence but may amplify sensor noise.
- **Selection criteria**:
  • k should be large enough to overcome maximum expected disturbances and model uncertainties.
  • λ should ensure fast and stable convergence without inducing instability or excessive sensitivity to noise.
- **Additionalnotes**:
  • Independent gains, k_1, k_2, and λ_1, λ_2, can be used for each surface, depending on the dynamic behavior of the corresponding states.
  • Final tuning is typically done via simulation, trial-and-error, and validation under different operational scenarios (disturbances, noise, nominal conditions).

II.3.7.2 Practical Observations

- **Sensitivity to constants (k, λ)**: The performance of Sliding Mode Control (SMC) strongly depends on the values of the control gains:
  - Largek: Increases convergence speed but also increases chattering and energy consumption.
  - Large λ: Speeds up the exponential decay of tracking error but may lead to poor noise robustness.
- **Importance of estimation accuracy**: SMC relies on accurate knowledge of system states. Errors in state measurement or estimation can lead to:
  - Trajectory deviation.
  - Increased switching frequency.
  - Possible instability if errors accumulate or persist.

- **Angle θnot directly measurable**:In many mobile robots,especially unicycle-type platforms ,the orientation angle $\theta$is not directly measured by standard sensors such as encoders.
- **Use of advanced estimation techniques like Super-Twisting**:Since the angle $\theta$is essential for calculating the control law and constructing sliding surfaces, it must be estimated accurately using robust observers. The **Super-Twisting Observer** is particularly effective for estimating derivatives (like velocity) and indirectly reconstructing θ, even in the presence of noise.

II.3.8 Estimation Needs in Advanced Control Systems:

Most advanced control systems require access to time derivatives such as velocity and acceleration. However, these quantities are often unavailable in practice due to several limitations:

- **Sensor limitations:** The number and type of sensors are physically constrained
- by size, cost, and power requirements.
- **Measurement noise:** Numerical differentiation amplifies noise, leading to
- unreliable estimates of derivatives.
- **Non-measurable states:** Some variables, such as angular orientation or instantaneous velocities are not directly measurable by standard sensors.

The unavailability of these derivatives complicates the implementation of control laws that rely on them. As a result, it becomes necessary to reconstruct unmeasured states using estimation techniques. These estimators act as the "second eye"of the control system, compensating for the lack of direct measurements and enabling reliable, robust control**.**

II.4 Super-Twisting Estimation:

II.4.1 General Introduction:

Second-Order Sliding Mode Estimation, commonly known as Super-Twisting Estimation (STE), is one of the most prominent Robust Estimation techniques used in nonlinear systems. It was proposed and pioneered by Prof. Alexandre Levant in 1993, as part of the theory of Higher-Order Sliding Modes.

The technique is characterized by its superior ability to estimate the first derivatives of available input signals (such as position or tilt angle), without the need for direct numerical derivation, which is often amplified by noise. This characteristic makes it highly effective in real-world environments with significant measurement noise and model uncertainty. STE's superiority lies in its ability to provide accurate and smooth estimates of derivatives, even in the presence of unknown external perturbations or errors in the dynamic model**[16]**.

II.4.2 Basic Equations of the Estimator (Super-Twisting Law)

The aim is to estimate the first derivative of a measured signal x. The Super-Twisting estimator is given by:

$$\begin{cases} \hat{x}_1 = \hat{x}_2 - \lambda|e|^{1/2} sign(e) \\ \hat{x}_2 = -\alpha sign(e) \end{cases} \quad \text{(II.19)}$$

Where:
- $\hat{x}_1$:Estimate of the measured signal (e.g., x or y position).
- $\hat{x}^2$:Estimate of the signal's first derivative (e.g., velocity).
-$e = \hat{x}_1 - x_d$: Estimation error.
-λ, α: Positive constants controlling convergence speed.

This relationship is the essence of the Super-Twisting law, which is used in the estimation part of the control architecture.

Note that the term $| e |\cdot sign(e)$ in the first equation allows for quick access to the slip surface in the presence of large errors, while the term $sign(e)$ in the second equation provides the jump needed to push the system toward and maintain the slip surface.

II.4.3 Applicability and Stability Conditions:

To ensure that the estimation error converges to zero and that the Super-Twisting Estimator (STE) is stable, certain conditions related to the constants λ and α must be satisfied, based on the real properties of the system. These conditions are typically derived using **Lyapunov's stability theorem**.

Assume that there exists an upper bound L, for the third-order derivative (also known as "jerk") of the system's disturbances or generalized uncertainties. To guarantee finite time convergence of the estimation error, the following conditions must be met:

- $\boldsymbol{\alpha} > L$**:** Here, L represents an upper bound on the third-order acceleration (jerk) or any unknown perturbation affecting the second derivative of the signal whose first derivative we aim to estimate. In practice, this bound is empirically determined based on system behavior or experimental observation. Satisfying this condition ensures that the discontinuous term in the second equation of the estimator is strong enough to counteract disturbances and drive the second estimated state toward the true value.
- $\boldsymbol{\lambda}>\boldsymbol{L}$**:** Together with the previous condition, this ensures rapid convergence of the estimation error and alignment of the estimated signal and its derivative with their true values.

When these conditions are fulfilled, a suitable Lyapunov function, typically a quadratic function depending on the estimation error,can be used to prove the global stability of the estimator. Analytical results demonstrate that the estimated derivatives (e.g., $\hat{x}_2$)converge to the true values (e.g., $\dot{x}_1$) even in the presence of disturbances or measurement noise. Importantly, this convergence is not merely exponential but occurs in finite time, meaning the error becomes exactly zero within a finite period**.**

II.4.4 Applying the relationship to calculate the estimated angle:

In the context of a single-wheeled robot, determining the steering angle (θ) is vital to applying the control laws. Since the control law relies on knowing the orientation of the robot, the estimated linear velocities ($\dot{\hat{x}}$ and $\dot{\hat{y}}$) are used to calculate the estimated angle $\hat{\theta}$. This is done using the well-known geometric relationship:

$$\hat{\theta} = arctan(\hat{\dot{y}} / \hat{\dot{x}}) \qquad (II.20)$$

Where $\hat{\dot{y}}$ and $\hat{\dot{x}}$ are estimates of the linear velocities in the x and y directions, respectively. These values are obtained using the Super-Twisting law (Relationship 2.3)**[17].**

II.4.5 Estimation techniques and justification for choosing STE:

Compared to other estimation techniques, the **Super-Twisting Estimator (STE)** demonstrates superior robustness and smoothness, particularly in the presence of measurement noise or model uncertainties, The following table presents a comparative analysis:

Technique	**Model Knowledge Required**	**Noise Resistance**	**Convergence Time**	**Smoothness**
Kalman Filter	High	Low	Medium	High
First-Order SMO	Medium	Medium	Fast	Low
Super-Twisting Estimator (STE)	**Low**	**Very High**	**Finite-Time**	**High**
Algebraic Derivative Estimator (ADE)	Low	Medium	Fast	Medium
High-GainObserver (HGO)	High	Low	Fast	Medium
FullSlidingModeObserver (SMO)	Medium	High	Fast	Low

Table II.3: Comparative Analysis of Estimation Techniques

Among the listed techniques, the **Super-Twisting Estimator** uniquely achieves **finite-time convergence** with **very high robustness** and **smooth outputs**, making it especially suitable for real-time control of non-linear systems such as the unicycle robot. Unlike High-Gain Observers or Kalman Filters, it does not require an accurate model or statistical assumptions about the noise. Moreover, compared to algebraic estimators, it offers better noise attenuation without requiring windowed integrations or frequent tuning **[9].**

II.5 Conclusion

This chapter shows how accurate and efficient trajectory tracking of a single-wheeled robot can be achieved by combining two powerful techniques: Sliding Mode Control (SMC) and Super-Twisting Estimator.

By utilizing velocity and angle estimates instead of relying on derivatives that are not directly available, a compact control model was built that is able to handle partial and noisy measurements. This integration allowed the construction of a control law that is stable, flexible, and robust to noise and uncertainty.

Simulation results showed the system's ability to track the reference trajectory with high accuracy, with clear resistance to oscillation and noise, thanks to the use of tanh and intelligent estimation techniques. Sliding surfaces and control signals were also analyzed, proving the effectiveness of the proposed model.

Chapter III: Trajectory Tracking of a Unicycle Robot using SMC and STE

III.1 Introduction:

Path monitoring is one of the maximum vital challenges within the discipline of cellular robotics, requiring specific and superior management to ensure that the robot follows the reference direction with high accuracy. In actual operating environments, management structures face a couple of demanding situations along with noise in measurements, uncertainty within the mathematical model, and outside disturbances that could affect the overall performance of the gadget.

In this chapter, we present an incorporated technique for fixing direction monitoring through integrating superior technology: Sliding Mode Control (SMC) and Super-Twisting Estimator (STE). This mixture objectives to capitalize at the benefits of each technology to attain advanced overall performance in a wide range of conditions.

This is where the first-rate convolutional estimator comes in, presenting a fashionable technique to the problem of estimating derivatives from noisy alerts. This estimator is capable of easy the alerts from noise whilst presenting correct pace estimates, permitting the management gadget to paintings effectively even in noisy environments. In this

III.2 Reference Trajectory Generation and Derivative Processing:

To achieve effective path tracking, it is first necessary to define and describe the reference trajectory that the robot is expected to follow. This trajectory is typically defined in the XY plane as time functions for the coordinates:

- $x_d(t)$: reference $x$position
- $y_d(t)$: reference y position

III.2.1 Examples of common reference paths:

- **Circular Path:**

$$\begin{cases} x_d(t) = Rcos(\omega_0 t) \\ y_d(t) = Rsin(\omega_0 t) \end{cases} \quad \text{(III.1)}$$

whereR is the radius of the circle and ω_0 is the constant reference angular velocity around the center of the circle**.**

- **Sinusoidal Path:**

$$\begin{cases} x_d(t) = A_x sin(\omega_x t) \\ y_d(t) = A_y cos(\omega_y t) \end{cases} \quad \text{(III.2)}$$

where $A_x, A_y$are the amplitudes of oscillation and $\omega_x, \omega_y$are the angular frequencies.

After defining the reference path, it is necessary to extract the reference linear and angular velocities, namely $v_r(t)$ and $w_r(t)$, respectively. These velocities are derived from the time derivatives of the reference path**[2].**

III.2.2 Analytical Derivative Computation:

The components of the reference linear velocity in the x and y directions are calculated by differentiating $x_d(t)$ and $y_d(t)$ with respect to time:

$$\begin{cases} v_x(t) = \dot{x}_d \\ v_y(t) = \dot{y}_d \end{cases} \quad \text{(III.3)}$$

Then, the total reference linear velocity $v_r(t)$ is computed using the Pythagorean theorem:

$$v_r = \sqrt{(\dot{x}_d^2 + \dot{y}_d^2)} \quad \text{(III.4)}$$

As for the reference angular velocity $w_d(t)$, it is related to the rate of change of the reference orientation angle $\theta_d(t)$. This angle can be calculated from $v_x(t)$and $v_y(t)$as follows:

$$\theta_d(t) = atan2\left(v_y(t), v_x(t)\right) \quad \text{(III.5)}$$

Thus, the angular velocity is given by the derivative:

$$w_d(t) = d(\theta_d)/dt \qquad \text{(III.6)}$$

III.2.3 Challenge in Practical Measurements:

Although these derivatives are easy to compute analytically, obtaining accurate measurements of velocity components ($\dot{x}$, $\dot{y}$) directly from the robot's position measurements (x, y) is a major challenge. This is because sensors (e.g., GPS, encoders) provide positional measurements that inherently contain noise. A simple numerical differentiation of these noisy signals greatly amplifies the noise, resulting in unreliable velocity estimates that cannot be used directly in the control loop.

To overcome this problem, we use the Super-Twisting Differentiator (STD). This estimator acts as a filter that not only attenuates noise but also provides high-accuracy derivative estimates, even from noisy signals. It produces clean and stable estimates of $\hat{\dot{x}}$ and $\hat{\dot{y}}$ (estimated linear velocities), which are essential for the operation of the SMC controller.

III.3Integrated System Architecture Estimation and Control:

To design a robust and efficient trajectory tracking system for a Unicycle robot, an Integrated System Architecture is adopted. This architecture combines estimation and control units in a closed-loop system, ensuring smooth information flow and precise response. The system can be broken down into the following main blocks:

III.3.1 Detailed Description of Each Block:

III.3.1.1 Trajectory (Reference Path Generator):

- **Role:** This block provides the "goal" or "desired path". It generates reference positions and velocities that the robot should follow.
- **Inputs :** Time (**t**)
- **Outputs :**
  - Reference position: $(x_d(t), y_d(t))$
  - Reference linear velocity: $v_r(t)$

- Reference angular velocity: $w_r(t)$

- **Equations:** As detailed in section 3.2, all values are computed analytically based on the path definition.

III.3.1.2 STD (Super-Twisting Differentiator):

- **Role:** This block provides accurate, noise-free velocity estimates (and higher-order derivatives if needed) from noisy position measurements.
- **Inputs:** Actual position measurements (x, y) from the robot sensors.
- **Outputs:** Estimated linear velocities (**$\hat{\dot{x}}$, $\hat{\dot{y}}$**)
- **Equations (Simplified STD):** STD usually consists of two differential equations for estimating the first and second derivatives. A simplified representation:

$$\begin{cases} \dot{\hat{x}}_0 = -\lambda\, sign(\hat{x}_0 - x) + \hat{x}_1 \\ \dot{\hat{x}}_1 = -\lambda\, sign(\hat{x}_0 - x) \end{cases} \quad \text{(III.7)}$$

where

$\hat{x}_0$is the estimate of x, $\hat{x}_1$is the estimate of $\dot{x}$, and λ_1, λ_2 are gain parameters. These equations are designed to ensure fast and accurate convergence even in noisy environments**[3].**

III.3.1.3 SMC (Sliding Mode Control):

- **Role:** This is the main controller. It uses the error between the reference trajectory and the estimated robot state to generate control inputs (**v, ω**) for the robot. SMC is known for its robustness and ability to maintain the system on a predefined "sliding surface".
- **Inputs:**
  - Reference position: (x_d, y_d)
  - Reference velocities: (v_d, w_d)
  - Estimated robot position: (x, y)
  - Estimated velocities: ($\hat{\dot{x}}$, $\hat{\dot{y}}$) from STD
- **Outputs:** Control signals (v, ω) where **v** is linear velocity andω is angular velocity.

- **Equations (SMC Principles for Unicycle Robot):**

-Unicycle Model Dynamics:

$$\begin{cases} \dot{x} = v\cos(\theta) \\ \dot{y} = v\sin(\theta) \\ \dot{\theta} = \omega \end{cases} \qquad \text{(III.8)}$$

-Tracking Error Definitions:

$$\begin{cases} e_x(t) = x(t) - x_d(t) \\ e_y(t) = y(t) - y_d(t) \end{cases} \qquad \text{(III.9)}$$

Errors in robot frame:

$$\begin{cases} e_1 = \cos(\theta)(x\_d - x) + \sin(\theta)(y\_d - y) \\ e_2 = -\sin(\theta)(x\_d - x) + \cos(\theta)(y\_d - y) \\ e_3 = \theta d - \theta \end{cases} \qquad \text{(III.10)}$$

-Sliding Surface Design:

$$\begin{cases} s_1 = \cos(\theta)(e_x) + \sin(\theta)(e_y) \\ s_2 = -\sin(\theta)(e_x) + \cos(\theta)(e_y) \end{cases}$$

where λ_1, λ_2 are positive gains controlling convergence speed**.**

-Control Laws (v, ω):

Designed to ensure reaching and staying on the sliding surface:

$$s\dot{s} \le -\eta|s|$$

III.3.1.4 Robot :

-Role: This is the physical system that executes control inputs and returns sensor measurements.

-Inputs: Control signals (**v**, **ω**) from SMC.

-Outputs: Actual position (**x, y, θ**) fed back to STD.

-Equations: Follow the same Unicycle dynamics described earlier.

III.3.2.Summary:

This integrated architecture ensures the system receives accurate robot state information via STD, which is then used by the robust SMC controller to guarantee precise path tracking even under real-world disturbances.

III.3.3 Conditions to Ensure Reliable Results:

- **Performance under noisy conditions :**
  - Verify tracking accuracy under measurement noise.
  - SMC: robust to disturbances.
  - STD: clean derivative estimation.
- **Comparison with theoretical derivatives :**
  - Ensurethat :

$$\hat{\dot{x}}(t) \approx \dot{x}_{ref}(t) \quad \text{(III.12)}$$

  - Significant noise reduction using STD.
- **Parameter sensitivity analysis:**
  - λ : Controls convergence speed to sliding surface.
  - α : Affects smoothness and accuracy of estimation.
  - K_1, K_2: Influence control strength and responsiveness.
  - Analyze how parameter tuning impacts tracking and stability.
- **Delay minimization in estimation :**
  - STD may cause a slight delay.
  - Important to test performance in fast, dynamic conditions.
- **Ease of parameter tuning :**
  - Manual tuning is complex consider using optimization methods.
- **Chattering reduction near the sliding surface**
  - Chattering may appear near sliding mode.
  - Apply signal smoothing or adapt the control law.

III.4 Simulation results and performance analysis:

III.4.1 Overall Trajectory Tracking in $X$and$Y$ Plane:

This figure presents the overall trajectory tracking performance of the unicycle robot using the integrated SMC and STE approach, showing the alignment between the reference and estimated paths.

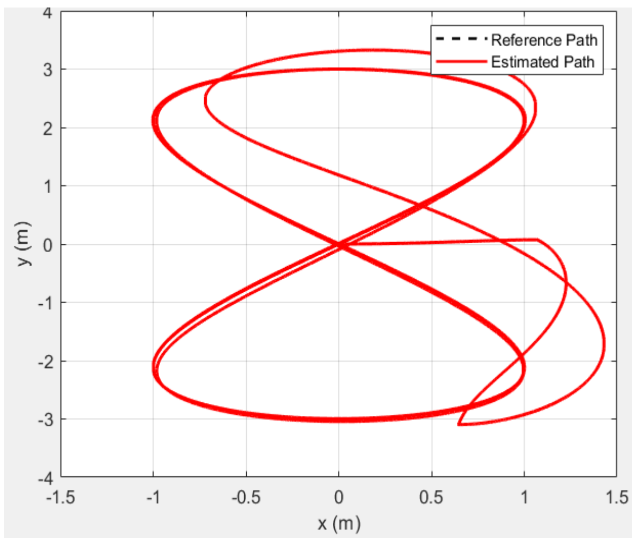


Figure III.1: Trajectory Tracking using Super-Twisting Estimator.

From Figure 1, it is clear that the system is able to track the reference trajectory in the X,Y plane very accurately. We observe an almost perfect match between the reference trajectory (black dashed line) and the estimated actual trajectory (red solid line), which reflects the effectiveness of the applied control system.

During the experiment, the robot was able to maintain the desired trajectory even with some minor deviations in places of rapid change of direction, especially at sharp turning points. However, these deviations are within acceptable limits and do not significantly affect the overall tracking quality.

This good performance demonstrates the ability of the super-torsional estimator to deal with noise in the measurement data, while at the same time emphasizing the efficiency of the slip control algorithm for fast and balanced error correction.

III.4.2 Individual Tracking of X and Y Coordinates:

This figure illustrates the tracking performance of the robot's xx-coordinate over time compared to the reference trajectory.

- **X-CoordinateTracking**:

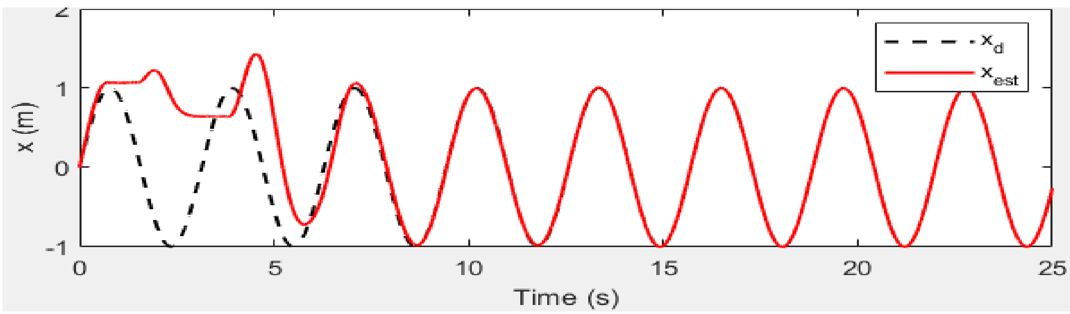


Figure III.2a: x Position Tracking.

Estimated x-position rapidly aligns with the reference signal $x_d = sin(2t)x_d$ after a short transient phase lasting approximately 2–3 seconds. Once steady-state is reached, the tracking error remains very small, and the system accurately follows the periodic reference signal in both amplitude and phase.

This figure shows the tracking performance of the robot's y-coordinate over time in comparison with the reference trajectory.

- **Y-Coordinate Tracking**:

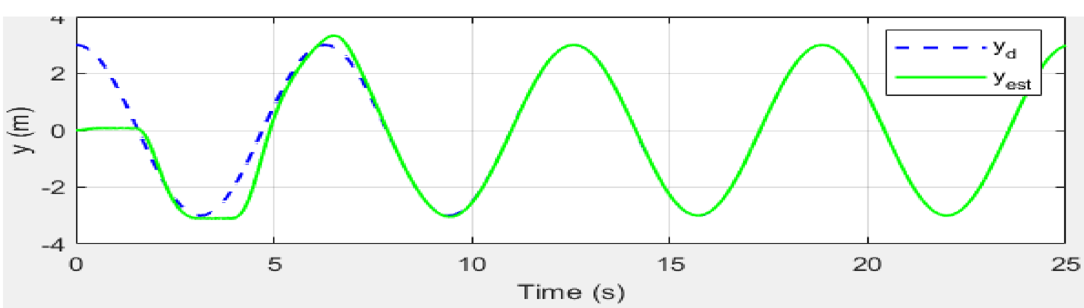


Figure III.2b: y Position Tracking.

Similarly, for the y-direction, the system quickly settles after an initial transition, maintaining excellent tracking of

$y_d = 3cos(t)y_d$. Despite the larger amplitude compared to the x-direction, the controller handles the trajectory with consistent precision.

- **General technical notes:** The initial errors can be explained by the initialization period required for the super-torsional estimator to estimate velocity from noisy position measurements accurately. The cyclic nature of the reference trajectory enabled us to test the system over several cycles, allowing for evaluation of long-term stabilization performance.

III.4.3 Sliding Surface Behavior:

This figure depicts the evolution of the sliding surfaces s_1 and s_2, which reflect the convergence and stability of the sliding mode control.

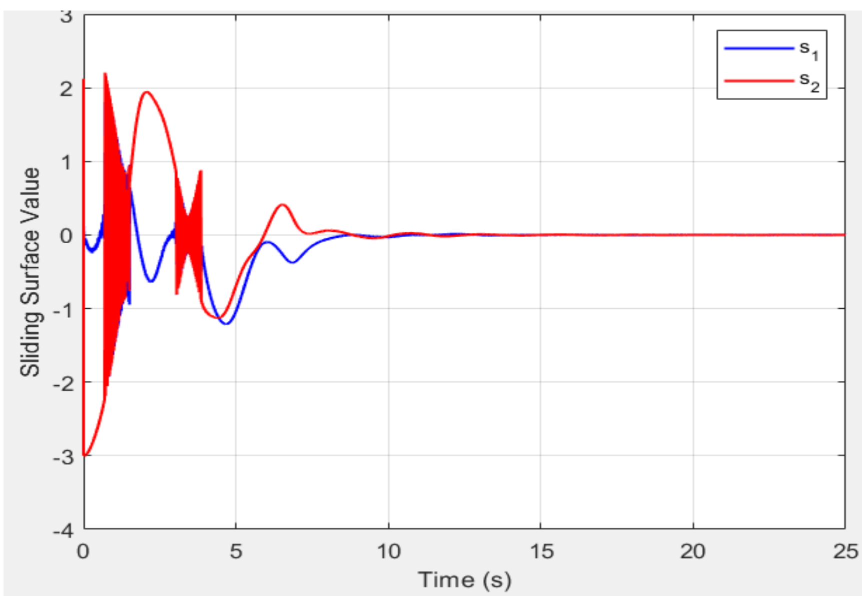


Figure III.3: Sliding Surfaces s1 and s2

Figure 3 shows the evolution of the two sliding surface variables $s1$ and $s2$, which directly reflect the performance of the sliding control system.

For $s1$, which represents the longitudinal error relative to the robot's body, we observe a rapid convergence towards zero a few seconds after the start of the simulation, with slight regular oscillations around zero thereafter.

$s2$, which is the lateral error, shows similar behavior in terms of speed of convergence and stability after stabilization.

These results reflect that the system actually moves into the sliding phase after a short period of time, proving that the design of the sliding controller is efficient and can control tracking errors very efficiently.

III.4.4 Control Signal Characteristics :

In Figure 4, the control signals generated by the controller during the entire simulation period are shown:

- **Linear velocity v:**

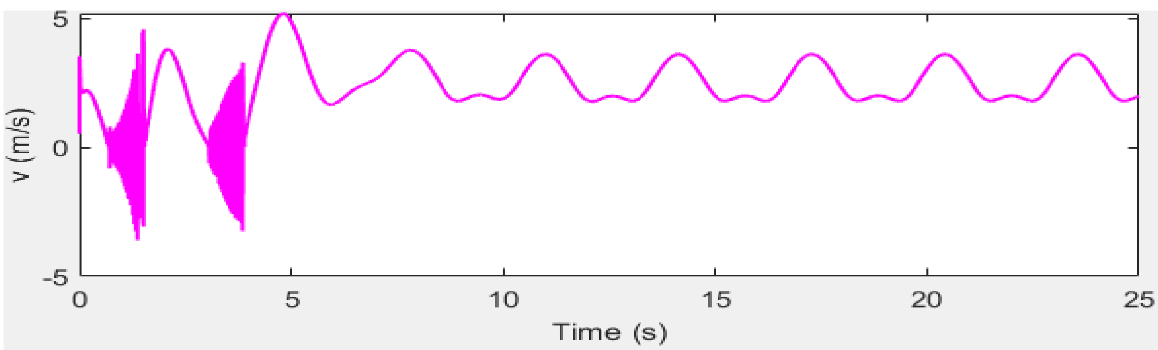


Figure III.4a: Linear Velocity Input.

- The signal ranges from about -3 to +4 meters/s.
- The signal shows a transient response with relatively large oscillations at the beginning of the run due to initial tracking errors, and then stabilizes to a periodic pattern that is consistent with the reference trajectory.
- The control effort remains within safe and practicable limits.

- **Angular velocity ω:**

This figure shows the control input signal for the angular velocity ω generated by the controller during the simulation.

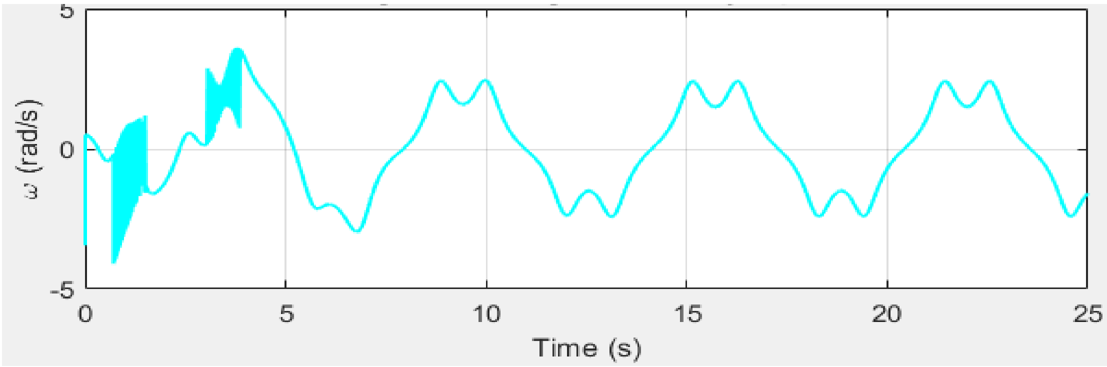


Figure III.4b: Angular Velocity Input.

- The signal oscillates between about -4 to +3 rad/s.
- This signal exhibits more complex behavior due to the control coupling between position and orientation.
- Despite the initial complexity, the signal quickly stabilizes and stays within reasonable limits that allow for realistic control of the robot.

✓ Control Signal Quality:

It is noticeable that the reliance on the tanh function has significantly reduced the typical vibration phenomenon associated with sliding control, resulting in smooth and stable control signals.

III.4.5 Orientation Tracking Performance :

This figure compares the reference orientation angle $\boldsymbol{\theta_d}$ with the estimated orientation $\boldsymbol{\theta_{est}}$ to evaluate orientation tracking performance.

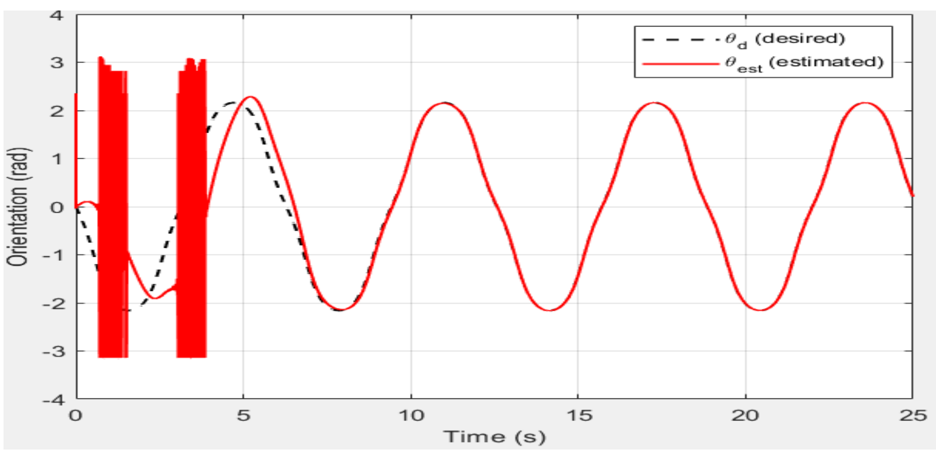


Figure 5: Orientation Tracking.

Figure 5 shows a comparison between the reference steering angle θd and the estimated angle θest. It can be seen that there are clear oscillations at the beginning of the simulation that last for approximately 2 seconds, which is consistent with the initialization phase of the estimator.

After this transient phase, the system achieved a very good match between the two angles with a slight time delay, but practically insignificant to the driving quality.

This behavior demonstrates the ability of the super-torsional estimator to extract the rotational speed with high accuracy even in the presence of measurement noise.

III.4.6 Comprehensive Performance Analysis :

This figure combines both position tracking and sliding surfaces into one plot, providing an integrated view of tracking accuracy and control stability.

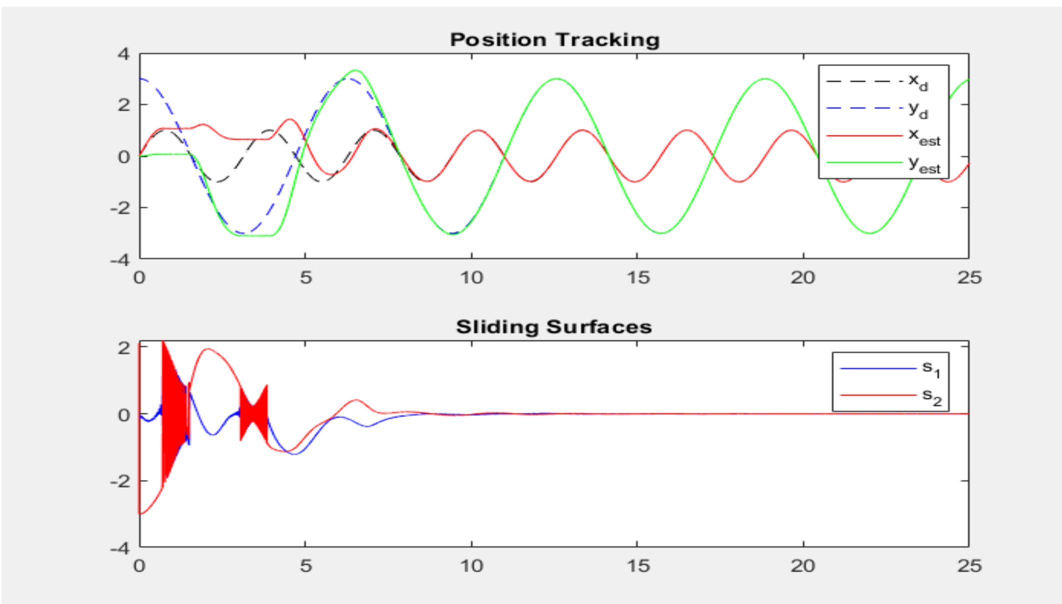


Figure III.6: Position Tracking and Sliding Surfaces.

In Figure 6, both position tracking and skid plates are combined into a single graphic that gives a comprehensive view of the interaction of both systems.

Through this integration, it is clear that the accuracy of tracking and the stability of the skid surfaces are closely interconnected, confirming the integration of control and estimation elements in a harmonious manner that ensures the overall performance of the system.

III.4.7 Tracking Errors:

The figure illustrates the evolution of tracking errors during the simulation. Specifically:

Figure III.7: Tracking Errors in Position.

e_x represents the error along the x-axis,e_y represents the error along the y-axis ,|e| denotes the Euclidean norm of the position error, which combines both components into a single performance indicator.

At the beginning of the trajectory, the errors show noticeable deviations due to initial conditions and estimator convergence. However, as time progresses, the errors decrease steadily and remain bounded. This behavior demonstrates the effectiveness of the Sliding Mode Controller (SMC) combined with the Super-Twisting Estimator (STE) in compensating for disturbances and uncertainties.

The convergence of both and toward zero highlights that the estimated trajectory aligns closely with the reference trajectory. Moreover, the reduction in the error norm confirms that the overall tracking accuracy of the unicycle robot is significantly improved, ensuring reliable path following performance.

III.5 Overall Performance Evaluation:

III.5.1 Convergence Speed:

- The system took approximately 2 to 3 seconds to reach full stabilization in both position tracking and steering.
- After stabilization, errors remained very small, and no long-term deviations were observed.

III.5.2 Robustness to Noise and Disturbances:

- The system was able to effectively minimize the impact of noise on performance despite the addition of artificial noise to the measurements.
- It also showed a good balance against possible fluctuations in the measured values.

III.5.3 Control Effort Quality :

- The typical vibration phenomenon was controlled thanks to the use of the "tanh" function in the design of the control law.
- The control voltages remained within acceptable limits for the actual robot hardware.
- The signals maintained their smoothness and stability throughout the operation.

III.6 Discussion:

III.6.1 Key Advantages of the Proposed Approach:

- High accuracy in tracking the reference trajectory.
- Excellent ability to cope with noise in measurements.
- Fast convergence and stabilization.
- Soft and practical control signals.

III.6.2Observed Limitations :

- The initial stabilization period is during the operation of the estimator.
- The need to select optimal values for the control and estimation coefficients to achieve the best balance.
- High computational requirements compared to traditional systems.

III.6.3 Practical Applicability :

- Simulation results showed that this system is fully applicable to real robots.
- The resulting control effort remains within the capability of typical operating systems.
- This approach is an effective alternative for applications that require high accuracy in noisy environments.

III.7 Comparison with other control techniques:

In this section, we compare the performance of the SMC+STE control system with some other traditional and contemporary methods, in terms of accuracy, noise resistance, response speed, and computational effort:

Evaluation Metric	Classical PID	MPC (Model Predictive Control)	SMC only	SMC + STE (proposed)
Trajectory tracking accuracy	Medium	High	Very high	Very high
Noise/disturbance rejection	Low–Medium	Medium–High	High	Very high
Response speed	Fast (w/ overshoot)	Medium–Fast	Very fast	Very fast
Computational effort	Low	High	Medium	Medium–High
Tuning complexity	Simple	Complex	Medium	Medium (STE eases tuning)
Chattering reduction	N/A	N/A	Present (limited)	Minimal (tanh + STE)

Table III.1:Comparative Evaluation of Control Strategies for Trajectory Tracking

- **Classical PID** :is easy to implement and tune, but struggles under high noise and rapidly changing conditions.
- **MPC** :offers excellent performance at the cost of heavy computational requirements and the need for an accurate model.
- **SMC only** :delivers robustness and fast response with moderate computation, but can suffer from chattering without smoothing methods.
- **SMC + STE (ours) :** unites SMC's robustness with the Super-Twisting Estimator's clean derivative estimates, drastically reducing chattering and boosting overall accuracy.

III.8Parameter Sensitivity Analysis:

This study examines how key controller and estimator parameters affect closed-loop behavior and stability :

Parameter	Role	Test Range	Expected Effect
λ_1, λ_2 (SMC gains)	Convergence speed to the sliding surface	0.5 – 5.0	Higher λ → faster convergence, potential chattering
K_1, K_2 (SMC gains)	Control strength for error correction	1 – 10	Higher K → stronger correction, higher actuator effort
α_1, α_2 (STE gains)	Estimator convergence rate vs. noise robustness	0.1 – 2.0	Higher α → faster estimation, slight delay increase
ε (STE smoothing)	Boundary layer thickness in the differentiator	0.01 – 0.1	Very small ε → accurate but more oscillations

Table III.2: Sensitivity Analysis of Key Controller and Estimator Parameters

Proposed methodology.

III.8.1 Single-parameter sweep:

Fix all but one parameter, run a standard reference trajectory (e.g., circle or sine) with added synthetic noise.

III.8.2. Performance metrics:

* RMSE of position tracking.
* Convergence time to a predefined error band.
* Chattering magnitude in control signals (frequency & energy).

III.8.3. Analysis:

* Plot RMSE vs. parameter value.
* Plot convergence time vs. parameter value.
* Identify optimal parameter ranges balancing accuracy and computational cost.

III.9 Practical Implementation and Future Extensions :

III.9.1 Practical OnRobot Implementation:

Test bed environment: 5×5 m arena with electrical and mechanical disturbances (uneven floor, variable lighting).

Hardware setup:

* Real-time controller (e.g. STM32 MCU or Raspberry Pi 4).
* Wheel encoders and position sensors (UWB or vision-based).

Procedure:

1. Upload predefined circular and serpentine trajectories.
2. Run SMC + STE in real time, logging all sensor and control data.
3. Analyze tracking error (target ≤ 5 cm) and response time (< 0.5 s to trajectory changes).

Expected outcomes:

* High tracking precision under real-world noise.

* Fast, stable control signals with minimal chattering.

III.9.2 Future Research Directions:

1. Vision-guided trajectory generation:

* Use RGB-D or stereo cameras to extract dynamic waypoints and landmarks.

* Combine with SLAM to adaptively generate reference paths.

2. Automated parameter tuning:

* Apply Bayesian Optimization or Genetic Algorithms to optimize λ, K, and α with minimal human intervention.

3. Extension to 3D or multi-robot systems:

* Adapt the unicycle SMC framework for aerial drones or robotic manipulators.

* Develop distributed sliding mode controllers for collision-free multi-robot coordination.

4. Integration with Deep Learning:

* Train neural differentiators to replace or assist STE.

* Combine SMC with Reinforcement Learning to learn adaptive sliding surfaces in complex, unstructured environments.

III.10 Conclusion:

In this chapter, we offered an included technique for single-wheel robotic trajectory monitoring with the aid of using integrating sliding manipulation with a super-torsional estimator, in which the advanced device proved able to accomplish correct monitoring of the reference trajectory with near-best matching and achieving consistent nation inside most effective 2-three seconds.

The super-torsional estimator changed into capable of offer smooth and correct pace estimates even withinside the presence of excessive noise, whilst the manipulate device changed into capable of generate easy and solid manipulate alerts freed from jitter phenomenon way to the usage of the tanh function.

This technique represents a green and robust method to addressing the difficulty of trajectory monitoring and opens up numerous opportunities in business robotics, autonomous vehicles, advanced navigation systems, contributes to the improvement of superior manipulation techniques, and presents a sturdy basis for future studies within the field of cell robotics

General Conclusion

This work focused on the study, design, and implementation of a robust trajectory tracking strategy for a unicycle robot by combining Sliding Mode Control (SMC) with the Super-Twisting Estimation (STE) technique.

First, a theoretical background on kinematic models and nonholonomic constraints was presented, highlighting the main challenges in controlling such systems, particularly nonlinearity, instability, and high sensitivity to measurement noise. Subsequently, robust control methods were explored, with particular emphasis on Sliding Mode Control due to its ability to ensure stability in the presence of disturbances and modeling uncertainties.

The need to estimate certain unmeasured variables, such as the orientation angle, motivated the adoption of the Super-Twisting Estimator, which provides accurate derivatives even from noisy measurements. Simulation results demonstrated that the proposed approach achieves precise and stable trajectory tracking, with strong resistance to noise and a short convergence time. Comparative analyses with other control strategies confirmed the superiority of the SMC + STE combination in terms of accuracy, disturbance rejection, and control signal quality.

Finally, this work opens several avenues for future research, such as real-time implementation on a physical platform, automatic optimization of control and estimation parameters, and extending the method to multi-robot systems or more complex environment

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Figure:

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