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Identification of hydraulic conductivities tensor for heterogeneous confined aquifers using swarm intelligence

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Dedication

I dedicate this modest work to:

To my parents, Mom & Dad that I can never thank enough for all the help they gave me;

To my sister and my brothers: Mohamed, Riad, Kamel and my only sister that I love the most Meriem, for being supporters and always by my side;

To my soul 'alma' for staying strong and always by my side;

To my cousin: Salima;

To my dear friends: kaouthar, Saliha, madjda and mebarka.

Thanks

Above all, I wish to thank God for giving me the strength and the ability to finish this work.

I would like to thank my Supervisors, ***Mr.Tadj walid*** *and* ***Mr. Mouattah kadddou****r for their patient guidance and their generous supports and encouragements.*

I thank from the bottom of my heart and with great love my parents who never stopped believing in me during all my years of study.

Thanks also to my sisters and brothers, and to the whole family that always has me Encouraged.

My thanks to myself and my soul 'alma'.

I thank the members of instructors ***Mr. Chettih Mohammed*** *and* ***Mr. Bouaach Mohammed.***

ملخص

ينتمي تحديد النفاذية الهيدروليكية الموزعة مكانيا إلى المسائل العكسية المعقدة والتي تكون طبيعة معادلاتها غير خطية بحيث لا يمكن حلها بفعالية باستخدام تقنيات التحسين التقليدية، لذلك فإن استخدام تقنيات التحسين العشوائية أكثر إثارة للاهتمام. في هذا السياق، تقترح هذه الدراسة ا نهج الربط بين خوارزمية التحسين ونماذج العناصر المنتهية. يكمن دور خوارزمية التحسين في تقدير النفاذية الهيدروليكية التي تقلص الفارق بين قيم الضغط المقاسة والمحسوبة باستخدام نموذج العناصر المنتهية. تم اختبار فعالية هذا الربط على ثلاث مسائل افتراضية في ظل شروط حدية معلومة ومجهولة. النتائج المتحصل عليها مرضية حتى في ظل وجود قياسات ضغط غير دقيقة

الكلمات المفتاحية: المسائل العكسية، خوارزمية بحث الغراب، الضبط التلقائي،مصفوفة التوصيلات الهيدروليكية.

Abstract

The identification of spatially distributed hydraulic conductivities belongs to the inverse problems which are complex, highly nonlinear and cannot be effectively solved using traditional optimization techniques. The use of stochastic optimization techniques is, therefore, more interesting. As such, the present study proposes a linked optimization-simulation approach for estimating hydraulic conductivities of confined heterogeneous aquifers. This linkage is based on combining a recent swarm optimization technique called crow search algorithm (CSA) with a finite element model. Here, the role of CSA is to estimate the optimal set of hydraulic conductivities that minimize an objective function that measures the discrepancy between the measured hydraulic heads and those computed by the finite element model. The effectiveness of our approach was tested on three synthetic aquifer problems under both known and unknown boundary conditions. The obtained results were found satisfactory even under noisy hydraulic heads measurement.

Key-words: Inverse-problems, Crow search algorithm, Automatic calibration, Tensor of hydraulic conductivities.

Résumé

L'identification des conductivités hydrauliques spatialement distribuées fait partie des problèmes inverses qui sont complexes, hautement non linéaires et ne peuvent pas être résolus efficacement en utilisant les techniques d'optimisation traditionnelles. L'utilisation des techniques d'optimisation stochastique est donc plus intéressante et efficace. De ce fait, la présente étude propose une approche couplée optimisation-simulation pour l'estimation des conductivités hydrauliques des nappes captives hétérogènes et anisotrope. Ce couplage est basé sur la combinaison d'une récente technique d'optimisation en essaim appelée ‘Crow search algorithm (CSA)’ avec un modèle aux éléments finis. Ici, le rôle de CSA est d'estimer l'ensemble optimal de conductivités hydrauliques qui minimise une fonction objective mesurant l'écart entre les charges hydrauliques mesurées et celles calculées par le modèle aux éléments finis. L'efficacité de cette technique a été testée sur trois cas des nappes synthétiques avec des conditions aux limites connues et inconnues. Les résultats obtenus ont été jugés satisfaisants même avec des mesures bruitées.

Mots-clés : Problèmes inverses, Crow search algorithm, Calage automatique, Tenseur des conductivités hydrauliques

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General introduction

Groundwater flows offer an important research area, as they involve the study of fluids in porous media. However, the heterogeneity of the medium and its complex geometry pose several problems that cannot be solved by simple analytical methods and require appropriate numerical methods.

Among the most widely used methods for solving groundwater flow equations, mathematical modelling.

Mathematical modelling has become a standard approach in the field of hydraulics. In order to carry out reliable simulations using mathematical models, two types of problems must first be solved: the direct problem, which constitutes the simulation phase, and its inverse problem called the calibration phase. The first predicts unknown states of the system by solving the equations governing the studied phenomenon, while the second determines the unknown physical parameters and/or other conditions of the system by fitting the calculated states to the observed states. Therefore, the parameters of the mathematical model must be identified by inversion and then injected into the mathematical model in order to perform the required simulation. An inverse problem is often presented as an optimisation problem, where the difference between the state variables calculated via the mathematical model and the observed state variables must be minimised. Provided that the treated problem is well posed from a mathematical point of view, the quality of the physical parameters of the mathematical model depends, then, only on the optimisation technique adopted. Several optimization algorithms exist, which can be classified into two categories: based on the solutions produced, namely the classical or deterministic algorithms and the non-classical or the stochastic algorithms. The stochastic algorithm can be further classified into the heuristic and metaheuristic techniques. Metaheuristic techniques are capable of using search experience intelligently to explore and exploit the search space in a randomized manner, and the solution methods are inexact and near optimal. Metaheuristics are uncountable, they are classified in four categories: physics-based, swarm intelligence, evolutionary algorithms and bio-inspired.

In this study, we tried to take advantage of the benefits offered by metaheuristic by applying it to a hydraulic problem, which is ‘The identification of hydraulic conductivities tensor for heterogeneous confined aquifers using swarm intelligence’.

Crow search algorithm was chosen for this optimisation problem. This algorithm belongs to the category of swarm intelligence algorithms and relies on the cleverness of crows and their strong memory.

General introduction

This study has three chapters:

The first chapter presents the soil characteristics and flow equations.

The second chapter is dedicated to optimisations problems and the metaheuristic used in this work.

The third chapter is devoted to the coupling of the metaheuristic used with finite element models, in order to identify the physical parameters of three heterogeneous confined aquifers.

Finally, we end this work with a conclusion, which recalls the objectives of this study and the results obtained.

Chapter one: Flow in porous media

1.1 Introduction

This chapter aims to introduce some essential notions that will be important to the remainder of this study.

Groundwater is an important source of drinking water. The term groundwater refers to water that exists beneath the surface of the earth, however for a hydrologist or geotechnical engineer, it implies the water that saturates the porous medium (Muhammed 2015)

Porous environments refer to materials for which the solid phase is strongly intertwined with the fluid phase. There are many natural materials on this category: soils, sedimentary layers and most rocks.

In general, porous media are defined by two criteria:

• The material must contain small empty spaces, called pores, demarcated through a solid matrix;

• The material must be permeable to a flow of fluid (gas or liquid).

The main characteristics of porous media are: porosity, which present the vacuum fractions and permeability (hydraulic conductivity), which indicates the ability of a porous environment to be crossed by a flow (Maxime 2003).

Aquifers and rock bodies are assumed to be porous media with void spaces and solid matrix in the groundwater flow field.

1.2 Classification of aquifers

Groundwater exists mainly in aquifers, which are capable of yielding great amount of water. (Vedat 1998).

The aquifer is a geological formation, bounded to the lower part by a waterproof base called substratum and the top can be either free or covered with another waterproof layer called a roof (Bailly 2009; Marsily1981).

Figure 1.1 presents the distribution of subsurface water.

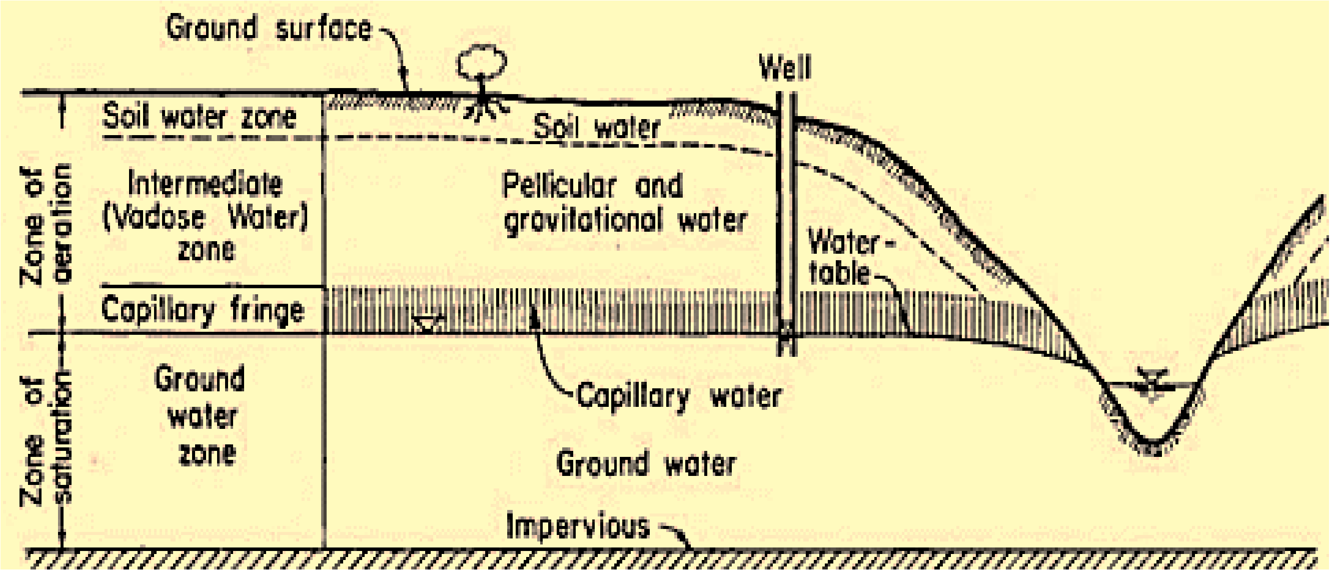


Figure 1.1: The distribution of subsurface water (Bear 1972)

In general, the aquifers can be classified into three main categories: confined, leaky and unconfined aquifers. These three types are schematically shown in Figure (1.2).

1.2.1 Confined aquifer

A confined aquifer is a fully saturated layer of soil with upper and lower boundaries that are impermeable. The pore pressure in such aquifer is quite above the atmospheric pressure. The imaginary surface of the confined aquifer is called piezometric surface (Muhammed 2015).

1.2.2 Unconfined and Perched Aquifers

An unconfined aquifer is also known as phreatic aquifer or water table aquifer. The piezometric surface in an unconfined aquifer is exposed to the atmospheric pressure. This surface can rise or stoop freely into permeable hydrogeological formation (Bailly 2009).

The perched aquifers are essentially unconfined aquifers that standoff on clay lenses of limited areal extent. The perched aquifers are located in the unsaturated zone between the ground surface and the main water table (Muhammed 2015).

1.2.3 Leaky aquifers

If an aquifer (confined or unconfined) loses or receives water through either or both of adjacent semipervious layers from above or below, it's a leaky aquifer.

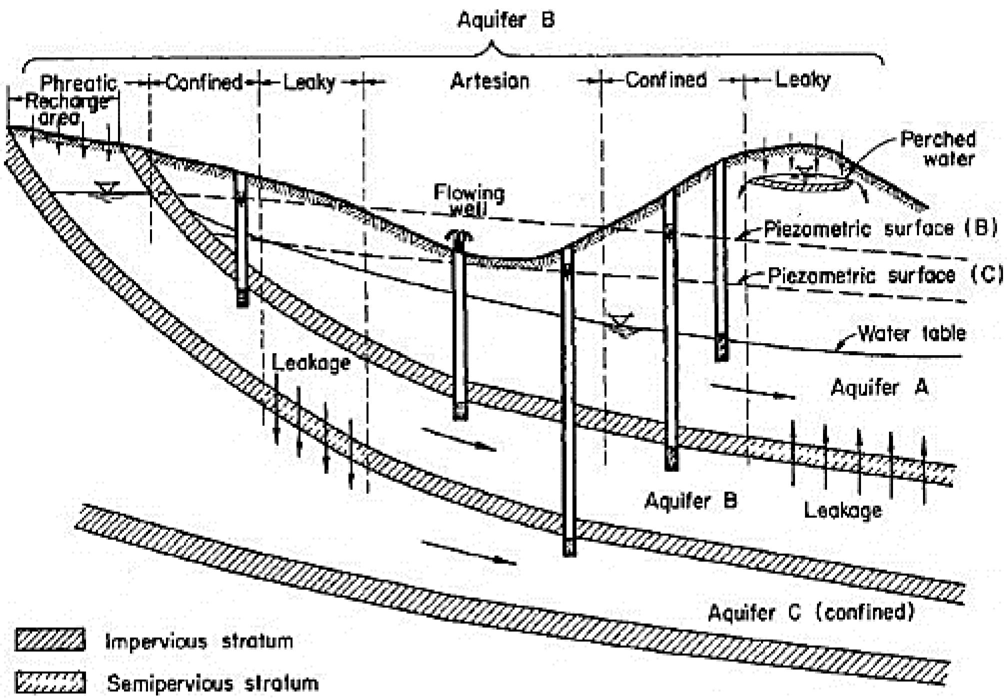


Figure 1.2: Types of aquifers (Bear 1979)

1.3 Aquifer characteristics

Hydraulic head, drawdown, porosity, hydraulic conductivity (permeability), transmissivity and storativity are the key hydrogeological quantities used to characterize aquifers. These quantities are used for almost every aquifer problem **(**Vedat 1998).

1.3.1 Hydraulic Head

It's also called piezometric head or simply head. Hydraulic head is the fluid pressure of formation water produced by the height above a given point. Geometrically, the hydraulic head at a point of an unconfined aquifer and a confined aquifer is shown in figures (1.3) and (1.4) **(**Vedat 1998).

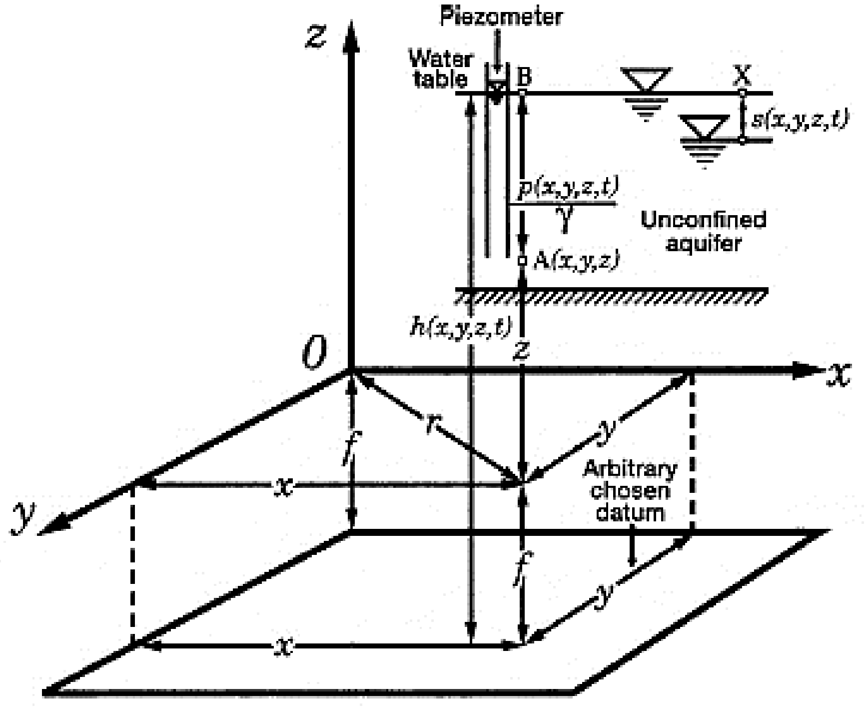


Figure 1.3: Hydraulic head in an unconfined aquifer (Vedat 1998)

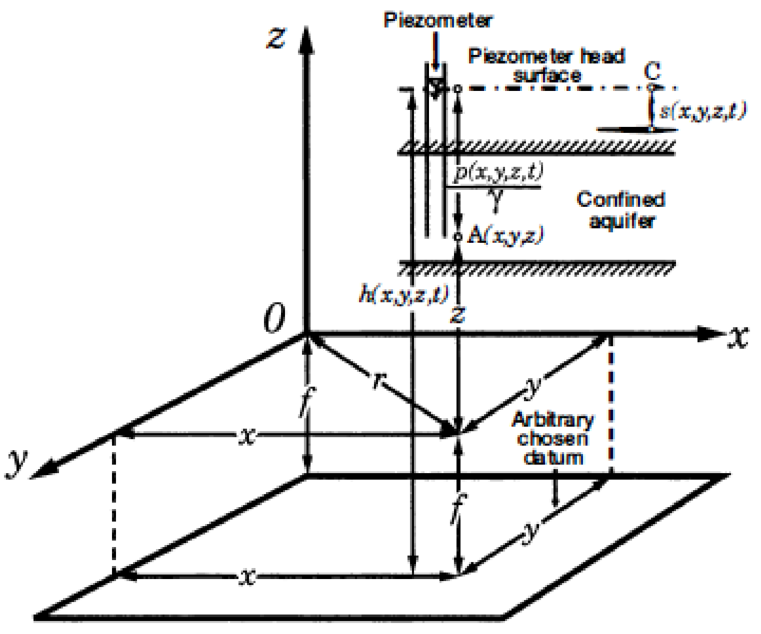


Figure 1.4: Hydraulic head in a confined aquifer (Vedat 1998)

In figure (1.3) (the unconfined case), we see that the water level in the piezometer has the same elevation as the water-table, whereas for a confined aquifer case the water level in a piezometer and the piezometric surface has the same elevation of the aquifer at the same point (see Figure 1.4) **(**Vedat 1998).

As shown in Figures 1.3 and 1,4 the hydraulic head, *h* (*x*, *y*, *z*, *t*) at point *A (x*, *y*, *z*), consists of the pressure head, *p* (*x*, *y*, *z*,*t*)/*γ*, and the elevation of the bottom of the piezometer, z, above an arbitrarily chosen datum elevation. The pressure head *p* (*x*, *y*, *z*, *t*)/*γ*, is the height of the column of water in the piezometer above its bottom or above point A (*x*, *y*, *z*). As a result, the hydraulic head is defined as (Hantush 1964).

$$h\,(x\,,y,z,t\,) = \frac{p(x,y,z,t)}{\gamma} + \; z \; + \; f \qquad (1.1)$$

which *x*, *y*, and *z* are the cartesian coordinates.

γ: is the unit weight of water,

f: is the elevation of the x-y plane above an arbitrarily chosen datum elevation,

t: is the elapsed time since a reference time.

The hydraulic head is generally expressed as "meters of water" (Vedat 1998).

1.3.2 Drawdown.

Drawdown is defined as the distance between the initial hydraulic head at a point in the aquifer and the lowered position of the hydraulic head for the same point. For an unconfined aquifer the drawdown is presented from the water level at point C on the water table dropped by a vertical distance *s* (*x*, *y*, *z*, *t*). (see Figure 1.3). For a confined aquifer case, the drawdown is defined with respect to the piezometric surface (see Figure 1.4). Drawdown is considered only when the flow is transient.

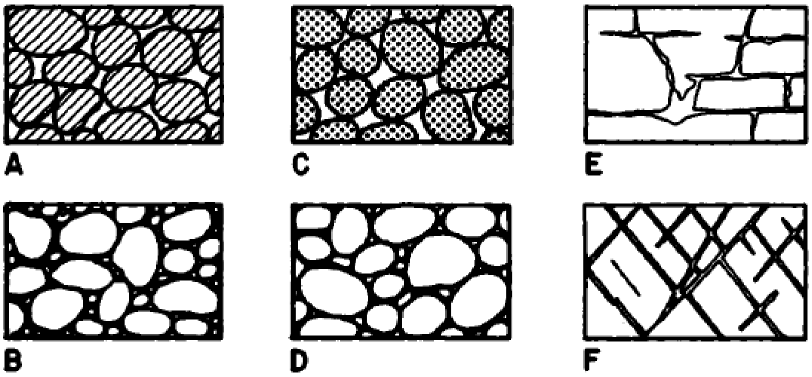
1.3.3 Porosity and effective porosity

Porosity (*n*), or volumetric porosity is considered as a macroscopic porous medium property, it is the ratio of volume of the void space (U_v) to the bulk volume (U_b) of a porous medium.

$$n = \frac{U_V}{U_b} \qquad (1.2)$$

If we consider all the pore space of a medium filled with water is not open for water flow, then the effective porosity is defined as the portion of pore space in a saturated porous material in which water flow occurs. In other words, it is the volume of the interconnected voids of a sample that contributes to fluid flow (Bear 1972).

Figures (1.5) represent a diagram showing several types of Rock Interstices



Figures 1.5: Diagram showing several types of Rock Interstices. A. Well-sorted sedimentary deposit having high porosity; B. Poorly sorted sedimentary deposit having low porosity; C. Well sorted Sedimentary deposit consisting of pebbles that are themselves porous, so that the deposits a whole has a very high porosity; D. Well-sorted sedimentary deposit whose porosity has been diminished by the deposition of mineral matter in the interstices; E. Rock rendered porous by solution; F. Rock rendered porous by fracturing (Bear 1972).

The effective porosity *ne* reads:

$$ne = \frac{\text{volume of the interconnected voids of a sample}}{\text{total volume of the sample}} \qquad (1.3)$$

table 1.1 shows some Porosity of various materials

Table1.l: Porosity of various materials (Vedat 1998)

Group	Porous material	Range of porosity (*n*)
Igneous rocks	Weathered granite	0.34-0.57
	Weathered gabbro	0.42--0.45
	Basalt	0.03-0.35
Sedimentary materials	Sandstone	0. 1 4-0.49
	Siltstone	0.2 1--0.41
	Sand (fine)	0.26--0.53
	Sand (coarse)	0.3 1--0.46
	Gravel (fine)	0.25--0.38
	Gravel (coarse)	0.24-0.36
	Silt	0.34-0.6 1
	Clay	0.35--0.57
Metamorphic rocks	Limestone	0.07--0.56
	Schist	0.04-0.49

1.3.4 Hydraulic conductivity

The ability of the aquifer material to conduct water through it under hydraulic gradients is indicated by the hydraulic conductivity (permeability) *k*. It is considered as a combined property of the porous medium and the fluid flowing through it.

When the flow in the aquifer is essentially horizontal, the aquifer transmissivity indicates the ability of the aquifer to transmit water through its entire thickness. It is the product of the hydraulic conductivity and the thickness of the aquifer (Vedat 1998).

1.3.5 The storativity

The concept of storage in the study of underground flows indicates the ability of a soil to store or release a certain volume of water, in another words it is the relationship between the changes in the quantity of water stored in an aquifer and the corresponding changes in the elevations of the piezometric surface (or the water table in an unconfined aquifer) **(**Bonnet 1982). Storage coefficient is considered in transient flows.

1.3.6 Homogeneity

Homogenous formation is which the hydraulic conductivity *k* is independent of position (constant physical characteristic in the field of flow). If the hydraulic conductivity *k* is dependent on the position within a geologic formation (figure 1.6), the formation is heterogeneous (Elango 2005).

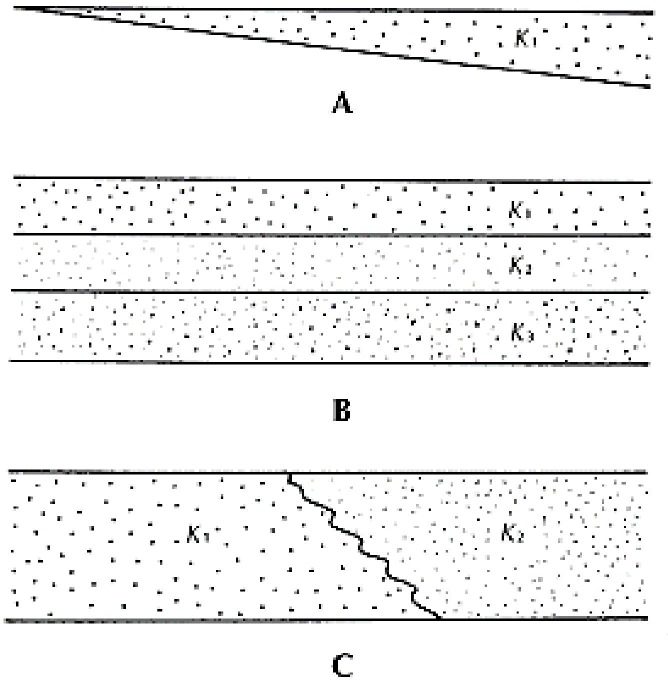


Figure 1.6: A. Heterogeneous formation consisting of a sediment that thickens in a wedge. B. Heterogeneous formation consisting of three layers of sediments of differing hydraulic conductivity. C. Heterogeneous formation consisting of sediments with different hydraulic conductivities lying next to each other (Fetter 2014).

1.3.7 Isotropy

The unit is called isotropic when the porous medium is made of spheres of the same diameter packed uniformly and the geometry of the voids is the same in all directions. Thus, the permeability of the unit is the same in all directions**.**

Conversely, if the geometry of the voids is not uniform, there may be a direction in which the permeability is greater. The medium is thus anisotropic **(**Fetter 2014).

For example, a porous medium composed of book-shaped grains arranged in a subparallel manner would have a greater permeability parallel to the grains than crossing the grain orientation (Figure 1.7) (Fetter 2014).

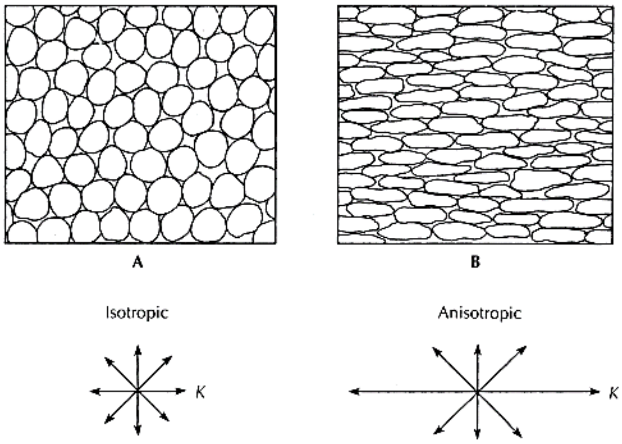


Figure 1.7: Grain shape and orientation can affect the isotropy or anisotropy of a sediment. (Fetter 2014)

1.4 Field equations of flow through a porous medium

1.4.1 Darcy's law

In 1856 a series of experimental studies were conducted to investigate the quantitative behavior of flow of water through homogeneous filters of sands in connection with the fountains of the city of Dijon, France, by a French hydraulic engineer Henry Darcy.

In his experimental setup shown in figure 1.8, water moved vertically down through the

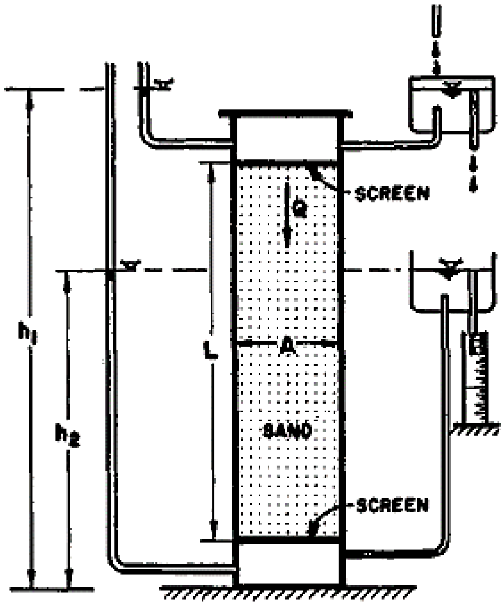


Figure 1.8: Darcy's experiment (Bear 1972).

sand column, and mercury was used as the manometer fluid. From his experiments, Darcy concluded that the rate of flow (volume per unit time) Q is proportional to the constant crosssectional area, A, proportional to (h_2 - h_1) and inversely proportional to the length L. When combined, these conclusions give the famous Darcy formula (Bear 1972):

$$Q = -k \times A \frac{(h2 - h1)}{L} \tag{1.4}$$

where k is a coefficient of proportionality. This constant of proportionality can be assigned a physical significance: it quantifies the ease with which the porous medium permits the fluid to flow. It is therefore called the coefficient of permeability or hydraulic conductivity. $\frac{(h2-h1)}{L}$ is the hydraulic gradient. the negative sign is due to the negative slope of the piezometric level.

The Darcy's experiments were limited to one dimensional flow. for a homogeneous isotropic medium, when the flow is three dimensional the formal generalization of Darcy's law is:

$$\vec{q} = -k \times \overrightarrow{gradh} \tag{1.5}$$

Where q is the specific flux vector, the preceding equation can also be represented by three equations:

$$q_x = k_x \times \frac{\partial h}{\partial x} \tag{1.6.a}$$

$$q_y = -k_y \times \frac{\partial h}{\partial y} \tag{1.6.b}$$

$$q_z = -k_z \times \frac{\partial h}{\partial z} \tag{1.6.c}$$

1.4.2 Conservation of mass (continuity equation)

To solve the precedent equations mathematically, we need a fourth equation for the four unknown parameters q_x,q_y,q_z,h. (Muhammed 2015).

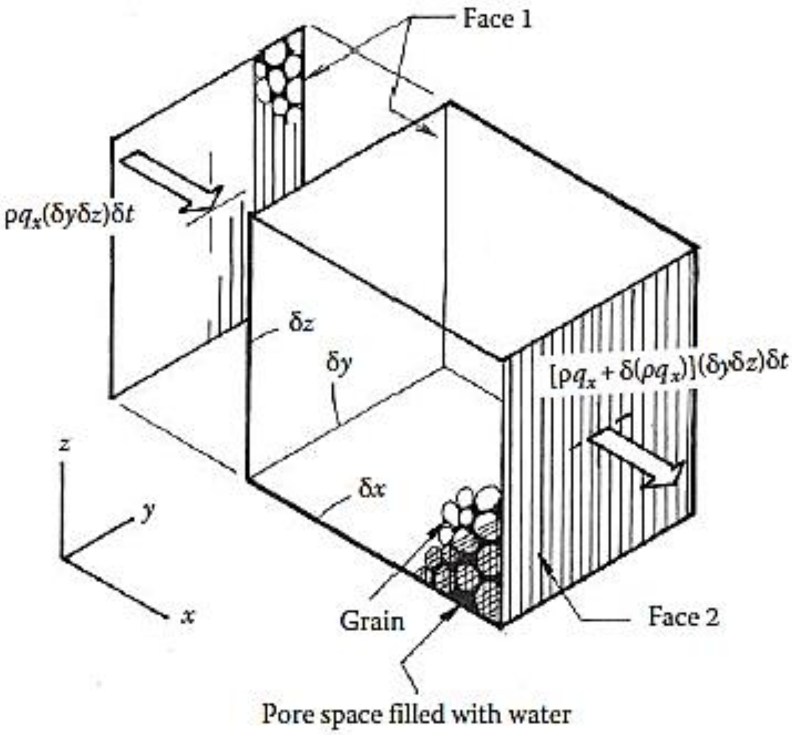


Figure 1.9: Definition sketch for conservation-of-mass equation (Muhammed 2015).

As shown in figure 1.9 and by the principle of mass conservation, the transport of mass across Face 1 into the parallelepiped during a small (infinitesimal) duration of time δt is:

$$\rho \times q_x(\delta y \times \delta z)\delta t \quad (1.7)$$

The transport of mass across Face 2 out of the parallelepiped is:

$$[\rho \times q_x + \delta(\rho q_x)](\delta y \times \delta z)\delta t \quad (1.8)$$

Thereby, the net gain of mass flow across the two faces, Face 1 and Face 2 is given by the following:

$$\rho \times q_x + \delta(\rho q_x) - [\rho \times q_x + \delta(\rho q_x)](\delta y \times \delta z)\delta t = -\delta(\rho q_x)(\delta y \times \delta z)\delta t \quad (1.9)$$

Replacing Taylor's series, ignoring higher order terms the net gain will be:

$$-\delta(\rho q_x)(\delta y \times \delta z)\delta t = -\frac{\partial(\rho q_x)}{\partial x}\delta x(\delta y \times \delta z)\delta t \quad (1.10)$$

If the mass transport takes place across all (six) faces, the net gain becomes:

$$net\ gain = -\left[\frac{\partial(\rho q_x)}{\partial x} + \frac{\partial(\rho q_y)}{\partial y} + \frac{\partial(\rho q_z)}{\partial z}\right] \delta x \times \delta y \times \delta z \times \delta t \qquad (1.11)$$

We assume that the mass of fluid inside the parallelepiped at any time be *m*(*t*).as it is known that only the pore space contain fluid, so for a completely saturated medium this mass should be given by the following (Muhammed 2015):

$$m(t) = -\rho(n \times \delta x \times \delta y \times \delta z) \qquad (1.12)$$

With

n: the porosity; ($n\ \delta x\delta y\delta z$) is the volume of the total pore space contained by the parallelepiped. The increment of mass *m*(*t*) during time interval δt thus becomes:

$$\delta m = \frac{\partial(\rho n)}{\partial t}\delta t(\delta x \times \delta y \times \delta z) \qquad (1.13)$$

Because the volume of the parallelepiped does not change with time, and δx, δy, δz, δt are arbitrary, the net gain to the increment of *m* yields the final equation:

$$-\left[\frac{\partial(\rho q_x)}{\partial x} + \frac{\partial(\rho q_y)}{\partial y} + \frac{\partial(\rho q_z)}{\partial z}\right] = \frac{\partial(\rho n)}{\partial t} \qquad (1.14)$$

Finally, if the density ρ is a constant, Eq (1.14) reduces to (Muhammed 2015):

$$\frac{\partial(q_x)}{\partial x} + \frac{\partial(q_y)}{\partial y} + \frac{\partial(q_z)}{\partial z} = 0 \qquad (1.15)$$

1.4.3 Laplace equation

Combining the mass continuity equation and Darcy's Law produces the governing equation for groundwater flow. By inserting the components of specific discharge vector *q* of the three equations of Darcy into the continuity equation we obtain (Muhammed 2015):

$$\frac{\partial}{\partial x}\left(\frac{\partial(-k_x h)}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial(-k_y h)}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial(-k_z h)}{\partial z}\right) = 0 \qquad (1.16)$$

When the medium is isotropic, the coefficient of permeability *k* is the same in all directions, consequently Eq (1.16) reads:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \nabla^2 h = 0 \qquad (1.17)$$

This last equation is known as Laplace equation with ∇^2 is called the Laplacian operator (Bear 1972). It is the basic elliptic partial differential equation governing steady incompressible groundwater flow.

1.5 Conclusion

In this chapter we have touched some essential theoretical foundations for groundwater flows that are relevant for analyzing incompressible steady-state groundwater flow in confined aquifers.

Chapter two: Swarm intelligence

2.1 Introduction

Many optimization problems in engineering and science applications in general are highly complex and challenging and thereby require novel problem-solving approaches **(**Yang et al. 2017)

Therefore, during the last three decades, the water resources engineering field specially has received a huge increase in the development and use of meta-heuristic algorithms like evolutionary algorithms (EA) and swarm intelligence (SI) algorithms for solving several kinds of optimization problems. (Reddya, and Kumarb 2020**).**

Swarm intelligence (SI) is a motivational area of the population-based metaheuristic algorithms. The concepts of SI were first introduced in 1993 (Beni and Wang 1993).
SI is ‘‘the emergent collective intelligence of groups of simple agents’’ according to
(Bonabeau et al. 1999). The SI techniques were inspired mostly by natural colonies, birds, animals, and schools of fishes. Some of the most popular SI techniques are ant colony optimization (ACO) (Dorigo and Birattari 2010), particle swarm optimization (PSO) (Kennedy and Eberhart 1995), and artificial bee colony (ABC) (Basturk 2006**) (**Asef1and et al. 2021**).**

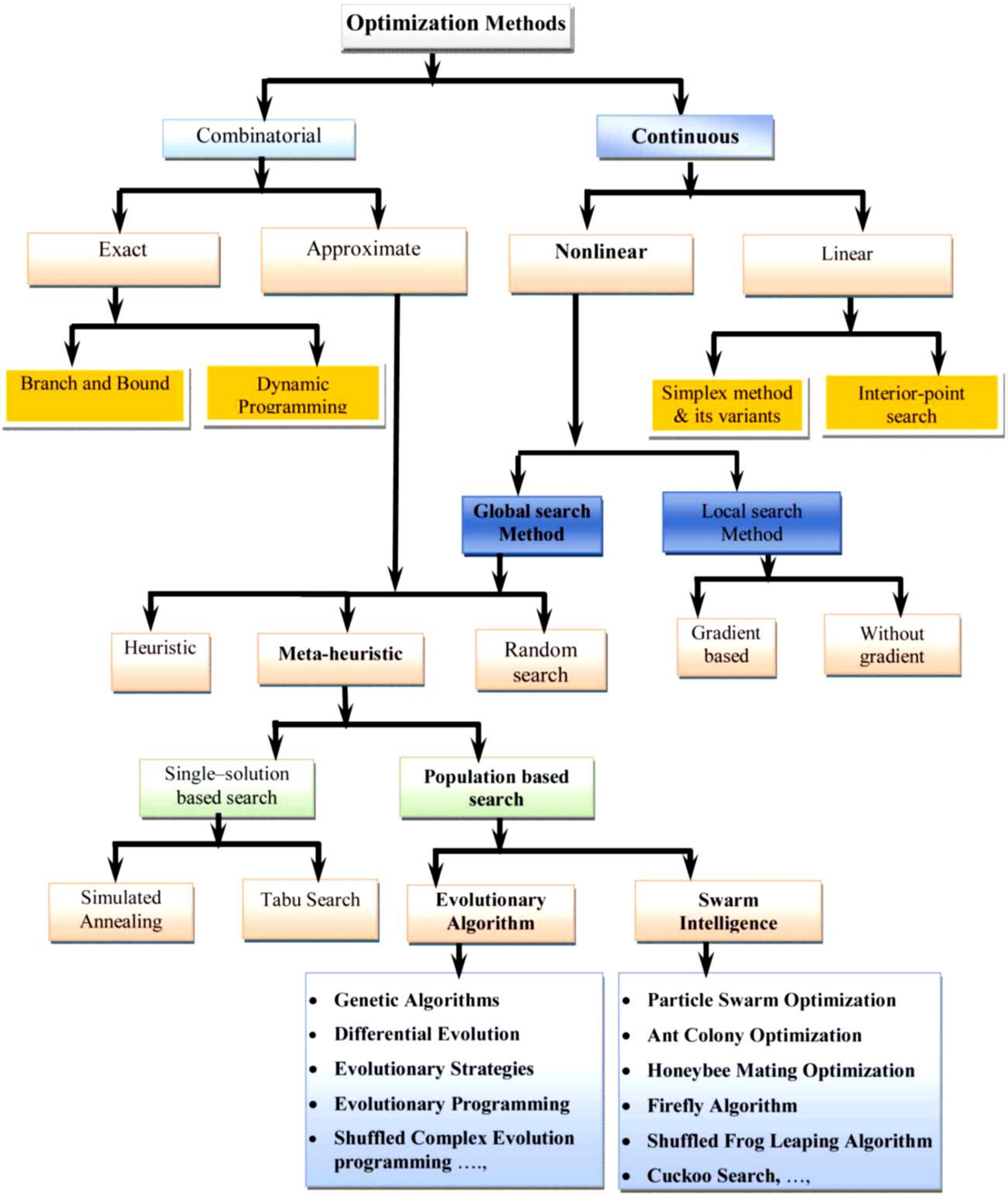


Figure 2.1: Taxonomy of optimization methods **(**Reddya and Kumarb 2020**).**

2.2 Optimization methods

Mathematical optimization is a section of applied mathematics and computer sciences which deals with the choice of the optimal solution for a particular mathematical function (or problem) with the aim of either minimizing or maximizing the output of such function. In other words, optimization could be described as the process of selecting of the best element(s) from among a series of available alternatives to get the best possible results when solving a particular problem **(**Galinier et al. 2013; Cuevas et al. 2016).

The hydraulic problems can be formulated as problems to optimize as well, in this case, we call them *the inverse problems*. Solving an inverse problem is observing the available solutions, and trying to determine the values of the parameters of the model governing the phenomenon under consideration; in other words, it is a calibration operation.

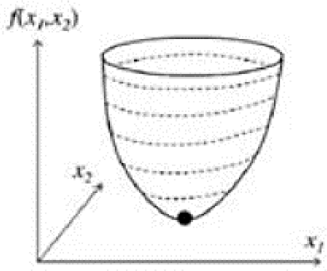
An optimization problem is often represented as follows:

Minimize: $f(x), x_i^L < x_i < x_i^U \ , i = 1,2, \dots \dots .. Npar$ (2.1)

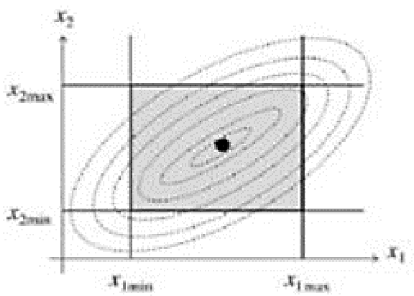
With *f*(x) is the objective function to be minimized (Figure. 2.2a and Figure. 2.2d),

Npar is the number of parameters to be identified,

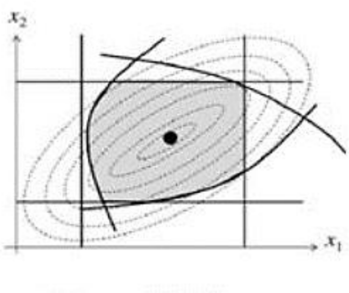
$x = [x_1, x2, \dots .. xNpar]^T$ is the vector of Npar dimensional parameters. The values of the xi parameters are located in the search space, limited by the lower boundsx_i^L and upper bounds x_i^U (Figure. 2.2b). **(**Tadj 2019**)**



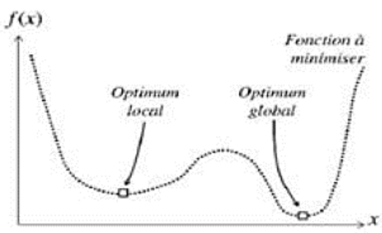
(a) Objective function



(b) Unconstrained research



(c) Search space with constraints



(d) Local and global optimum

Figure 2.2: Schematization of optimization problems **(**Bruyneel and al. 2010**).**

We can categorize optimization problems into two types based on the solutions produced, namely the classical or deterministic algorithms and the non-classical or non-deterministic also called the stochastic algorithms. The stochastic algorithm can be further classified into the heuristic and metaheuristic techniques **(**Okwu and. Tartibu 2021**)**

If we try to classify optimization problems according to the number of objectives, then there are two categories: mono-objective and multi-objective. Multi-objective optimization is also referred to as multicriteria or even multi-attribute optimization in the literature. In real-world problems, most optimization tasks are multi-objective **(**Yang, 2014**).**

The following table summarise the classification of optimization problems based on some other options:

Table 2.1: Classification of optimization problems **(**Tadj 2019**)**

Optimization	Objective	• Mono-objective • Multi-objective
	Constraints	• Constrained • Unconstrained
	Landscape of *f(x)*	• Unimodal • Multimodal
	Behavior	• Linear • Non-linear
	Variables	• Discretes • Continues
	Techniques	• Deterministics • Stochastics

An optimization process often combines an optimization technique with a numerical simulation model (Figure.2.2). The optimization technique generates the *x* parameter vector, and injects it into the simulation model in an iterative way in order to minimize a given objective function. It is an operation that transfers information from observed state variables to estimated parameters. The objective function represents the differences between the state variables calculated by the simulation model and those observed. It is expressed according to the parameters to be identified. The objective function can take several forms depending on the nature of the problem being addressed, such as the sum of squared errors (SSE) (Eq. 2.2).(Tadj 2019).

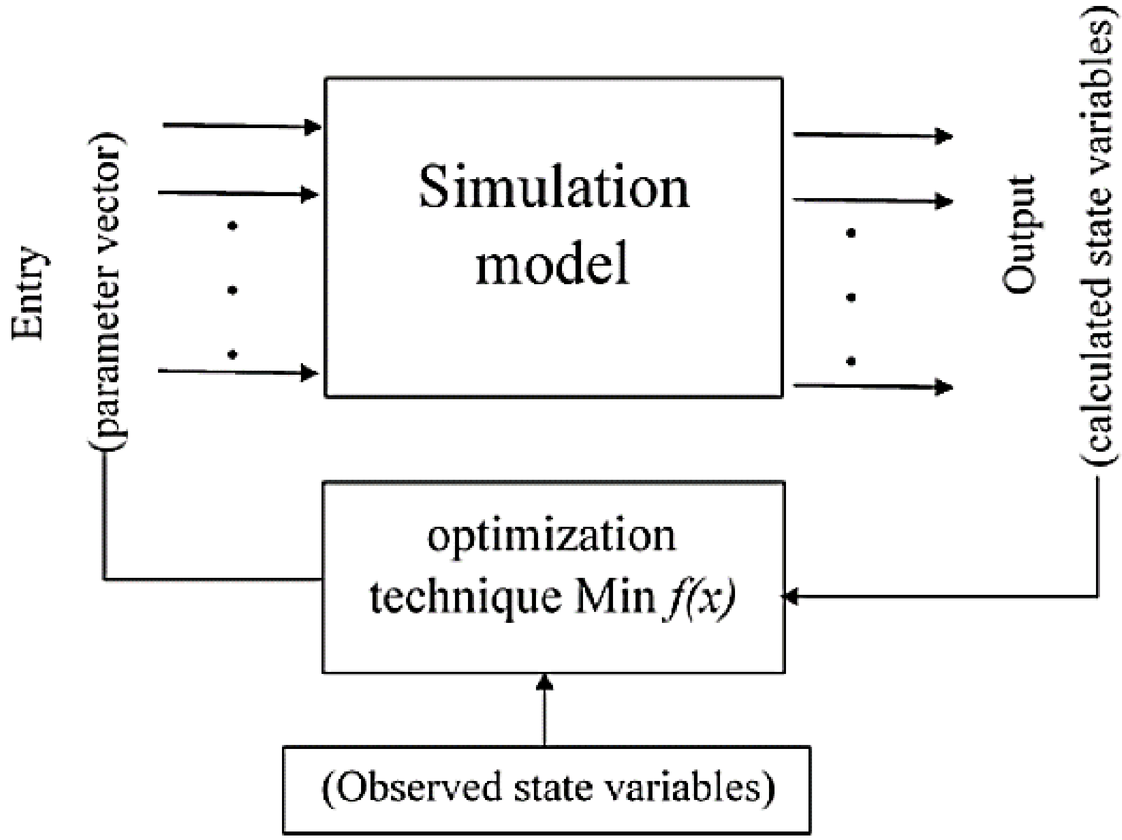


Figure 2.3: Optimization process (Tadj 2019).

$$f(x) = SSE = \sum_{i=1}^{N}(y_i^0 - y_i^c)^2 \quad (2.1)$$

With N is the number of observed points, y_i^0 are the observed variables,$y_i^c$are the variables computed by the simulation model. Depending on the inverse problem complexity and on the availability of data, the problem of optimization process can be simple, slightly difficult, moderately difficult or very difficult (Sun and Sun 2015). There is no general rule that dictates the amount of data (observed variables) required for the parameter identification process, but the more complex the model is, the more data are needed.

2.2.1 Optimization techniques

The oldest optimization technique, which is still in use, is the trial-and-error one, it is a manual and subjective technique based on trial and error guided by the experience of the operator, it is very slow and lacks precision, consequently various algorithms optimization was created, and the choice of which one is appropriate for a given problem remains a very active area of research. As the Theorem (No-Free Lunch) indicates, no optimization technique is superior to all others for all problems **(**Wolpert and Macready 1997); there is no universal optimization algorithm. For different types of optimization problems, we frequently have to use different optimization techniques, as some algorithms are more suited to certain types of optimizations than others. In an optimization process, the ideal is to find eligible solutions within a short period of time (Tadj 2019).

As we discussed earlier, optimization algorithms are divided into two categories: deterministic algorithms and stochastic algorithms.

a) Deterministic algorithms

Deterministic optimization algorithms (eg., descent gradient, Gauss-Newton, Levenberg-Marquardt. etc.) start their search from an initial guess value. As the name suggests, these techniques follow the same path of research, that means from the same starting values we will always have the same solutions. Take, for example, the downward or ascending gradient algorithm. It is a gradient-based algorithm, also called the steepest slope algorithm. The idea of this algorithm is simple, and can be summed up in the following steps **(**Kruse et al. 2016):

1. The algorithm starts at a initial point $x^{(0)} = (x_1^{(0)}, \cdots x_{Npar}^{(0)});$
2. Calculate the gradient at current point $\nabla_x f\left(x^{(k)}\right) = \left\{\frac{\partial}{\partial x_1} f\left(x^{(k)}\right), \cdot \cdots, \frac{\partial}{\partial xNpar} f\left(x^{(k)}\right);\right.$
3. Calculating the next position $x^{(k+1)} = x^{(k)} + \eta \nabla_x f\left(x^{(k)}\right)$, η : discretisation step
4. Repeat steps 2 and 3 to move through the search space in (or against) the direction of the steepest slope of the objective function, until you reach an optimum;
5. Stop condition: The algorithm stops when a maximum number of iterations is reached, or the value of the gradient becomes below a certain threshold.

As shown in Figure 2.4, the effectiveness of this gradient-based deterministic algorithm depends on the parameters of the initial estimate $x^{(0)}$ and the step discretisation value η.

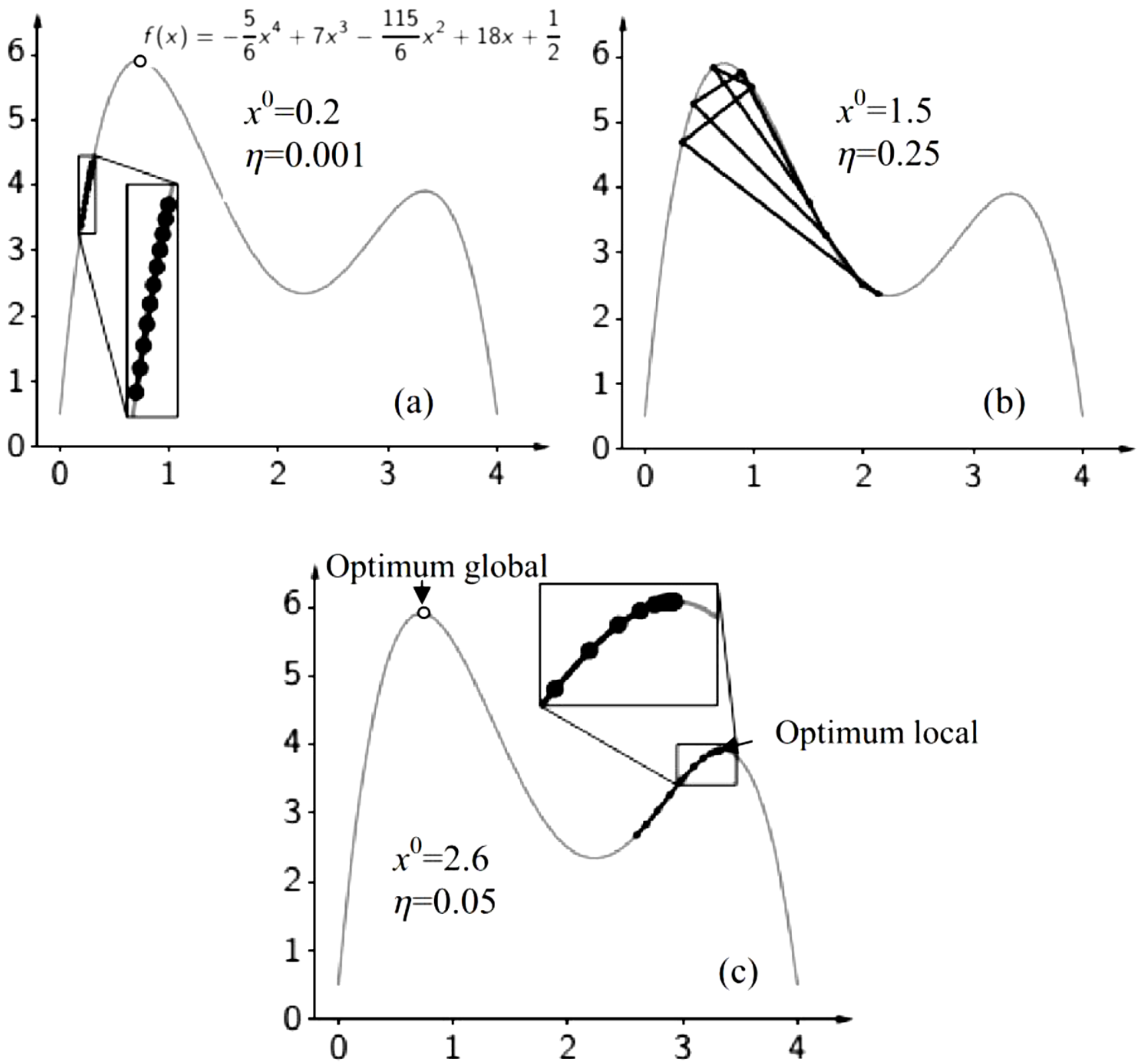


Figure 2.4: Maximizing an f(x) function by the ascending gradient algorithm **(**Tadj 2019**).**

For small value of η (Figure.2.4a), the convergence of the algorithm is very slow requiring high computational time. When the η value is high (Figure 2.4b), oscillations are made observed, the solution jumps back and forth.

When the initial estimate $x^{(0)}$ is far from the global optimum (Figure.2.4c), in this case the solution thus corresponds to a local optimum.

Furthermore, deterministic techniques are gradient-based, that means, the evaluation of the derivatives of the objective function in relation to the different parameters is essential, which adds a derivability condition to the objective function. These deterministic techniques are appropriate for convex problems. Unfortunately, hydraulic problems do not usually present this convexity property, however solutions obtained using such techniques can be trapped in local optimal. Sometimes the user of this type algorithms is forced to do multiple executions with different starting points when the objective function has many local optimal. (Tadj 2019)

b) Stochastics algorithms

Stochastic optimization algorithms are an ensemble of artificial intelligence methods to solve optimization problems deemed difficult. These are gradient-free methods that require only direct evaluation of the objective function, they have a random character. Stochastic algorithms are called heuristics or metaheuristics **(**Yang 2010**).**

Heuristic algorithms use a trial-and-error approach in generating new solutions, while the metaheuristic algorithms are a higher-level heuristic with the use of memory, solution history and other forms of 'learning' strategy. Nowadays, most metaheuristic algorithms are nature-inspired algorithms and most such algorithms are based on swarm intelligence **(**Yang 2018**).**

2.3 Metaheuristics

Metaheuristic algorithms are now used all over, to solve complex optimization problems. Most of these algorithms are based on biological phenomena and behaviour of animals.

Metaheuristic optimization algorithms are growing in popularity in engineering applications for several reasons:

First, they depend on rather simple concepts and are easy to implement;

Secondly, they require no gradient information;

Third, they can bypass local optima;

Finally, they can be utilized in a set of problems in different disciplines **(**Asef 2021**).**

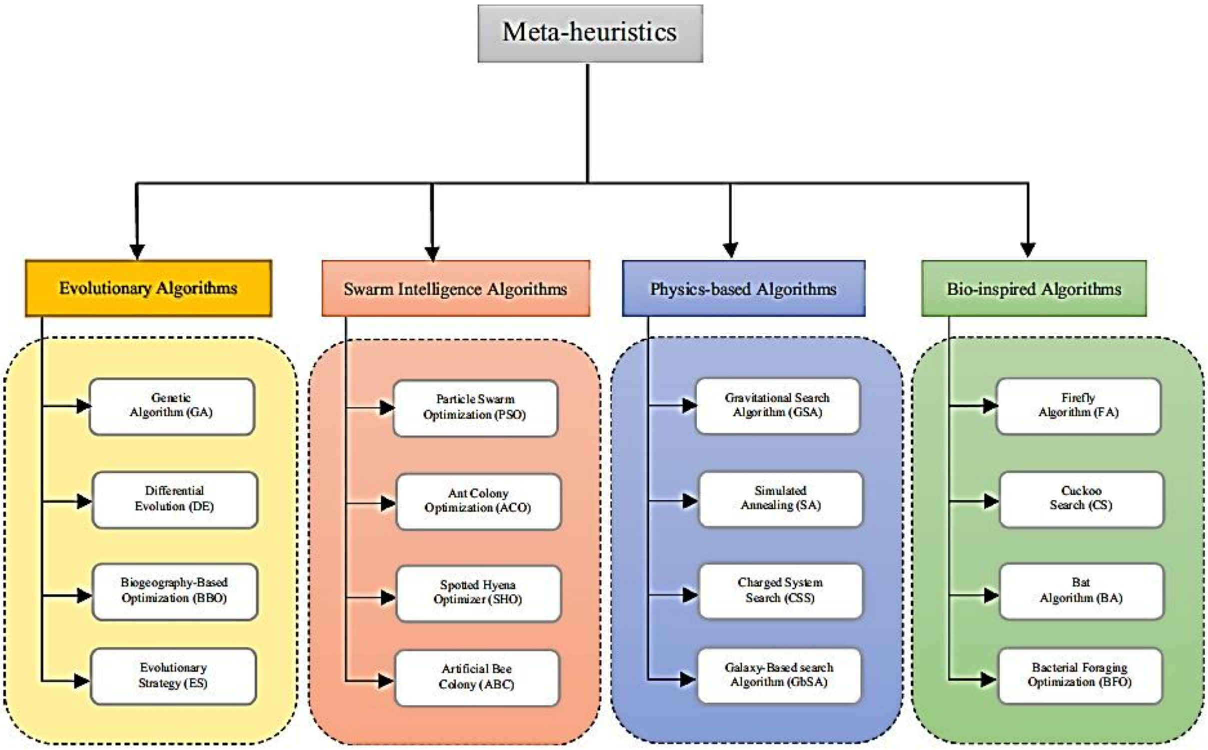


Figure 2.5: The classification of the metaheuristic algorithms **(**Dhiman and Kumar 2018**).**

Figure 2.5 shows the population-based metaheuristic algorithms are classified as four categories called physics-based algorithms, swarm intelligence algorithms, evolutionary algorithms and bio-inspired algorithms **(**Dhiman and Kumar 2018).

Obviously, it is not possible to include even a good fraction of these algorithms. Thus, our emphasis is on the algorithms that can be considered as representatives, especially those who have been applied in the area of water resources.

2.3.1 Evolutionary Algorithms

The EAs are rapidly expanding in the area of artificial intelligence research. Over the past two decades, there has been increasing attention in algorithms, which are based on the principle of natural evolution (i.e., the survival of the fittest) **(**Fogel et al. 1966**).** The EAs are the population-based random search techniques guided with some heuristics (also called as meta-heuristic techniques). The EAs consist of a population of individuals, each representing a search point in the space of feasible solutions and is exposed to a collective learning process which proceeds from generation to generation **(**Brownlee 2011**).** The population is randomly initialized and then subjected to the process of selection, recombination, and mutation through various generations, in such a way that the newly created generations evolve towards more favourable regions of the search space. The progress in the search is achieved by evaluating the fitness of all individuals in the population, choosing the individuals with a better fitness value, and combining them to create new individuals with an increased likelihood of improved fitness. After some generations, the program converges, and the best individual represents the optimum (or near-optimum) solution. There exist many EAs, but the basic structure of any evolutionary algorithm is presented in Figure 2.6.

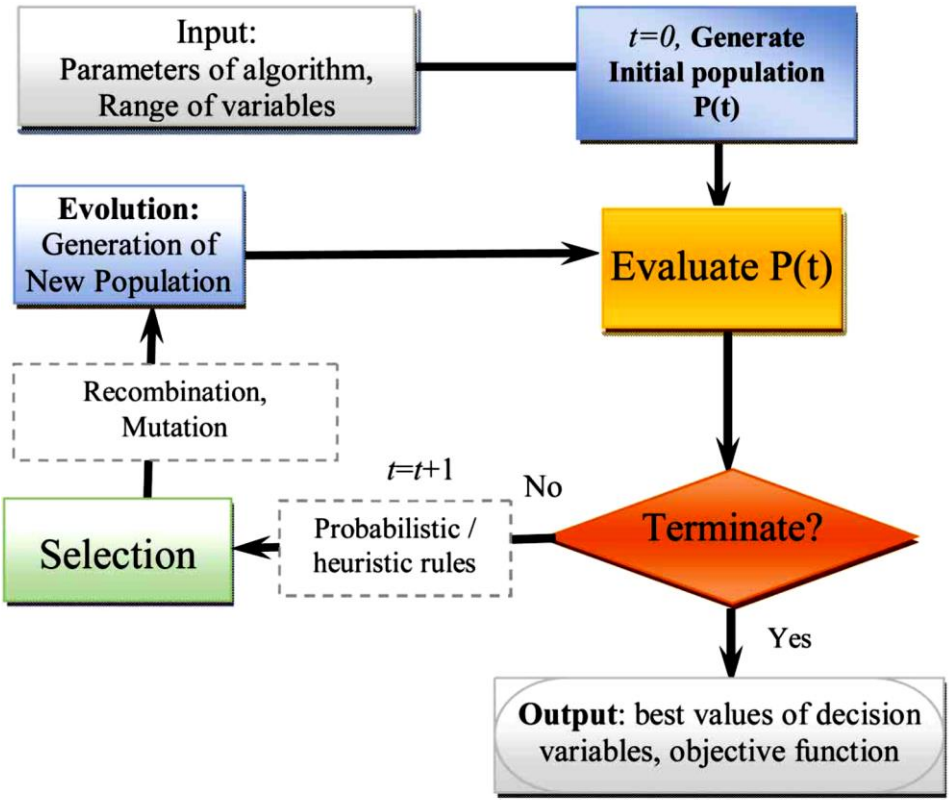


Figure 2.6 The basic structure of an evolutionary algorithm (EA) (Reddy and Kumar 2021**).**

The key steps involved in EA include (Reddy and Kumar 2021**):**

1. Seeding the population using random generation;

2. Rate the fitness of each individual in the population;

3. Repeat the evolution steps until stopping criterion satisfied:

(a) Choose the individuals for reproduction;

(b) Perform genetic operations to generate the offspring;

© Evaluate the individual fitness of the offspring;

(d) Exchange the least fit individuals with new best fit individuals.

4. Report the best solution of the fittest individual.

a) Genetic Algorithms

Genetic algorithms (GA) are one of the earliest metaheuristics inspired by the concepts of natural selection and evolution and are one of the best, successful evolutionary algorithms (EA) because of its conceptual simplicity and easy implementation **(**Mitchell 1996**).** GA was initially developed by John Henry Holland in 1960 with the purpose to understand the phenomenon of natural adaptation, and how this mechanism could be implemented into computers systems to fix complex problems.

In GA, a population of N solutions $\mathbf{x}i = [x_{i,1}, x_{i,2}, \ldots, x_{i,npar}]$ is first initialized; each of such solutions (called *chromosomes*). Every iteration (also called generation) of GA's evolution process, the chromosome population is modified by applying an ensemble of three evolutionary operators, namely: crossover, selection and mutation. For the selection operation, GA randomly selects a pair of chromosomes from within the entire chromosome population, based on their individual selection probabilities. For the crossover operation, the selected chromosomes (now called parents) are recombined to produce two new chromosomes (referred as *offspring*). Lastly, for the mutation operation, some parameters of the newly generated offspring are perturbed. Mutation can occur over each parameter in the chromosome with a particular probability. This process of selection, crossover, and mutation of individuals takes place until a population of N new chromosomes has been produced, and then, the N best chromosomes among the original and new populations are taken for the next generation, while the remainder individuals are discarded. (Fausto et al. 2019).

2.3.2 Swarm intelligence

Swarm intelligence can arise in multi-agent systems, and it is not clear yet what mechanisms are responsible for the emergence of collective behaviour in a swarm. However, swarm-intelligence-based algorithms have been evolved and utilised in a wide number of applications in optimization, machine learning, engineering, data mining and image processing. **(**Yang et al. 2017)

Swarm intelligence is a natural or artificial self-adaptive, decentralized system comprising of interaction of individuals with one another and their environment. The most common source of inspiration is from nature, especially biological systems. As a part of this domain, we explore further into countless algorithms as it is cited in Table 2.2 **(**Goel 2020**).**

Table 2.2: List of SI algorithms (Goel 2020).

Algorithm	Author	Year
Swarm intelligence		
Ant colony optimization	Dorigo et al.	1992
Artificial bee colony	Karaboga et al.	2005
Artificial fish swarm	X. Li et al.	2002
Bat algorithm	X.S. Yang	2010
Bees algorithm	Pham et al.	2009
Biogeography-based optimization	Simon et al.	2008
Charged system search	Kaveh et al.	2010
Firefly algorithm (FA)	Yang	2007
Cuckoo search	Yang X.S. et al.	2009
Cuttlefish optimization	Eesa et al.	2014
Duelist algorithm	Biyanto et al.	2016
Flower pollination	Yang et al.	2012
Glowworm swarm optimization	Kaipa et al.	2009
Gravitational search	Rashedi et al.	2009
Grey wolf optimizer	Mirajlili et al.	2014
Hydrological cycle	Wedyan et al.	2017
Imperialist competitive algorithm	Atashpaz-Gargari and Lucas	2007
Intelligent water drop	Shah H. et al.	2007
Particle swarm optimization	Kennedy et al.	1995
Shuffled frog leaping (SFLA)	Eusuff et al.	2006
Simulated annealing (SA)	S. Kirkpatrick et al.	1983
Spiral optimization (SOA)	Tamura et al.	2011
Stochastic diffusion search (SDS)	Bishop J.M.	1989
Whale optimization (WOA)	Mirajlili et al.	2016
Artificial algae algorithm (AAA)	M. Beskirli et al.	2017
Rain fall optimization (RFO)	S. Hr. Kaboli et al.	2017
Killer whale algorithm (KWA)	Totok Biyanto et al.	2017
Artificial flora (AF)	L. Cheng et al.	2018
Plant intelligence behavior optimization (PIBO)	Godfrey	2018
Squirrel search algorithm (SSA)	Mohit Jain et al.	2019
Artificial innovative gunner	P. Pijarski et al.	2019
Harris hawk optimizer (HHA)	Heidari et al.	2019
Class topper optimization algorithm (CTO)	Pranesh Das et al.	2019
Butterfly optimization algorithm	Arora and Singh	2019

a) Particle swarm optimization

Particle swarm optimization, or PSO, was evolved by Kennedy and Eberhart in 1995 and has become amongst the most widely applied swarm-intelligence-based algorithms by reason of its simplicity and flexibility. instead of using the mutation/crossover or pheromone, it uses real-number randomness and global communication among the swarm particles. Therefore, it is also easier to implement due to the fact that there is no encoding or decoding of the parameters into binary strings as with those in genetic algorithms where real-number strings can also be used. **(**Yang 2014).

The PSO algorithm searches the space of an objective function by adjusting the trajectories of individual agents, called *particles*, as the piecewise paths formed by positional vectors in a quasi-stochastic manner. The movement of a swarming particle consists of two major components: a stochastic component and a deterministic component. Each particle is attracted toward the position of the current global best g^* and its own best location x^*i in history, while at the same time it has a tendency to move randomly. When a particle finds a location that is better than any previously found locations, updates that location as the new current best for particle *i*. There is a current best for all *n* particles at any time *t* during iterations. The intention is to find the global best from among all the current best solutions until the objective no longer improves or after a certain number of iterations. Figure 2.7 represent the schema of the movement of particles, where $x_i^{*(t)}$ is the current best for particle *i*, and g^* is the current global best at *t* (Yang 2014**).**

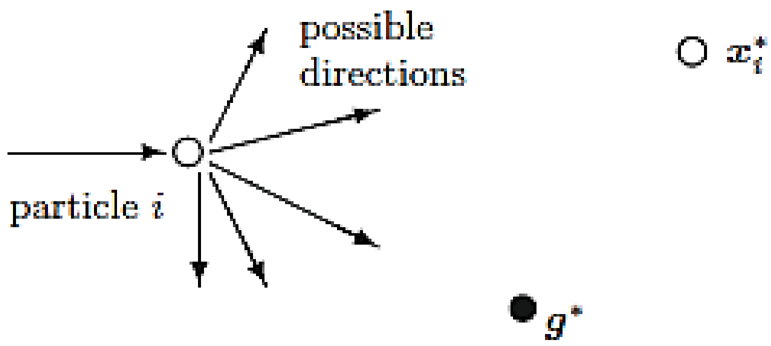


Figure 2.7: Schematic representation of the motion of a particle in PSO moving toward the global best g^* and the current best x_i^* for each particle *i* **(**Yang 2014**).**

Figure 2.8 present the pseudo code of the essential steps of the particle swarm optimization (note that *d* denotes *Npar*).

Particle Swarm Optimization

Objective function $f(x)$, $x = (x1, ..., x\ d)^T$

Initialize locations xi and velocity vi of n particles.

Find g^* from $\min\{f(x1), ..., f(xn)\}$ (at $t = 0$)

while (criterion)

  for loop over all n particles and all d dimensions

  Generate new velocity v_i^{t+1}

  using equation $v_i^{t+1} = v_i^t + \alpha\epsilon_1[g^* - x_i^t] + \beta\epsilon_2[x_i^{*(t)} - x_i^t]$,

  Calculate new locations $xt+1$

  $i = x_i^t + v_i^{t+1}$

  Evaluate objective functions at new locations x_i^{t+1}

  Find the current best for each particle x_i^*

  end for

  Find the current global best g^*

  Update $t = t + 1$ (pseudo time or iteration counter)

end while

Output the final results x_i^* and g^*

Figure 2.8: Pseudo code of particle swarm optimization **(**Yang 2014**).**

b) Ant colony optimization (ACO)

One of the most popular, successful and standard versions of the SI techniques is based on the ant system. It has been observed that the capabilities of a single ant are very limited however an ant system has very complex behaviour. Each ant is collectively cooperating in this ant system without knowing their cooperative behaviour. By laying the pheromone on the ground, they are helping their fellows to take that path. The aim of the ACO technique is not to simulate the complete ant colony but instead to use the collective behaviour of ants to develop an optimization tool for complex real-life optimization problems **(**Slowik 2020).

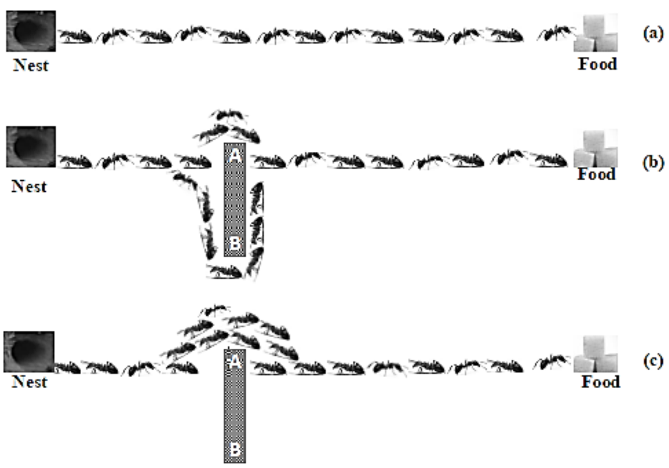


Figure 2.9: Ant system **(**Slowik 2020).

In Figure 2.9 (a), ants follow the shortest path between nest and food source. intended for demonstrate the path seeking behaviour of ants, as shown in Figure 2.9(b)a block is placed in this path. Unawares, ants scatter to find the path to their food source or nest. Now they have two paths to follow: paths through edge A and B. The ants which are going through edge A will take less time to return than ants following edge B. As a result, more pheromone concentration will be deposited on the path through edge A. Because of this increasing pheromone concentration all ants will follow this shortest path, as shown in Figure 2.9(c). This discussed behaviour of the ant system is adopted in the ACO algorithm. (Slowik 2020)

The longer it takes for ants to travel backward and forward along a specific course, the lower the pheromone density as a consequence of the evaporation factor. On the subject of optimization using ACO, the pheromone evaporation is useful because it avoids locally optimal solution and minimizes the attractiveness of existing potential locally optimal solution (Okwu and Tartibu 2021).

C) Crow search algorithm (CSA)

Crows (crow family or corvids) are considered one of the most intelligent birds. They contain the biggest brain relative to their body size. Based on a brain-to-body ratio, their brain is slightly lower than a human brain. Proofs of the cleverness of crows are numerous. They have demonstrated self-awareness in mirror tests and have tool-making ability. Crows have the ability to remember faces and warn each other when an unfriendly one approaches. Furthermore, they are able to use tools, communicate in complicated ways and recall their food's hiding place up to numerous months later.

It is known that crows watch other birds, watch where the other birds hide their foods, and then once the owner leaves, the crow steal it from it. If a crow has committed thievery, it must take extra precautions such as moving hiding places to protect their caches and also to avoid being a future victim. Actually, they use their own experience of having been a thief to predict the behaviour of a pilferer, and can determine the safest course to protect their caches from being pilfered **(**Askarzadeh 2016**).**

Crow Search Algorithm (CSA) is one of the latest algorithms developed by Alireza Askarzadeh in 2016, which simulates the crow actions in storing their food and retrieving it when they need it. Since its appearance, CSA has been widely used and utilised to different optimization issues **(**Hussien et al. 2020)

The principles of CSA are listed as follows:

- Crows lives in the form of flocks;

• Crows memorize the position of their hiding places;

• Crows go behind each other to do thievery;

• Crows defend their caches from being pilfered by a probability.

Figure 2.10 shows the exploration and exploitation of CSA

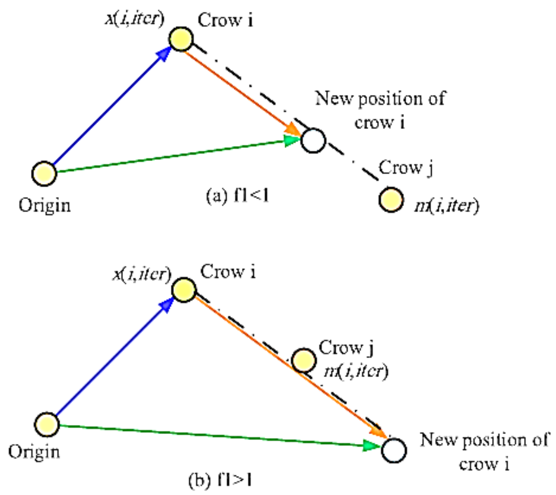


Figure 2.10: Exploration and exploitation of CSA (Askarzadeh 2016)

igure 2.11 represent the flowchart of CSA. ss

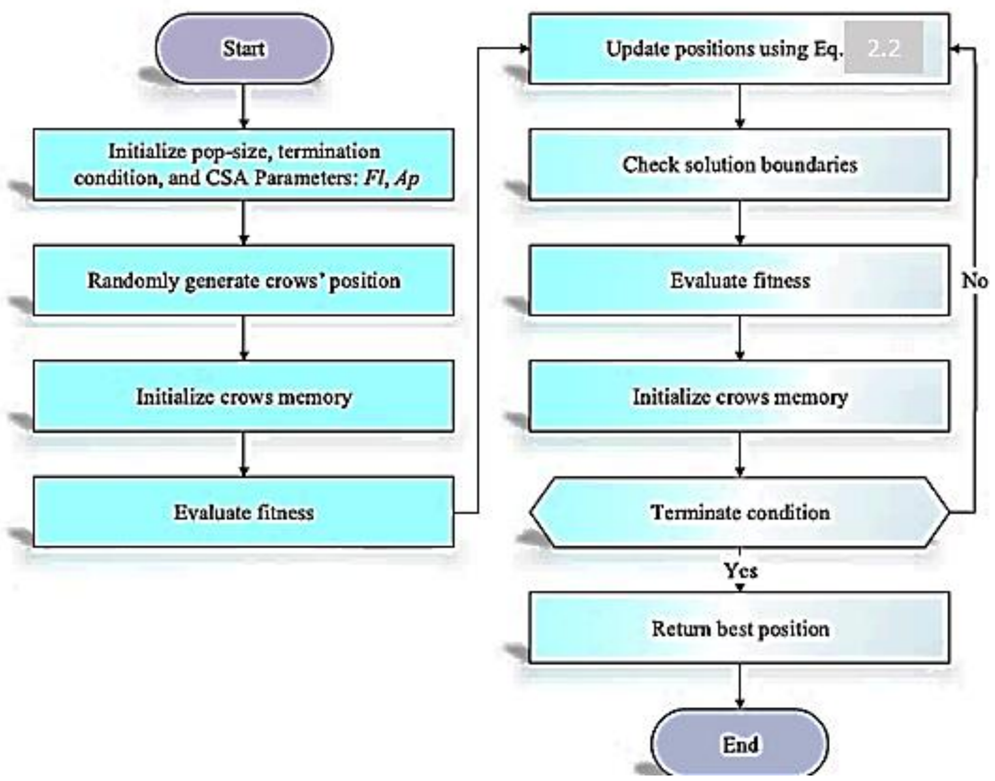


Figure 2.11: The flowchart of CSA (Hussien et al. 2020).

The main phases of CSA can be shown as follows **(**Askarzadeh 2016**):**

1) Initializing crows swarm randomly;

2) A fitness function is used to evaluate each crow, and its value is put as an initial memory value. Each crow stores its hiding place in its memory variable m_i;

3) Crow updates its position by selecting a random another crow, it means x_j and generating a random value. if this value is greater than Awareness Probability `AP', then crow x_i will follow x_j to know m_j;

4) Crow *i* updates its position by selecting a random other crow i.e x_j and following it to know m_j. Then new x_i is calculated as follows:

$$x_{i,ter+1} = \begin{cases} x_{i,jter} + r_i \times fl \times \left(m_{j,iter} - x_{i,iter} \right) & \text{if } r_j \geq AP \\ \text{a random position} & \text{otherwise} \end{cases} \tag{2.2}$$

- AP refers to crow *j* awareness probability;
- *iter* refers to iteration number;
- r_i; $r_j$refers to random numbers;
- fl is the flight length.

5) Updating memory:

$$m_{i,iter+1} = \begin{cases} x_{i,iter+1} & f(x_{i,iter+1}) < f(m_{i,iter}) \\ m_{i,iter} & otherwise \end{cases} \tag{2.3}$$

The figure 2.12 below presents the Pseudo code of CSA.

```
Algorithm    CSA: Crow Search Algorithm
Input: n Number of crows in the population.
itermax Maximum number of iterations.
Output: Optimal crow position
Initialize position of crows.
Initialize crows' memory
while iter < itermax do
for crow i belong to crows do
choose a random crow.
determine a value of awareness probability AP
Update  xi,iter+1
end for
Check solution boundaries.
Calculate the fitness of each crow
Update crows' memory
end while
```

Figure 2.12: Pseudo code of CSA (Hussien et al.2020).

2.4 Applications of metaheuristics in the field of water resources

Water is an exhausting, important and essential element of life, it ought to be managed in a way that optimal and responsible. Nowadays, water resource management is done using models with a number of parameters and conditions that are difficult to obtain.

SI and EA's methods have emerged as a powerful tool for optimization and management of water resources issues. There are numerous applications of EAs for water-related problems, such as water distribution systems design, reservoir operation, groundwater remediation, parameter estimation in hydrological modeling, watershed management, and fluvial systems, etc. Considering that there exist countless thousands of papers on applications of these algorithms, here, some of the essential applications in water resources are reviewed **(**Janga and Kumarb 2020).

2.4.1 Applications in water distribution systems

Water distribution systems (WDS) comprises a system of interconnected nodes, via pipes, supply sources, including tanks reservoirs and a series of hydraulic control elements, namely, pumps, valves, regulators, etc. The network of interconnected nodes, pipes, and other hydraulic control elements is collectively termed as a water distribution network (WDN). A typical WDN design is formulated as an optimization problem requiring minimization of cost, satisfying the minimum pressure and flow requirements at different nodes. A variety of EA were applied for design and rehabilitation of WDNs, like GAs (genetic algorithms), Ant Colony optimization (ACO), Differential Evolution (DE), Harmony search algorithm (HS), Tabu search algorithm (TS), Honey-bee mating optimization (HBMO), Cross entropy (CE) and Gravitational search algorithm (GSA) (Janga and Kumarb 2020).

2.4.2 Applications in urban drainage and sewer systems

Sewer systems and urban drainage require to be designed in such a manner that the required flow capacity is met at minimum cost. The consideration of networks where both stormwater and sewage are carried through the same channel makes the problem a little more involved. Multifarious studies used different EA's for the design of urban drainage and sewer systems like GA, SA, TS, DE method (Janga and Kumarb 2020).

2.4.3 Applications in reservoir operation and irrigation systems

Reservoirs and irrigation systems need to be operated in a cost-effective manner in order that the deficits are minimum as well the benefits achieved in terms of minimum cost or maximum energy production. Therefore, the optimization problem is formulated as determination of the optimal release or operating policies such that the deficits are minimum and the benefits are maximum. Several meta-heuristic techniques were utilised to resolve different problems (Rani and Moreira 2010).To solve such problems, the techniques used include GA, Shuffled complex evolution (SCE) algorithm, PSO (Janga and Kumarb 2020).

2.4.4 Applications in watershed management and fluvial systems

Watershed management and planning needs the modeling the hydrologic and fluvial characteristics properly and efficiently, with the intention that the required water management conditions can be achieved in a cost-effective manner. Furthermore, the various watershed management techniques, including the design of detention systems, flood management practices, and other best management practices, need to be designed considering cost minimization, efficiency maximization and system reliability. Many studies used EA's for solving the relevant optimization models like GA, PSO, DE (Janga and Kumarb 2020).

2.5 Conclusion

Swarm intelligence is an interesting and important area, and swarm intelligence-based algorithms have permeated into almost all areas of sciences and engineering. Chapter two presented an introduction to optimization world from the classification to the process of optimization. Then, we highlighted some EA and SI-based algorithms and subsequently presented their main components, characteristics and properties. We focused on the crow search algorithm (CSA) that will be used in finite element models updating in the next chapter. Finally, the applications of some optimization techniques in the field of water resources were also presented.

2 *Chapter two: Swarm intelligence*

Chapter three: Steady-state groundwater flow models updating using crow search algorithm

3.1 Introduction

Numerical simulation has become an essential tool in engineering. Numerical simulation involves solving the partial differential equations (PDE) governing the phenomenon under consideration, using numerical tools such as the finite differences method (eg., Zouhri et al. 2004; Karahan and Ayvaz 2005; Gurarslan and Karahan 2015), or the Finite element method (FEM) (eg., Smaoui 2018). PDE include a number of parameters with physical significance, such as hydraulic conductivity, which is often distributed in space and are not directly measurable, and whose values directly affect the quality of the solution. In practice, it is very difficult to build an accurate simulation model for a given phenomenon, such as underground flow. First, the equations governing the phenomenon may not be an appropriate description of physical reality. Secondly, the peculiarities of the analysed system, such as the geometry of the domain, and the physical parameters, as well as the boundary conditions that are difficult to measure accurately in the field. In general, the numerical simulation provides different results from those observed. For this reason, numerical models need to be calibrated to better reproduce the observed factors. The calibration is the process of changing the physical parameters and/or boundary conditions of a mathematical model until the calculated solutions correspond fairly well to the observed variables. The problem of identifying physical parameters is called the inverse problem. The state variables of a groundwater system can be measured. These measurements, such as water levels are usually obtained from wells, boreholes, or by consulting documents from previous measurement campaigns. Once the state variables are available, the calibration can be done by adjusting the state variables calculated by the model to the observed variables. The quality of the simulation is judged by the difference between the calculated and observed values. But we have to be careful, because we can run into the problem of non-uniqueness solution. Which means that several combinations of physical parameters and boundary conditions can lead us to the same calibration quality. An inverse problem is an optimization problem, usually solved by coupling the direct simulation model with an appropriate optimization technique. The optimization technique iteratively generates the parameters of the model, then injects them into the simulation model to calculate the state variables.

In this chapter, a recent metaheuristic algorithm called 'Crow Search Algorithm (CSA) ' is adopted as an optimization technique linked to a model of the finite elements to estimate the tensors of hydraulic conductivity of confined aquifers.

To test the effectiveness of the proposed approach, several hypothetical cases will be analysed. The aim is to develop an optimization framework based on a CSA/FEM coupling, for the identification of physical parameters relating to the cases dealt with.

3.2 Well-posed and ill-posed problem

The importance of solving the inverse problems is obvious. Although direct simulation accurate results, if the physical parameters and boundary conditions used in the model are not correct, the results obtained will not be reliable.

Therefore, the resolution of inverse problems is indeed a crucial step in modeling underground flows. Unfortunately, as we will see in the following, there are several essential difficulties related to the inverse problems (Tadj 2019).

Hadamard (1902) introduced the notion of a well-posed problem. This is a problem for which:

- a solution exists;
- the solution is unique;
- the solution is continuously dependent on the data.

A problem that does not meet at least one of these conditions, is called ill-posed problem. Direct Models results are always stable, as they are based on the solution of an PDE, however, the inverse problems often tend to be unstable (ill-posed).

The inverse problems are generally characterized by the non-uniqueness and instability of the parameters identified. The instability of the inverse solution comes from the fact that small errors in the observed state variables will cause large errors in the parameters identified.

Intuitively, the existence of an inverse solution does not seem to be a problem, since physical reality is a solution. In practice, despite that, the observation error (or the noise) of the state variables cannot be avoided. Consequently, an exact solution to an inverse problem may not exist (Tadj 2019).

Figure 3.1 represent the diret problem and its inverse . *f* is the operator of the direct problem.

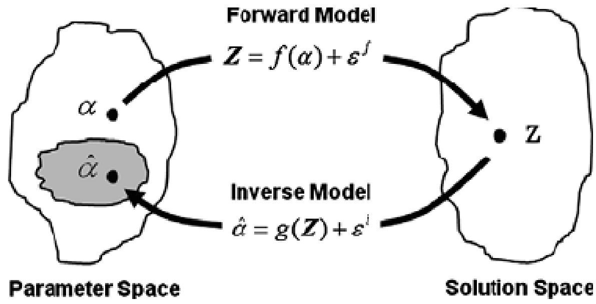


Figure 3.1: The direct and the inverse problems (Ayvaz et al. 2007).

ε^f is the error made by the numerical simulation technique or an error in measuring the state variable. The operator *g*, is the operator of the inverse problem, with, *i* is the inversion error. When $(\varepsilon^f, \varepsilon^i) \rightarrow 0$, the values of the identified parameters through an optimization technique, approaches the values of the used parameters in the direct simulation (Case of a synthetic problem) or the values of the actual parameters for a real case. $(\alpha = \hat{\alpha})$ (McLaughlin and Townley 1996; Ayvaz and al. 2007). Marsily and al. (1992) have indicated in regard to the use of mathematical models in groundwater that the parameters of a model are uncertain, probably wrong in many cases and can easily be invalidated, but we can use them to make predictions, as long as they reproduce the observed behaviour of the system.

3.3 Parameterization

The problem of identifying spatially distributed parameters in systems has been studied over the past few years. The parameters are identified by observing the state variables collected in the flow area space. The number of observations is limited, while the dimension of the space domain is infinite (continue); for a heterogeneous medium, the size of physical parameters is theoretically infinite. The PDE governing the studied phenomenon is reduced to a set of algebraic equations when solved by the finite element method. State variables and physical parameters all reduce to finite-dimensional vectors. The state variables will be associated with the nodes and the physical parameters with the different sets of elements in the domain. The dimension of the parameters must be reduced into a finite set of sub-domains; this is possible using the zonation technique. The flow domain is divided into several sub-domains or zones, and a parameter constant value is assigned to characterize each zone (Tadj 2019).

3.4 Calibration of numerical models

The difference between the observed and simulated values is one of the indicators of the model quality; the objective function is used to quantify this difference. The purpose of calibration is to minimize a given objective function (Tadj 2019).

3.4.1 Trial and error setting

The trial-and-error method is the simplest technique to solve an inverse problem, for example, for the calibration of an underground flow model, we need:

- Some measurements of the state variable (piezometric heads);
- A direct simulation program;
- An experienced hydrogeologist with knowledge of the aquifer in question.

Figure 3.2 shows the trial-and-error calibration procedure. Steps 2, 4, 5, 6 are performed by an expert. This person can provide a good initial estimate of the model parameters. After analysing the simulation results, the expert knows how to modify the model parameters for a better fit between the observed state variables and those calculated, this step is repeated until an agreement is deemed satisfactory.

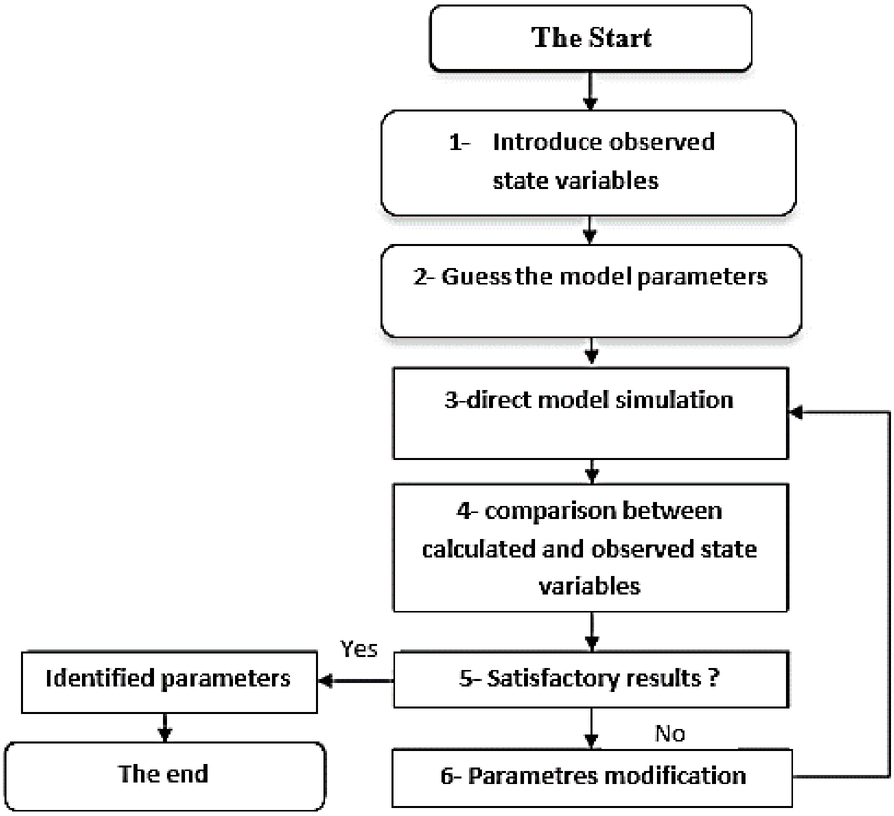


Figure 3.2: Trial and error calibration flowchart (Tadj 2019)

If a satisfactory fit cannot be obtained, modification of the mathematical model should be considered. Among the advantages of this manual technique is that there is no need for an optimisation technique for inversion. Furthermore, it can be used to solve any type of inverse problem. Expert judgement can be incorporated into the search procedure, which can help overcome the instability problem, which means when two sets of parameters generate similar state variables, the expert can select one set of parameters based on his experience. Because of these advantages, this primitive method is still widely used in practice, despite its slow execution.

3.4.2 Automatic calibration

The automatic calibration of numerical models consists of incorporating the simulation model into an optimization scheme. While the simulation model calculates the state variables, the optimization technique is responsible for the quality of the parameters to be identified. The flowchart of automatic calibration is shown in Figure 3.3.

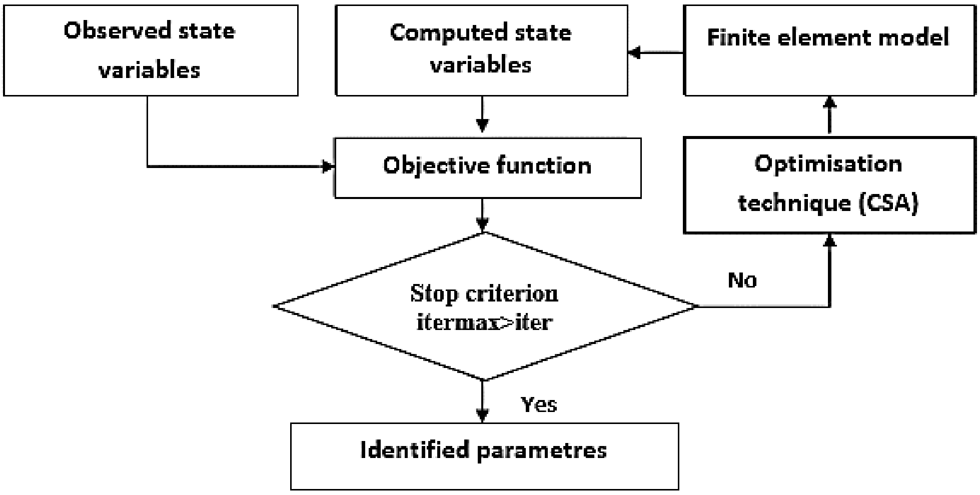


Figure 3.3: Simulation–optimization linkage (CSA / FEM) (Tadj 2019).

The success of parameter identification depends mainly on the performance of the adopted optimisation technique with which the simulation model is coupled. But it should not be forgotten that the quantity and quality of the observed state variables play a major role in the identifiability of the parameters. In this study we used a metaheuristic algorithm called 'crow search algorithm' which is known for its ability to solve complex problems, and its real coding makes it suitable for continuous optimization. The objective function to be minimized *f*(x) is the sum of squared errors (SSE) between the observed state variables and those computed by the considered model, x is the vector of parameters to be identified comprised between the bounds of the parameter search spaces x_i^L et x_i^U.

3.5 Simulation models

The finite element method was chosen to solve the PDE of the considered problems, namely: hydraulic conductivities identification for heterogeneous confined aquifers. We present in the following the mathematical model used in this work, which consists of (Tadj 2019):

(1) PDE governing the phenomenon, in our case it is an elliptic PDE;

(2) the flow domain;

(3) boundary conditions.

Note that the finite element simulation codes developed by Smith et al. (2014) in Fortran, have been modified to meet our technical requirements.

3.5.1 Permanent flows in porous media

Steady-state two-dimensional (2D) flows in heterogeneous and anisotropic porous media, such as flows in aquifers or through earth dams, are governed by an elliptic partial differential equation (Eq. 3.1), called the Laplace equation when the Q term is zero, or the Poisson equation otherwise (Tadj 2019).

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial h}{\partial x}\right) + \left(k_y \frac{\partial h}{\partial y}\right) = Q \qquad (3.1)$$

Where: k_x (*x*, *y*) and k_y (*x*, *y*) [L/T] are the diagonal elements of the permeability tensor, *h* (*x*,*y*) [L] is the hydraulic charge (head), *Q* [L^3/T] is the input (+) or output (-) term. For the uniqueness of the solution, equation 3.1 must be subjected to boundary conditions (BC), (Figure 3.4).

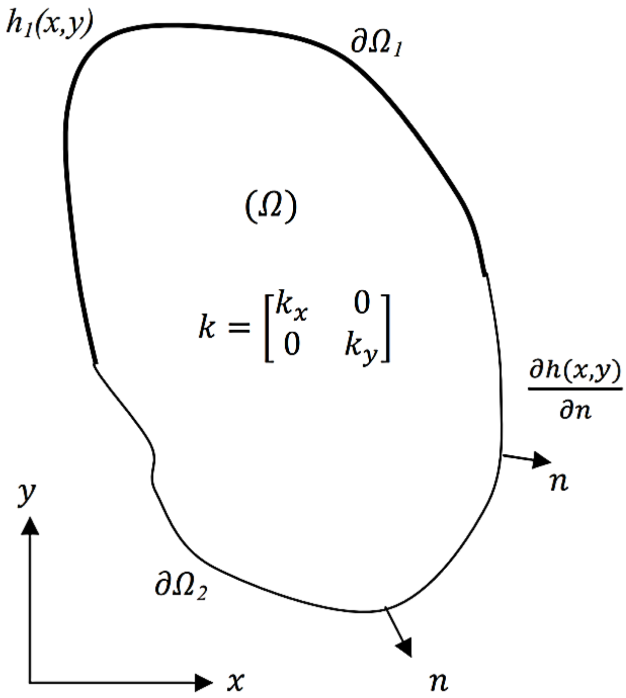


Figure 3.4: Steady-state flow domain in porous media, with boundary conditions (Tadj 2019).

Dirichlet BC: $h(x, y) = h_1(x, y) \quad x,y \in \partial\Omega_1$

Neumann BC: $\frac{\partial h(x,y)}{\partial n} = q(x, y) \quad x,y \in \partial\Omega_2$

With $\partial\Omega$ is the boundary of the flow domain (Ω), $\partial\Omega = \partial\Omega_1 \cup \partial\Omega_2$, h_1(x, y) is the hydraulic head (state variable) imposed on the boundary ∂Ω1, and *q* (*x*, *y*) represents the flow imposed on the boundary $\partial\Omega_2$ (Tadj 2019).

3.5.2 Finite element discretization

The finite element method is currently widely used to solve hydraulic problems. It was first used in the early 1970s to solve groundwater flow and solute transport problems (Istok 1989).

Solving a 2D problem by finite elements, consists in replacing the continuous system by an equivalent discrete system, in this case the flow domain (Ω) must be meshed. The result will therefore be a set of nodes and elements. There are different types of two-dimensional elements, namely: quadrilateral element (linear four-nodes Q4; quadratic eight-nodes Q8 and cubic twelve-node Q12) and triangular element (linear three-nodes T3; quadratic six-nodes T6 and cubic nine-nodes T9). It is up to the user to choose the appropriate type of element for their analysis, as there is no general rule for choosing the type of element. As for the size of the element, it should be smaller than the elementary representative volume of the domain, and refinement is always desirable in complex geometries and in areas of high gradients. In this study, quadrilateral elements with four nodes (Q4) were used. The values of physical parameters such as hydraulic conductivities are generally assumed to be constant in each element, but may vary from element to element. The state variables (hydraulic heads) are calculated at the nodes. The finite element formulations of the PDEs employed in this work were obtained using Galerkine's weighted residual method (Smith et al. 2014; Hutton 2004).

The finite element discretisation process reduces the PDE (3.1) to a system of linear equations of the form:

$$[K_c]\{h\} = \{q\} \tag{3.2}$$

Where $[K_c\ ]$ is the global conductivity matrix, $\{h\}$ is the global column vector of the nodal hydraulic heads and $\{q\}$ represents the column vector of the incoming or outgoing nodal flows (Tadj 2019).

3.6 Applications

In this section, we test the efficiency of the crow search algorithm (CSA) presented in the previous chapter to identify physical parameters of the partial differential equations governing the considered problems. The examples treated in this work are synthetic in order to eliminate the sources of uncertainty due to measurements and to the model choice. Direct finite element simulations with different sets of physical parameters provided the inversion measures, which are the hydraulic heads. These inversion measurements from the direct numerical simulations then represent observed variables without error, and their use allows us to test the ability of our programs to identify the physical parameters that were used to generate these variables. The measurements were then perturbed with Gaussian noise of mean μ and standard deviation (STD) σ, to check the stability of the identification process. The characteristics of CSA used in this chapter are the following:

Flight length *fl* =2, Awareness probability *AP*=0.1, Population size *N*= 60.

3.6.1 Tensor of hydraulic conductivities identification for synthetic aquifers

Three steady-state flow problems in heterogeneous and anisotropic confined aquifers were chosen.

Problem 1: The first problem is of a regular geometric form with known Dirichlet boundary conditions, also with 2 zones, $iter_{max}$=5000, 4 observed points). Figure (3.5) shows the flow domain of this case.

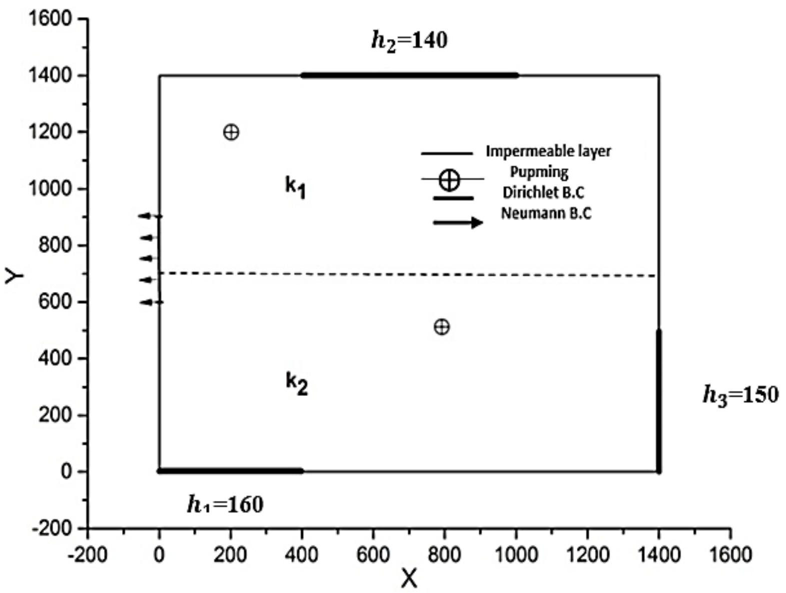


Figure 3.5: Flow domain of a synthetic aquifer of regular shape (Known BC, problem 1).

Problem 2 : The second problem is more complex, with an irregular shape and unknown boundary conditions , with 4 zones, $iter_{max}$=20000, 47 observed points, bounds of the search space is represented on table 1).

Table3.1: Bounds of the search space

Parameters	Lower bound x^L	Upper bound x^U
Hydraulic conductivity	1.00E-10	1.00E-01
Heads (BC)	1.00E-10	300.

The flow domain of problem 2 is shown in figure (3.6)

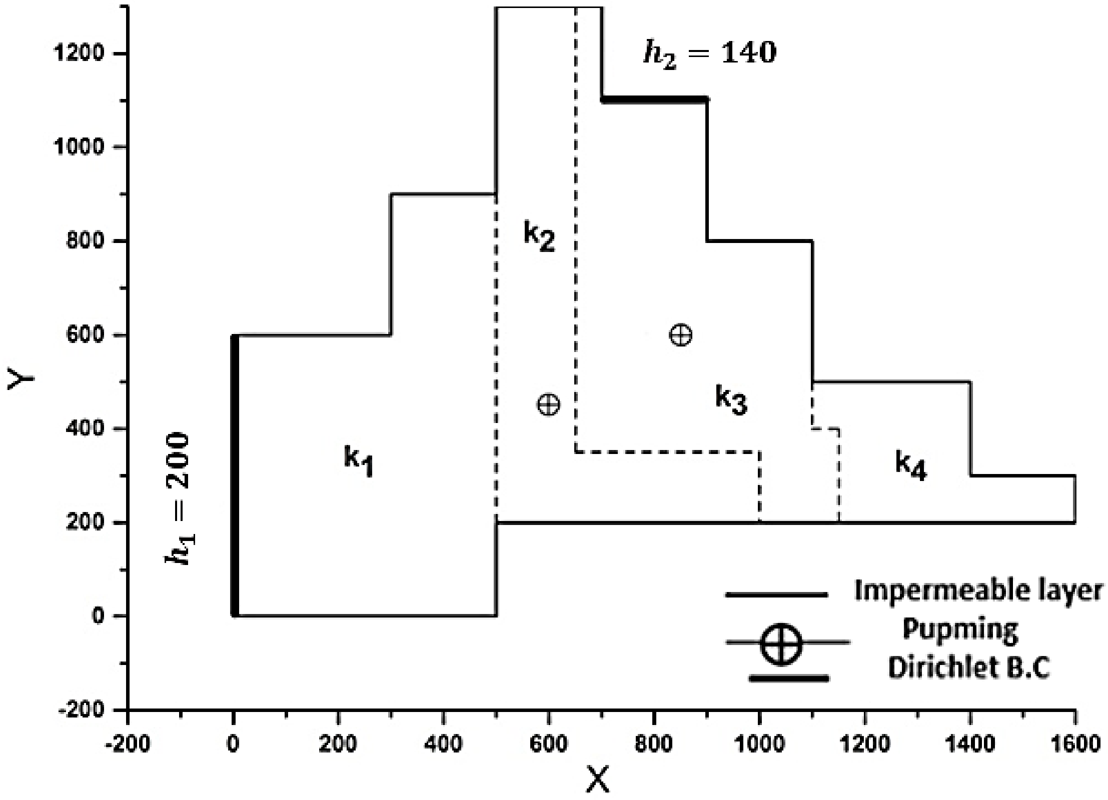


Figure 3.6: Flow domain of an irregularly shaped synthetic aquifer (Unknown BC, problem 2).

Problem 3: The third problem is almost with same characteristic of problem 2, only with a different shape and number of the observed points (60 points), Bounds of the search space are as well represented on table 1.

Figure 3.7 represent the flow domain of this aquifer

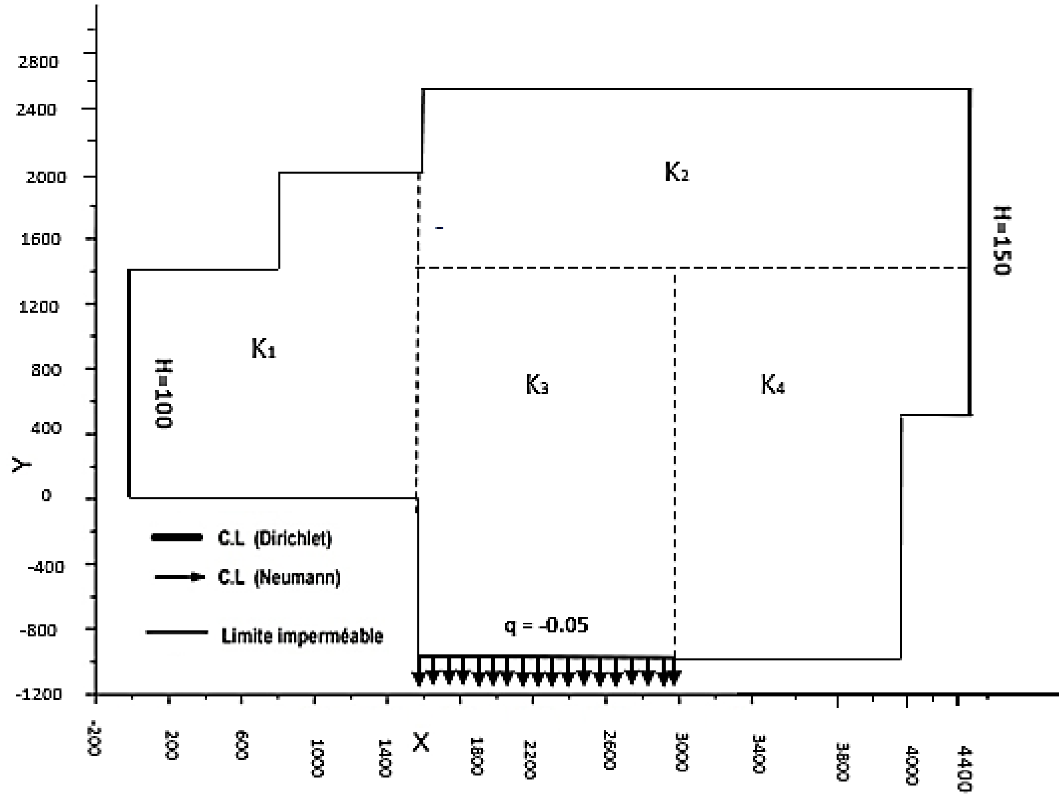


Figure 3.7: Flow domain of an irregularly shaped synthetic aquifer (Unknown BC conditions, problem 3).

The real (synthetic) physical parameters used in the direct simulation phase to generate the observed results and the parameters estimated by inversion of the three groundwater flow problems are detailed in tables (3.2), (3.3), (3.4).

Concerning the first problem with known boundary conditions, the hydraulic conductivities were successfully identified even for a limited number of observation points (4 points),in addition to this the identification process using CSA remained stable when the measurements were perturbed by white Gaussian noise of different magnitudes(σ=0.05,σ=0.1). As we see as well, the more the noise increase, the relative errors rise.

Table 3.2: Tensor hydraulic conductivities identification (Problem 1, CSA/FEM).

Noise	Search space	Number of Observed points	Actual parameters (Synthetic) (m/s)	Identified Parameters by CSA (m/s)	Relative errors (%)
μ=0. σ=0. h±0. m	$k_i \in [10^{-10},0.1]$	4	$k_{1x} = 0.02$ $k_{1y} = 0.05$ $k_{2x} = 0.06$ $k_{2y} = 0.01$	$k_{1x} = 0.0201$ $k_{1y} = 0.0496$ $k_{2x} = 0.0603$ $k_{2y} = 0.0101$	0.365 0.842 0.442 0.935
μ=0. σ=0.05 h±0.05m	$k_i \in [10^{-10}, 0.1]$	4	$k_{1x} = 0.02$ $k_{1y} = 0.05$ $k_{2x} = 0.06$ $k_{2y} = 0.01$	$k_{1x} = 0.0197$ $k_{1y} = 0.0549$ $k_{2x} = 0.0559$ $k_{2y} = 0.0088$	1.13 9.84 6.80 12.0
μ=0. σ=0.1 h±0.1m	$k_i \in [10^{-10}, 0.1]$	4	$k_{1x} = 0.02$ $k_{1y} = 0.05$ $k_{2x} = 0.06$ $k_{2y} = 0.01$	$k_{1x} = 0.0207$ $k_{1y} =0.0435$ $k_{2x} =0.0645$ $k_{2y} = 0.0118$	3.80 12.9 7.60 18.5

The convergence curves of the objective function of problem 1 are shown in the following figure.

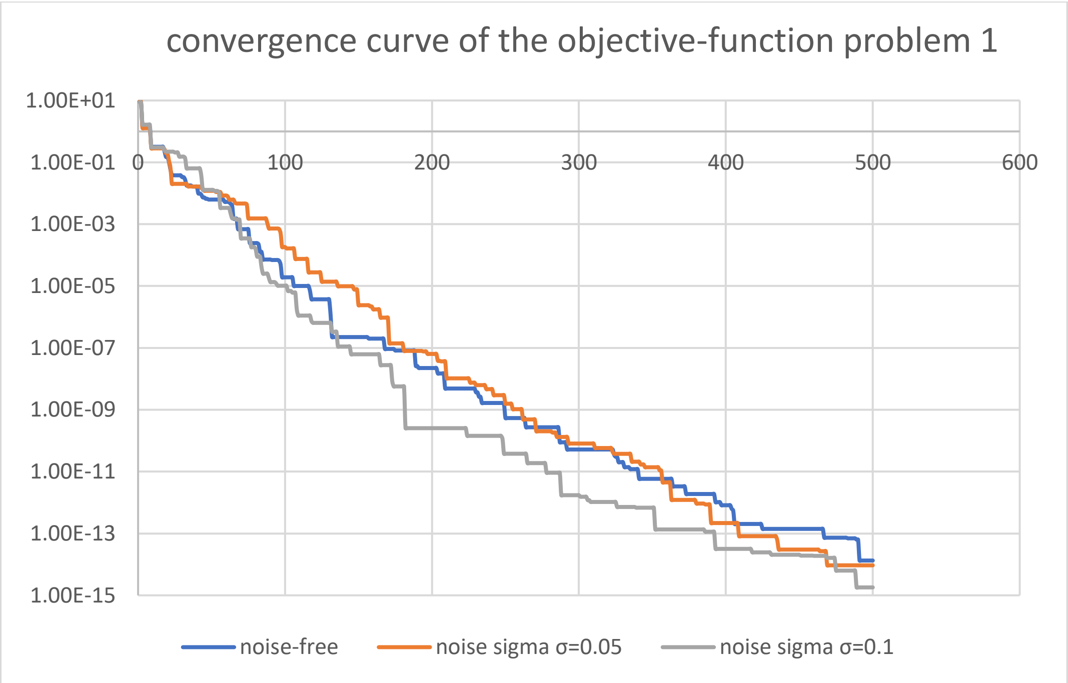


Figure 3.8: The convergence curves of problem 1.

In regards to second and third problem, the two has unknown boundary conditions thus, we had 10 parameters to be identified. The crow search algorithm (CSA), obtained promising results and was able to identify with high accuracy the Dirichlet boundary conditions as well as the hydraulic conductivities with a maximum relative error of 11.1% for a noise of (μ=0, σ=0.1) for the second problem, and concerning (problem 3) maximum relative error of 7.23% for a noise of (μ=0, σ=0.05).

Table 3.3: Tensor hydraulic conductivities identification (Problem 2, CSA/FEM).

Noise	Search space	Number of Observed points	Actual parameters (Synthetic) (m/s)	Identified Parameters by CSA (m/s)	Relative errors (%)
μ=0. σ=0. h±0.m	$k_i \in [10^{-10,}$ 0.1] h ∈ [0, 300.]	47	k_{1x} = 0.002	k_{1x} = 0.001998	0.0576
			k_{1y} = 0.006	k_{1y} = 0.00599	0.117
			k_{2x} = 0.008	k_{2x} = 0.008008	0.100
			k_{2y} = 0.004	k_{2y} = 0.00399	0.0444
			k_{3x} = 0.01	k_{3x} = 0.00996	0.341
			k_{3y} = 0.001	k_{3y} = 0.000997	0.207
			k_{4x} = 0.003	k_{4x} = 0.00299	0.130
			k_{4y} = 0.005	k_{4y} = 0.00506	1.28
			h_1= 200. m	h_1= 199.999m	0.0000392
			h_2= 140. m	h_2= 139.994m	0.00373
μ=0. σ=0 h±0.05 m	$k_i \in [10^{-10}$, 0.1.] h ∈ [0., 300.]	47	k_{1x} = 0.002	k_{1x} = 0.00199	0.168
			k_{1y} = 0.006	k_{1y} = 0.00597	0.372
			k_{2x} = 0.008	k_{2x} = 0.00797	0.324
			k_{2y} = 0.004	k_{2y} =0.00393	1.64
			k_{3x} = 0.01	k_{3x} = 0.00940	5.98
			k_{3y} = 0.001	k_{3y} = 0.00101	1.69
			k_{4x} = 0.003	k_{4x} = 0.00308	2.93
			k_{4y} = 0.005	k_{4y} = 0.00505	1.01
			h_1= 200. m	h_1= 200.004m	0.0025
			h_2= 140. m	h_2= 139.982m	0.0122
μ=0. σ=0 h±0.1m	$k_i \in [10^{-10}$, 0.1.] h ∈ [0., 300.]	47	k_{1x} = 0.002	k_{1x} = 0.00199	0.265
			k_{1y} = 0.006	k_{1y} = 0.005953	0.771
			k_{2x} = 0.008	k_{2x} = 0.00795	0.560
			k_{2y} = 0.004	k_{2y} =0.0038636	3.41
			k_{3x} = 0.01	k_{3x} = 0.00888	11.1

			k_{3y} = 0.001	k_{3y} = 0.001039	0.393
			k_{4x} = 0.003	k_{4x} = 0.00316	5.63
			k_{4y} = 0.005	k_{4y} =0.0050857	1.71
			h_1= 200. m	h_1= 200.00430m	0.00215
			h_2= 140 m	h_2= 139.9888 m	0.00797

The convergence curves of the objective function of problem 2 are shown in the following figure

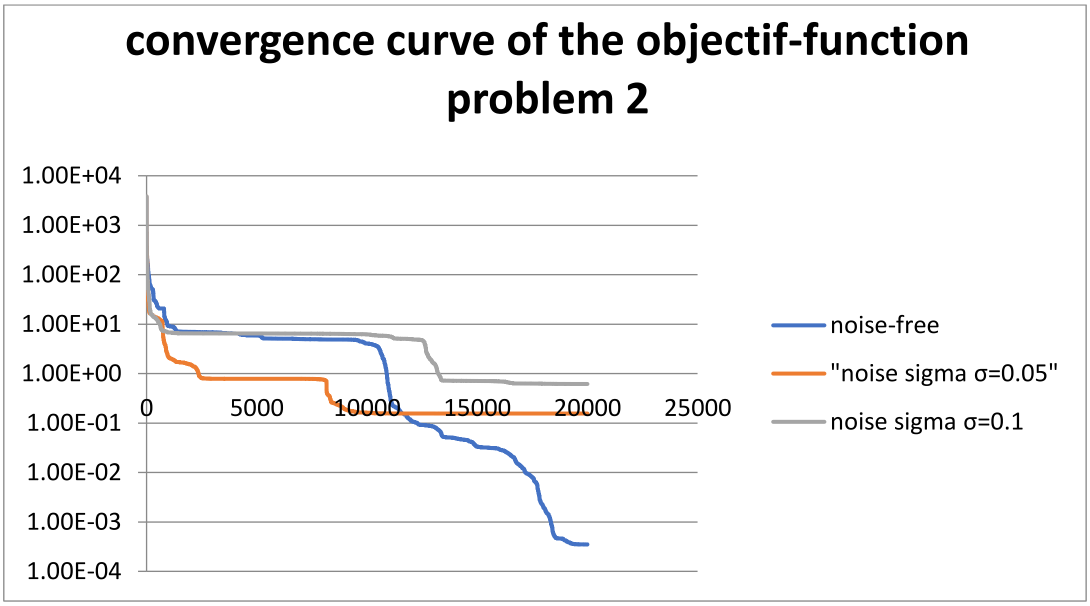


Figure 3.9: The convergence curves of problem 2

Table 3.4: Tensor hydraulic conductivities identification (Problem 3, CSA/MEF).

Noise	Search space	Number of Observed points	Actual parameters (Synthetic) (m/s)	Identified Parameters by CSA (m/s)	Relative errors (%)
μ=0. σ=0. h±0.m	$k_i \in [10^{-10}, 0.1]$ h ∈ [0, 300.]	60	k_{1x} = 0.01	k_{1x} = 0.01005	0.562
			k_{1y} = 0.02	k_{1y} = 0.02004	0.202
			k_{2x} = 0.03	k_{2x} =0.03012	0.433
			k_{2y} = 0.04	k_{2y} = 0.0403	0.851
			k_{3x} = 0.05	k_{3x} = 0.05009	0.199
			k_{3y} = 0.06	k_{3y} = 0.0600008	0.00137
			k_{4x} = 0.07	k_{4x} = 0.07004	0.0583
			k_{4y} = 0.08	k_{4y} = 0.08005	0.0741
			h_1= 100. m	h_1= 99.961m	0.0390
			h_2= 150. m	h_2= 149.9993m	0.000431
			k_{1x} = 0.01	k_{1x} = 0.0097	2.50
			k_{1y} = 0.02	k_{1y} = 0.0185	7.23
			k_{2x} = 0.03	k_{2x} = 0.02938	2.06

μ=0. σ=0 h±0.05m	$k_i \in [10^{-10}, 0.1.]$ h ∈ [0., 300.]	60	k_{2y} = 0.04	k_{2y} =0.040096	0.241
			k_{3x} = 0.05	k_{3x} = 0.050003	0.00696
			k_{3y} = 0.06	k_{3y} = 0.05958	0.695
			k_{4x} = 0.07	k_{4x} = 0.0696	0.555
			k_{4y} = 0.08	k_{4y} = 0.0798	0.167
			h_1= 100. m	h_1= 99.9656m	0.0344
			h_2= 150. m	h_2= 150.01754m	0.0117
μ=0. σ=0 h±0.1m	$k_i \in [10^{-10}, 0.1.]$ h ∈ [0., 300.]	60	k_{1x} = 0.01	k_{1x} = 0.01010	1.03
			k_{1y} = 0.02	k_{1y} = 0.01929	3.52
			k_{2x} = 0.03	k_{2x} = 0.0297	0.803
			k_{2y} = 0.04	k_{2y} = 0.04062	1.57
			k_{3x} = 0.05	k_{3x} = 0.04956	0.877
			k_{3y} = 0.06	k_{3y} = 0.0599	0.0529
			k_{4x} = 0.07	k_{4x} = 0.07058	0.829
			k_{4y} = 0.08	k_{4y} = 0.07832	2.09
			h_1= 100. m	h_1= 100.0968m	0.0969
			h_2= 150. m	h_2= 149.9986m	0.000804

The convergence curves of the objective function of problem 2 are shown in the following figure

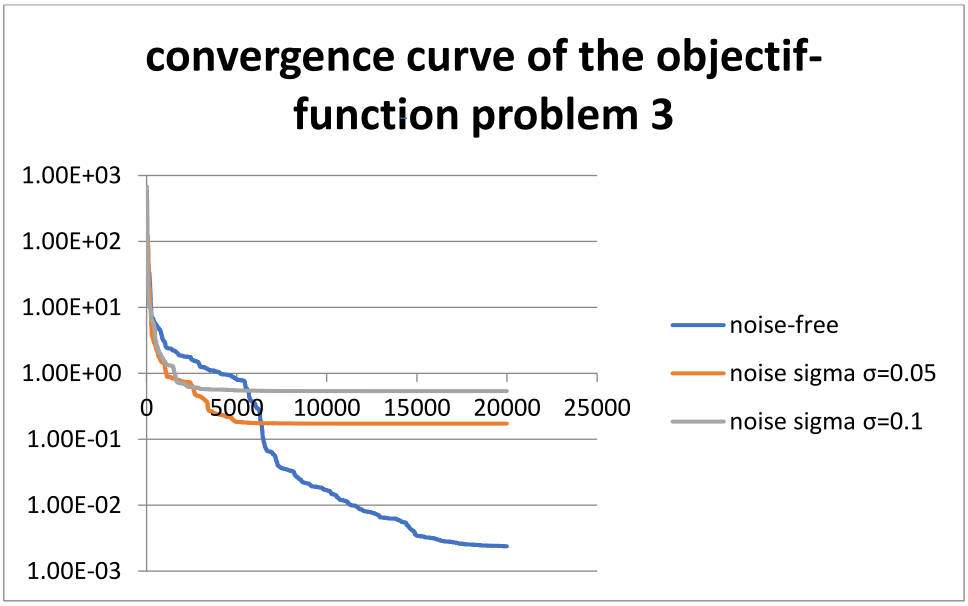


Figure 3.10: The convergence curves of problem 3

3.7 Conclusion

In this chapter, direct models simulating flows through porous media were inverted using a CSA/MEF coupling to identify their physical parameters and/or boundary conditions. The physical parameters were successfully identified thanks to the ability of the CSA used to locate the global optimum, unlike classical optimisation methods where the solution is getting stuck into a local optimum. The computational codes have been successfully validated for the synthetic problems that we used as Benchmarks, even in the presence of noise that simulates the errors made during the measurement of state variables. The coupling between deterministic methods (in this case finite elements) and techniques from artificial intelligence (CSA) has shown its effectiveness in this work.

General conclusion

General conclusion

The knowledge of physical parameters and boundary conditions is an essential and indispensable step in the construction of a reliable numerical groundwater flow model.

The process of identifying these parameters by measuring the state variables is called the inverse problem, which is usually difficult for deterministic optimisation techniques.

In this study, we have tried to take advantage of the benefits offered by artificial intelligence techniques, in this case, metaheuristics for the solution of inverse problems in hydraulics field.

A metaheuristic algorithm was used: called Crow search algorithm based on the cleverness of crows. The metaheuristic was coupled with finite element models of elliptic partial differential equations. These coupling were then used to solve the inverse problems of permanent flows in heterogeneous and anisotropic porous media. three synthetic cases have been used to test this metaheuristic algorithm, the first case: the permanent flow in a rectangular geometrical aquifer form with known Dirichlet boundary conditions. In addition, two other cases with irregular geometrical aquifers form and unknown boundary conditions. The measurements were then perturbed with Gaussian noise of mean μ and standard deviation (STD) (σ =0.05, σ=0.1)., to check the stability of the identification process.
The codes that we developed in Fortran 95 programming language, allowed us to successfully identify the physical parameters related to the treated problems, namely:

- Identification of hydraulic conductivity tensors;

- Simultaneous identification of hydraulic conductivity tensors and boundary conditions;

The algorithm has successfully identified the hydraulic conductivities of the three cases, and the boundary conditions of case two and three, with a really small relative error.

References

References

Arregui-Mena, J. D., Margetts, L., & Mummery, P. M. (2014). Practical application of the stochastic finite element method. Archives of Computational Methods in Engineering, 23(1), 171-190.

Asef, F., Majidnezhad, V., Feizi-Derakhshi, M. R., & Parsa, S. (2021). Heat transfer relation-based optimization algorithm (HTOA). Soft Computing, 1-30.

Ayvaz, M. T., Karahan, H., & Aral, M. M. (2007). Aquifer parameter and zone structure estimation using kernel-based fuzzy c-means clustering and genetic algorithm. Journal of Hydrology, 343(3-4), 240-253.

Bailly, D. (2009). Vers une modélisation des écoulements dans les massifs très fissurés de type karst: étude morphologique, hydraulique et changement d'échelle (Doctoral dissertation).

Batu, V. (1998). Aquifer hydraulics: a comprehensive guide to hydrogeologic data analysis. John Wiley & Sons.

Bear, J. (1972). Dynamics of fluids in porous media. Courier Corporation.

Bear, J. (1979). Hydraulics of groundwater. Courier Corporation.

Beni, G., & Wang, J. (1993). Swarm intelligence in cellular robotic systems. In Robots and biological systems: towards new bionics? (pp. 703-712). Springer, Berlin, Heidelberg.

Bonabeau, E., Marco, D. D. R. D. F., Dorigo, M., Théraulaz, G., & Theraulaz, G. (1999). Swarm intelligence: from natural to artificial systems (No. 1). Oxford university press.

Bonnet, M. (1982). Méthodologie des modèles de simulation en hydrogéologie. DOCUMENTS BRGM, (34).

Brownlee, J. (2011). Clever algorithms: nature-inspired programming recipes. Jason Brownlee.

Chakri, A., Ragueb, H., & Yang, X. S. (2018). Bat algorithm and directional bat algorithm with case studies. In Nature-Inspired Algorithms and Applied Optimization (pp. 189-216). Springer, Cham.

Craveur, J. C., Bruyneel, M., & Gourmelen, P. (2014). Optimisation des structures mécaniques : Méthodes numériques et éléments finis. Dunod.

Dhiman, G., & Kumar, V. (2018). Emperor penguin optimizer: a bio-inspired algorithm for engineering problems. Knowledge-Based Systems, 159, 20-50.

Dorigo, M., & Birattari, M. (2010). Ant colony optimization: overview and recent advances. Handbook of metaheuristics, 311-351.

Fetter, C. W. (2018). Applied hydrogeology. Waveland Press.

Fogel, L. J., Owens, A. J., & Walsh, M. J. (1966). Artificial intelligence through simulated evolution.

Galinier, P., Hamiez, J. P., Hao, J. K., & Porumbel, D. (2013). Handbook of optimization.

Goel, L. (2020). An extensive review of computational intelligence-based optimization algorithms: trends and applications. Soft Computing, 24, 16519-16549.

References

Gurarslan, G., & Karahan, H. (2015). Solving inverse problems of groundwater-pollution-source identification using a differential evolution algorithm. Hydrogeology Journal, 23(6), 1109-1119.

Hantush, M. S. (1964). Hydraulics of wells. Advances in hydroscience, 1, 281-432.

Haque, M. I. (2015). Mechanics of groundwater in porous media. CRC Press, Taylor & Francis Group.

Hussien, A. G., Amin, M., Wang, M., Liang, G., Alsanad, A., Gumaei, A., & Chen, H. (2020). Crow search algorithm: theory, recent advances, and applications. IEEE Access, 8, 173548-173565.

Istok, J. (1989). Finite Element Computer Programs. Groundwater Modeling by the Finite Element Method, 13, 255-302.

Janga Reddy, M., & Nagesh Kumar, D. (2020). Evolutionary algorithms, swarm intelligence methods, and their applications in water resources engineering: a state-of-the-art review. H2Open Journal, 3(1), 135-188.

Janga Reddy, M., & Nagesh Kumar, D. (2021). Evolutionary algorithms, swarm intelligence methods, and their applications in water resources engineering: a state-of-the-art review. H2Open Journal, 3(1), 135-188.

Karahan, H., & Ayvaz, M. T. (2008). Simultaneous parameter identification of a heterogeneous aquifer system using artificial neural networks. Hydrogeology Journal, 16(5), 817-827.

Maxime,N., (2003) Ecoulements dans les milieux poreux, thèse DEA mécanique énergétique, Ecole Doctorale Mécanique, Physique et Modélisation Université de Provence,Marseille.

McLaughlin, D., & Townley, L. R. (1996). A reassessment of the groundwater inverse problem. Water Resources Research, 32(5), 1131-1161.

Mitchell, M. (1996). An introduction to genetic algorithms. Cambridge.

Okwu, M. O., & Tartibu, L. K. (2020). Metaheuristic Optimization: Nature-Inspired Algorithms Swarm and Computational Intelligence, Theory and Applications (Vol. 927). Springer Nature.

Tadj, W. (2019). Apports de l'intelligence artificielle au calage automatique des modèles numériques en hydraulique. Ph.D thesis. University of Laghouat.

Rani, D., & Moreira, M. M. (2010). Simulation–optimization modeling: a survey and potential application in reservoir systems operation. Water resources management, 24(6), 1107-1138.

Reddy, M. J., & Kumar, D. N. (2012). Computational algorithms inspired by biological processes and evolution. Current Science, 370-380.

Slowik, A. (Ed.). (2021). Swarm Intelligence Algorithms (Two Volume Set). CRC Press.

Smaoui, H., Zouhri, L., Kaidi, S., & Carlier, E. (2018). Combination of FEM and CMA-ES algorithm for transmissivity identification in aquifer systems. Hydrological Processes, 32(2), 264-277

References

Sun, N. Z., & Sun, A. (2015). Model calibration and parameter estimation: for environmental and water resource systems. Springer.

Wolpert, D. H., & Macready, W. G. (1997). No free lunch theorems for optimization. IEEE transactions on evolutionary computation, 1(1), 67-82.

Yang, X. S. (2010). A new metaheuristic bat-inspired algorithm. In Nature inspired cooperative strategies for optimization (NICSO 2010) (pp. 65-74). Springer, Berlin, Heidelberg.

Yang, X. S. (2014). Nature-inspired optimization algorithms. Academic Press.

Yang, X. S., Deb, S., Zhao, Y. X., Fong, S., & He, X. (2018). Swarm intelligence: past, present and future. Soft Computing, 22(18), 5923-5933.

Zouhri, L., Gorini, C., Deffontaines, B., & Mania, J. (2004). Relationships between hydraulic conductivity distribution and synsedimentary faults, Rharb-Mamora basin, Morocco; Hydrogeological, geostatistical and modeling approaches. Hydrogeology Journal, 12(5), 591-600.