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Thème

Grey Wolf Optimizer for estimating confined and leaky aquifers parameters from transient time-drawdown data

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Promotion : 2020

Dedication

I dedicate this modest work to:

To my parents that I can never thank enough for all the help they gave me,

To all my brothers,

To all my colleagues in the 2019/2020 class without exception,

To all my friends,

To all my family members,

To all the people I know and have not mentioned.

Thanks

Above all, I wish to thank God for my support and for allowing me to complete this work.

I would like to thank my coach, **Mr.Tadj walid**, who spared no time and his efforts to guide me, for his objective remarks, his uninterrupted incentives and the pleasant working climate it creates.

I thank from the bottom of my heart and with great love my parents who never stopped believing in me during all my years of study.

Thanks also to my sisters and brothers, and to the whole family that always has me Encouraged.

My thanks also go to my friends who encouraged me to Carry out this work.

I thank the members of instructors **Mr. Chettih Mohammed** and **Mr. Bouach Mohammed**.

ملخص

إن معرفة خصائص طبقات المياه الجوفية مهمة في كثير من التطبيقات الهندسية للمياه الجوفية، عادة ما تكون هذه الخصائص مستمدة بشكل سيء من بيانات اختبارات الضخ من خلال المطابقة اليدوية للمنحنى.

في هذا العمل تم اقتراح طريقة تحليل تلقائي لبيانات اختبارات الضخ لطبقات المياه الجوفية والمحصورة ، وذلك من خلال الربط بين خوارزمية الذئب الرمادي واثنين من نماذج التحليل لإنخفاض سطح المياه الجوفية حسب طبقة الخزان الجوفي.

لقد قمنا باختبار النهج المقترح على ثلاث مجموعات بيانات لإنخفاض سطح المياه الجوفية ، وقارنا النتائج التي تم الحصول عليها بالنتائج التي حصلت باستخدام المطابقة اليدوية للمنحنى، النتائج التي تم الحصول عليها دقيقة وتتجاوز إلى حد بعيد النتائج التي تم الحصول عليها باستخدام المطابقة اليدوية للمنحنى.

Abstract

The knowledge of aquifer parameters is important in many groundwater engineering applications. Usually, these parameters are poorly derived from time-drawdown data by manual curve matching. In this study, we propose an automatic approach for estimating confined and leaky aquifers parameters from time-drawdown data. We linked the Grey Wolf Optimizer (GWO) with two analytical drawdown solutions, the Theis solution for analyzing time-drawdown data coming from confined aquifers, and Hantush and Jacob solution for those coming from leaky aquifers. We tested the proposed approach on three time-drawdown datasets, and we compared the obtained results with those obtained by using the manual curve matching. The obtained results are accurate and exceed by far those obtained by using the manual curve matching.

Résumé

La connaissance des paramètres de l’aquifère est importante dans de nombreuses applications d’ingénierie des eaux souterraines. Habituellement, ces paramètres sont mal dérivés des données de temps-rabattement par correspondance manuelle de courbe. Dans ce travail, nous proposons une approche automatique pour l’estimation des paramètres des aquifères captif et des aquifères semi-captif à partir des données de temps-rabattement. Nous avons couplé le Grey Wolf Optimizer (GWO) avec deux solutions analytiques de temps-rabattement, la solution de Theis pour analyser les données de temps-rabattement provenant d’aquifère captif, et la solution Hantush et Jacob pour ceux provenant d’aquifère semi-captif. Nous avons testé l’approche proposée sur trois séries des données temps-rabattement, et nous avons comparé les résultats obtenus avec ceux obtenus à l’aide de la correspondance manuel des courbes. Les résultats obtenus sont exacts et dépassent de loin ceux obtenus en utilisant la correspondance manuelle de la courbe.

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Introduction

Groundwater is the main source of drinking water for about 50% of worldwide population which must be appropriately managed. Generally, groundwater management is carried out by means of mathematical models that require physical parameters called aquifer parameters. which are not directly measurable. The pumping test is the most common practice for estimating these parameters. It is an in-situ technique based on extracting a constant flow rate from the pumping well, while measuring the response of the water level at different time steps at the observation well. The aquifer parameters are obtained by interpreting the data measured during the pumping test which are the time-drawdown vectors (transient time-drawdown data). Interpretation has traditionally been performed by manual curve matching method which is subjective and time consuming.

The number of aquifer parameters to be estimated depends on the nature of the considered aquifer system (confined aquifer, leaky aquifer, etc.) and on the adopted analytical drawdown solution (Theis, Hantush, etc.). The estimation of aquifer parameters can be formulated as an optimization problem which requires an appropriate method of resolution. Various computer techniques have been proposed to automatically interpret pumping test data. The interpretation process is based on determining the optimal aquifer parameters leading to the best fit between observed time-drawdown data and those computed using an analytical drawdown solution.

Optimization techniques can be classified as gradient-based or gradient-free methods. Gradient-based methods require the computation of the derivatives of the objective function with respect to the different parameters. Because of the nonlinear and nonconvex aspect of groundwater problems, these methods may converge only toward a local optimum, or may diverge when the initial estimates are far off. Gradient-free methods require only the evaluation of the objective function and do not require any derivative information. The success of the interpretation of the time-drawdown data is mainly related to the performance of the employed optimization technique.

In this work, we employ a new metaheuristic algorithm called Grey Wolf Optimizer (GWO) which belongs to the swarm intelligence algorithms. The GWO is linked to the Theis drawdown solution for analyzing time-drawdown data coming from confined aquifers, and to the Hantush and Jacob drawdown solution for those coming from leaky aquifers. Four datasets were analyzed, and the obtained results were compared to those obtained by using the classical curve matching method.

This thesis has three chapters:

- The first chapter is dedicated to the pumping tests;
- The second chapter, presents the Grey Wolf Optimizer;
- The third chapter presents the automatic interpretation approach, as well as the obtained results.

We end the manuscript by a conclusion and future recommendations.

CHAPTER 1: THE PUMPING TEST

1.1 Introduction

The pumping test is commonly practiced by hydrogeologists. It is an in-situ technique that involves pumping groundwater from a well at a constant rate, and measuring the change in the water level (drawdown) in the observation wells (piezometers). Pumping tests can last from hours to few days, depending on the purpose of the test, but traditional pumping tests generally last 24 to 72 hours. Generally, the data from a pumping test are the time-drawdown vectors, the pumping rate, and the distance between the pumping well and the observation well. The data from a pumping test must be interpreted in order to identify the hydraulic parameters of the pumped aquifer. The interpretation of a pumping test is done by choosing an analytical drawdown model that matches the nature of the studied aquifer system. In this work, we focus on the analysis of pumping tests data coming from confined and Leaky aquifers.

1.2 Conducting a pumping test

The purpose of a pumping test is to acquire data that allow the identification of certain physical constants of the investigated aquifer. Obtaining reliable data from a pumping test therefore involves a carefully planned program; involving measuring instruments, available personnel and taking into account the physical constraints of the land, such as the risk of land flooding that may be caused by the extracted water. Before starting the test, it is essential to check the status of equipments; this includes installing an appropriately sized pump. A suitable discharge control valve is required to control the pumping rate, as well as a flow meter. For water level measurements, a calibrated probe must be installed correctly before starting the measurements. The number of people involved in the pumping test depends on the objective and the duration of the test. For example, tests involving rapid changes in water levels with several nearby observation wells require more personnel than periodic measurements in a single well. Note that automatic data recording equipments exist on the market. The duration of the pumping test depends on the hydraulic properties of the aquifer, since the depression cone (i.e., the drawdowns) surrounding the pumping well depends on the time since the start of pumping, and the distance between the pumping well and the piezometer. At the start of the test, the depression cone spreads quickly because the pumped water comes from the immediate vicinity of the pumping well. As pumping continuous, the radial expansion of the cone occurs at a slow rate as larger volumes of water become available. The depression cone will continue to expand until the recharge of the cone is equal to the discharge of the well; in this case, the steady state is reached. In some wells, the equilibrium condition can be reached a few hours after the start of pumping, while in other cases it can occur after days, weeks, or perhaps never. It is not necessary to reach the steady state, because analytical drawdown solutions in transient regime exist in the literature. Table 1.1 presents the recommended time intervals for drawdown measurement.

Table 1.1: *Recommended time intervals for drawdown measurement.*

Time Since Pumping Started	Time Interval
0 - 10 min	2 min
10 - 30 min	5 min
30 - 1 h	10 min
1 h -12 h	30 min
12 h - 48 h	1 h

1.3 Different types of aquifers.

Aquifers are underground deposits that store and transmit water in usable quantities. Aquifers can be identified as free surface, confined or Leaky, depending on their configurations. In this thesis, we consider confined and Leaky aquifers.

1.3.1 Confined aquifers

A confined aquifer (Figure 1.1) is a fully saturated layer of soil with upper and lower boundaries that are impermeable. In a confined aquifer, the water is under pressure. The imaginary surface of the confined aquifer is called piezometric surface, this imaginary surface is above the upper limit of the confined aquifer and can be located in the observation wells. The hydraulic parameters to be identified from data from a pumping test carried out in a confined aquifer are the transmissivity (T) and the storage coefficient (S).

- Transitivity (T) represents the ease with which water flows through the thickness of the aquifer. In quantitative terms, transmissivity is the product of the hydraulic conductivity K of the aquifer and the saturated thickness of the aquifer b; T =K b

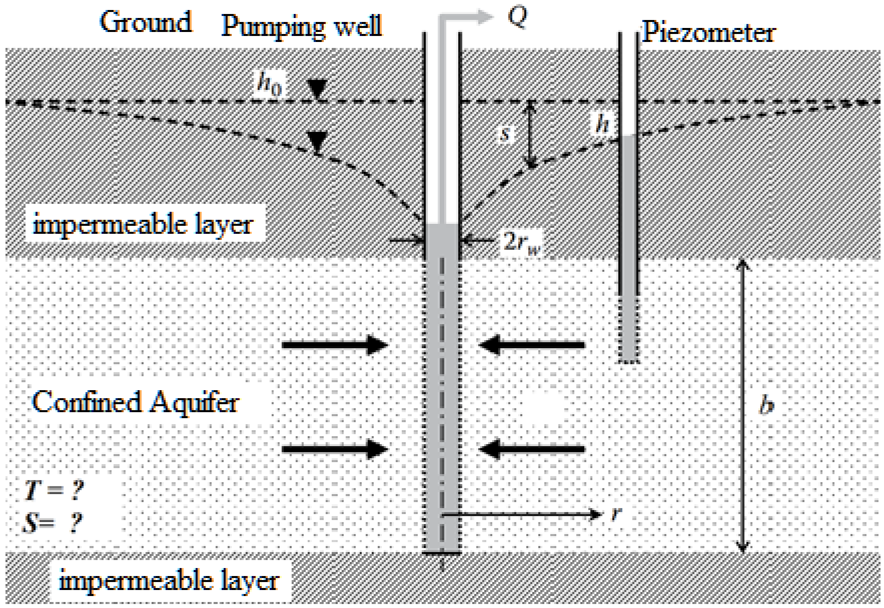


Figure 1.1*: Flow in a confined aquifer (Kresic, 2007).*

- The storage coefficient (S) represents the volume of water released by a unit volume of the porous medium due to a unit decrease in the hydraulic head, and it depends on the compressibility of the porous medium.

1.3.2 Leaky aquifers

A Leaky aquifer (Figure 1.2) is characterized by layers called Aquitards sufficiently permeable to allow vertical flow (leakage) from adjacent aquifers. The leak rate can increase considerably during pumping due to the increase in the hydraulic gradient between the pumped aquifer and the adjacent aquifer. For long pumping periods, the leakage rate balances the pumping rate, which stabilizes the drawdown at a fixed level, and the radius of influence of the well will stop expanding.

The hydraulic parameters to be identified from data from a pumping test carried out in a Leaky aquifer, are the transmissivity (T), the storage coefficient (S) and the leakage factor (B) of the aquitard; $B = \sqrt{Tb'/K'}$, with b' and $K'$are respectively the thickness and the vertical permeability of the aquitard

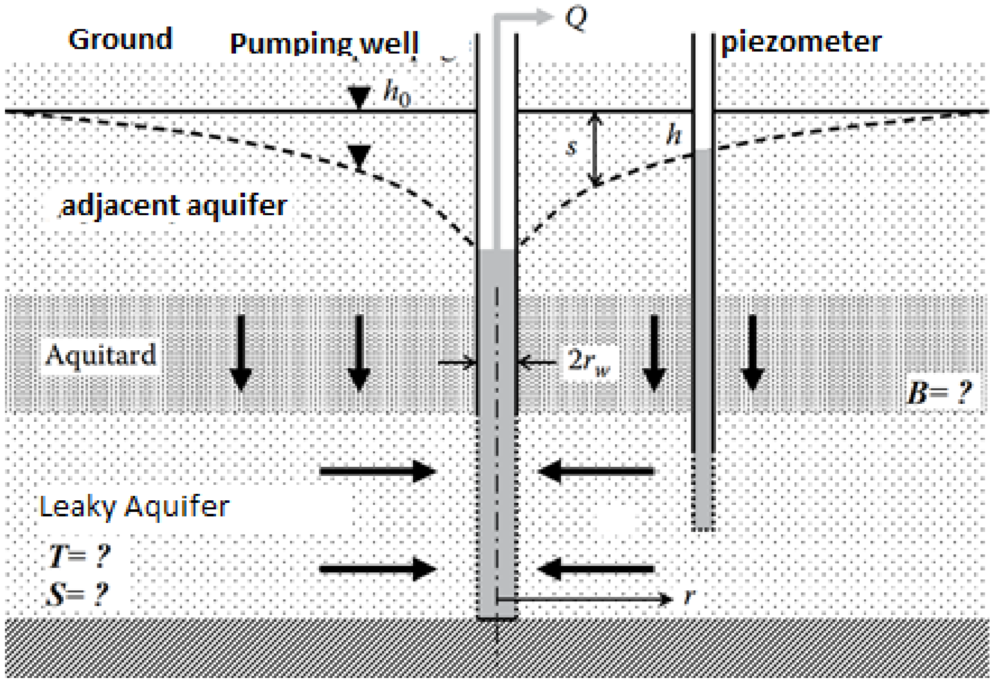


Figure 1.2*: Flow in a Leaky aquifer (Kresic, 2007).*

1.4 Analytical drawdown solutions

Many analytical drawdown solutions are available in the literature corresponding to the different configurations of aquifers. Each analytical solution was derived by considering a few simplifying assumptions. Generally, their mathematical expressions contain singular integrands. According to his experience and to the time-drawdown curve observed, the engineer must choose among the available analytical solutions the one that matches the studied aquifer system. In this section, we present the analytical drawdown solutions used in this work.

1.4.1 Theis solution

The Theis equation (1935) describes the transient flow of groundwater to a well that completely penetrates a confined aquifer. It forms the basis of most methods for analyzing transient pumping tests. The transmissivity and the storage coefficient can be determined from time-drawdown measurements without needing the establishment of the permanent regime. The assumptions on which Theis' solution is based are as follows:

- Confined aquifer;

- Homogeneous and isotropic medium with infinite dimensions having a constant thickness;
- Complete pumping well, the pumping well penetrates the entire aquifer;
- Transient flow regime;
- Horizontal flow to the well;

The Theis equation (Equation 1.1) calculates the drawdown *s* at any time after the start of pumping.

$$s = \frac{Q}{4\pi T} W(u) \qquad (1.1)$$

where Q [L^3 / T] is the pumping rate, kept constant during the test, T [L^2 / T] is the transmissivity, and *W (u)*is called the Theis function, or simply the well function. The dimensionless parameter u is given by:

$$u = \frac{r^2 S}{4Tt} \qquad (1.2)$$

where *r* [L] is the distance between the pumping well and the piezometer in which the drawdown is measured. S [-] is the storage coefficient, and *t* [T] represents the time since the start of pumping. The well function *W* (*u*) is given by the following equation:

$$W(u) = \int_u^\infty \frac{e^{-u}}{u} du \qquad (1.3)$$

Equation (1.3) is evaluated numerically using the following equation (Tseng and Lee, 1998):

$$W(u) = \begin{cases} -\gamma - \ln(u) + \sum_{i=1}^{\infty} \frac{(-1)^{i+1} u^i}{i \cdot i!}, \ u < 1 \\ \frac{e^{-u}}{u} \left(\frac{a_0 + a_1 u + a_2 u^2 + a_3 u^3 + u^4}{b_0 + b_1 u + b_2 u^2 + b_3 u^3 + u^4} \right), \ u \geq 1 \end{cases} \qquad (1.4)$$

With γ=0.57721566490153286, a_0=0.2677737343, a_1= 8.6347608925, a_2= 18.059016973, a_3= 8.5733287401, b_0= 3.9584969228, b_1= 21.0996530827, b_2= 25.6329561486, b_3= 9.5733223454. Note that the variable *u* is generally less than 1 (Maliva, 2016).

The values of the function $W(u)$ are tabulated or represented in the form of a graph $W(u) = f(u)$or $W(u) = f(\frac{1}{u})$ as shown in Figure 1.3 below.

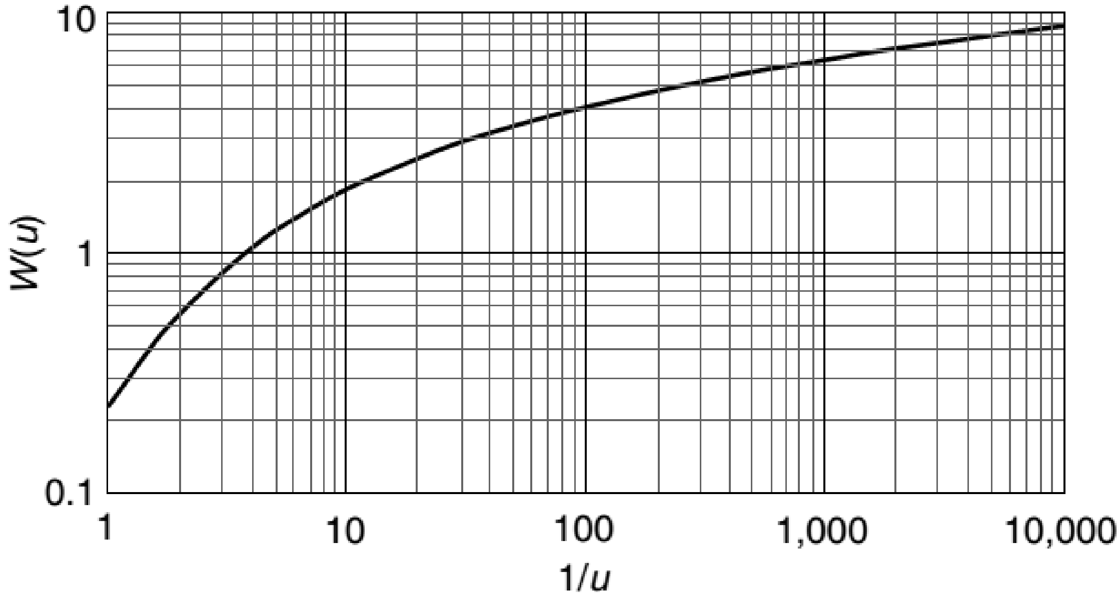


***Figure 1.3**: Theoretical curve of the well function, W(u)=f(1/u) (Kresic, 2007).*

The identification of hydraulic parameters from time-drawdown data is traditionally done manually in the following steps:

- The time-drawdown data are plotted on a transparent sheet having a scale identical to the theoretical curve in Figure 1.3;
- This curve is then superimposed on the theoretical curve;
- We make a manual adjustment between the two curves while keeping the axes parallel;
- Once a satisfactory adjustment is obtained, an adjustment point (match point) will then be chosen arbitrarily (Figure 1.4).

From equation (1.1) and the drawdown value s of the match point, we can write $T = \frac{Q}{4\pi s}W(u)$.

The storage coefficient S is calculated from equation (1.2), the value 1 / u of the adjustment point and time t; $S = \frac{4Ttu}{r^2}$.

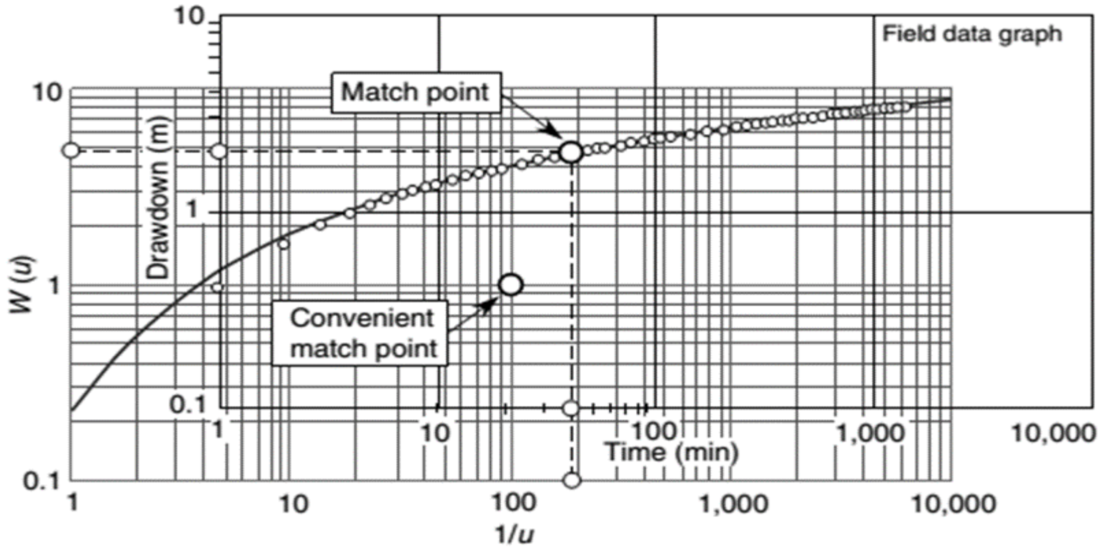


Figure 1.4*: Superimposition of the theoretical curve and the observed time-drawdown curve (Kresic, 2007).*

According to figure 1.4, the coordinates of the adjustment point are: $W(u) = 4.75$, $\frac{1}{u} = 180$, $u = 0.0055$, $s = 2.0$ m , $t = 39$ min $= 2340$ s. If the pumping rate $Q = 0.008$ m^3/s, and the distance $r = 40.5$m, So: $T = \frac{0.008}{4\times\pi\times2.0} \times 4.75 = 1.51 \times 10^{-3}$m^2/s et $S = \frac{4\times1.51\times10^{-3}\times2340\times0.0055}{(40.5)^2} = 7.4 \times 10^{-5}$.

1.4.2 Hantush and Jacob's solution

The data obtained from a transient pumping test carried out in a Leaky aquifer can be interpreted using the analytical solution of Hantush and Jacob (1955).

The hypotheses on which Hantush and Jacob's solution is based are the following:

- Leaky aquifer;
- Homogeneous and isotropic medium with infinite dimensions having a constant thickness;
- Leakage effect through the aquitard;
- The storage coefficient of the aquitard is negligible;
- Flow through the aquitard is vertical;
- Complete pumping well;
- Transient flow regime.

The Hantush and Jacob equation is given by the following equation:

$$s = \frac{Q}{4\pi T} W\left(u, \frac{r}{B}\right) \qquad (1.5)$$

$W(u, r/B)$ is the Hantush function with two arguments, given by:

$$W\left(u, \frac{r}{B}\right) = \int_u^{\infty} \frac{1}{y} \exp\left(-y - \frac{r^2}{4B^2 y}\right) d\,y \qquad (1.6)$$

The solution of Hantush and Jacob is a model with three parameters T, S and B. The values of the function $W(u, r/B)$ are tabulated or represented in the form of a graph $W(u, r/B) = f(1/u)$ as shown in Figure 1.5 below.

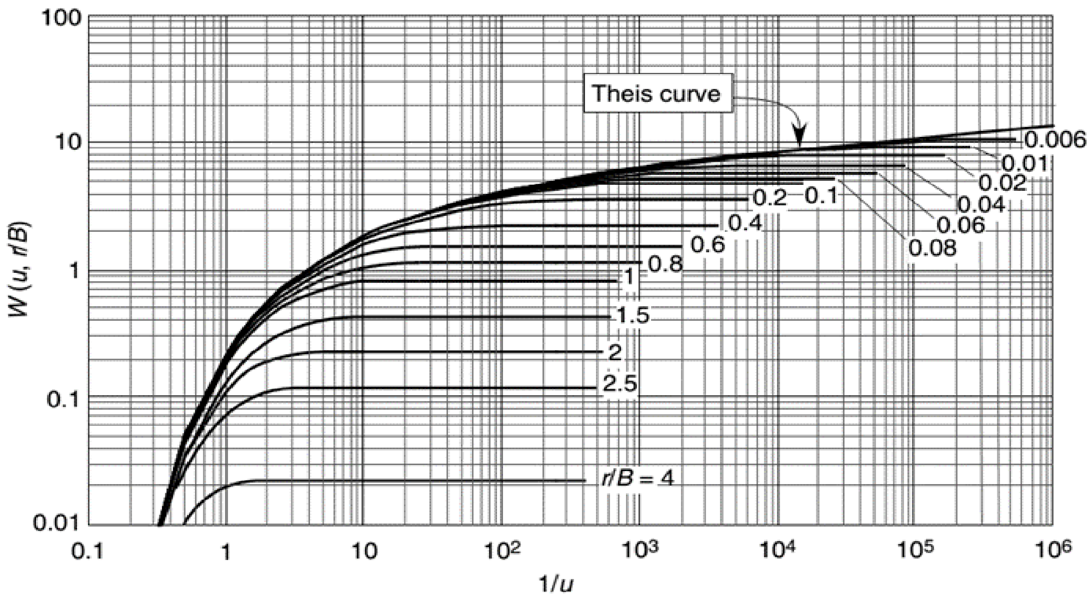


Figure 1.5*: Theoretical curves of the well function, W (u, r / B) = f (1 / u) (Kresic, 2007).*

The estimation of hydraulic parameters from time-drawdown data is done manually according to the following steps:

- The time-drawdown data are plotted on a transparent sheet having a scale identical to that of the theoretical curve in Figure 1.5;
- This curve is then superimposed on the theoretical curve;
- We make a manual adjustment between the two curves while keeping the axes parallel.

Once a satisfactory adjustment is obtained, an adjustment point will then be chosen arbitrarily (Figure 1.6).

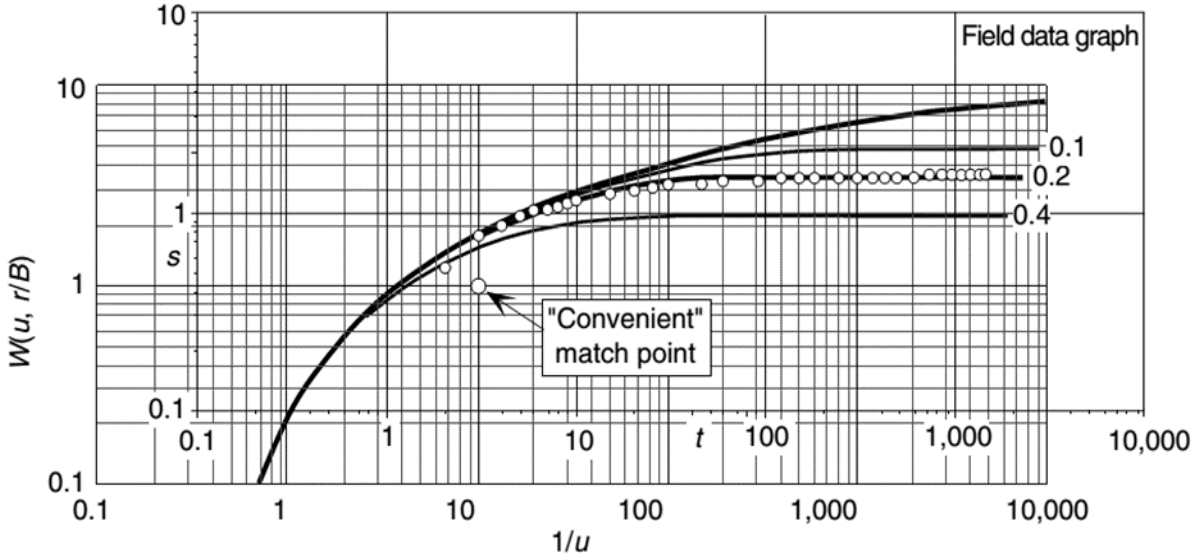


Figure 1.6*: Superimposition of the theoretical curve and the observed time-drawdown curve (Kresic, 2007).*

Five values will therefore be determined, *s*, *t*, *W(u,r/B)* and *1/u* and *r/B*. From equation (1.5), we can write $T = \frac{Q}{4\pi s}W(u, r/B)$. The storage coefficient $S = \frac{4Ttu}{r^2}$. The vertical permeability of the aquitard will then be: $K' = Tb^{'}(r/B)^2/r^2$

1.4.3 Numerical evaluation of the Hantush function

The evaluation of the Hantush function is fundamental to estimate the parameters of Leaky aquifers. The integration of the Hantush well function is usually carried out by evaluating its truncated shape (Chander and al., 1981, Samuel and Jha, 2003); this can lead to large errors when the calculation is performed in certain ranges of arguments (*u, r/B*) of the function (Yeh and Huang 2005). In this subsection, we briefly present the infinite series of Hunt (1977) as well as the rapid approximation of Veling and Maas (2010).

- **Infinite Hunt Series**

To calculate the Hantush well function, Hunt (1977) presented two analytical expressions in the form of convergent infinite alternating series given by:

$$W\left(u, \frac{r}{B}\right) = \sum_{n=0}^{\infty} \frac{(-\varepsilon/u)^n}{n!} E_{n+1}(u) \qquad for\ 0 \leq\ \varepsilon/u < \infty \qquad (1.7)$$

$$W\left(u, \frac{r}{B}\right) = 2K_0\left(2\sqrt{\varepsilon}\right) - \sum_{n=0}^{\infty} \frac{(-u)^n}{n!} E_{n+1}\left(\frac{\varepsilon}{u}\right) \qquad for\ 0 \leq\ u < \infty \qquad (1.8)$$

with $\varepsilon = (r^2/4B^2)$, K_0 is the modified Bessel function of second kind zero order, $E_n(u)$can be calculated explicitly from the exponential integral $E_1(u)=W(u)$ using the recurrence formula given by:

$$E_{n+1}(u) = \frac{1}{n}\left[e^{-u} - uE_n(u)\right] \quad (n = 1,2,\dots) \tag{1.9}$$

Hunt (1977) pointed out that equation (1.7) should be used when $u < \varepsilon/u$ and equation (1.8) when $\varepsilon/u < u$.

- **Veling and Maas rapid approximation**

Veling and Maas (2010) published an approximate formula for calculating the Hantush function as a function of the modified second-order Bessel function zero order K0 and the exponential integral E_1 (u) = *W (u).* Their formula is given by:

$$w(u,b) = \begin{cases} k_0(2\sqrt{b}) - \left[\left(\frac{E_1(2\sqrt{b}) - k_0(2\sqrt{b})}{E_1(2\sqrt{b}) - E_1(\sqrt{b})}\right) E_1\left(\frac{b}{u}\right) + \left(\frac{k_0(2\sqrt{b}) - k_0(\sqrt{b})}{E_1(2\sqrt{b}) - E_1(\sqrt{b})}\right) E_1\left(u + \frac{b}{u}\right)\right] & si\ u < \sqrt{b} \\ \left[\left(\frac{E_1(2\sqrt{b}) - k_0(2\sqrt{b})}{E_1(2\sqrt{b}) - E_1(\sqrt{b})}\right) E_1(u) + \left(\frac{k_0(2\sqrt{b}) - k_0(\sqrt{b})}{E_1(2\sqrt{b}) - E_1(\sqrt{b})}\right) E_1\left(u + \frac{b}{u}\right)\right] & si\ u \geq \sqrt{b} \end{cases}$$

(1.10)

1.5 Automatic interpretation of pumping tests

In the previous section, we have seen that the quality of the interpretation is based on a superposition of curves that the operator manually performs by visual inspection of the adjustment quality; this leaves him a part of subjectivity. From 1970 until now, various computer techniques have been proposed to automatically interpret the pumping test data. Several commercial interpretation software have been developed, and their use has considerably increased the precision of the hydraulic parameters and reduced the interpretation time. Among the most used software we can cite: Aquifer Test, AQTESOLVTM, MLU, ANSDIMAT…etc. Other free applications are available on the net such as the AQTESTSS spreadsheets from the U.S. Geological Survey (Halford and Kuniansky, 2002). Each of these software uses an optimization technique aimed to minimize the differences between the measured time-drawdown data and those calculated by an appropriate analytical drawdown solution. So the accuracy of the aquifer parameters depends on the efficiency of the adopted optimization technique.

Our work develops an optimization framework based on Grey Wolf Optimizer (GWO). The GWO supplies the analytical drawdown solution with the best hydraulic parameters which give the best fit between the observed and calculated values.

The next chapter presents the employed Grey Wolf Optimizer.

Conclusion

In this chapter we discussed the mechanism and the methods for analyzing transient time-drawdown data of pumping tests according to the type of aquifer, As with the number of aquifer parameters estimated so that it’s depends on the nature of the considered aquifer system.

The hydraulic parameters to be identified from data from a pumping test carried out in a confined aquifer are the transmissivity (T) and the storage coefficient (S), and for leaky aquifer are the transmissivity (T), the storage coefficient (S) and the leakage factor (B) of the aquitard.

We choose Theis solution for confined aquifers and Hantush and Jacob's solution for leaky aquifers.

CHAPTER 2: HYDRAULIC OPTIMIZATION AND GREY WOLF OPTIMIZER

2.1 Introduction

Optimization could be defined as the process of finding the best solution eligible for a given problem. It is an improvement operation that generally requires significant computing power. With the advent of the computer, optimization has now become a widespread practice in all fields of science, including hydraulics. The hydraulic problems can be formulated as problems to optimize, in this case, we called them 'inverse problems'.

Solving an inverse problem is observing the available solutions, and trying to determine the values of the parameters of the model governing the phenomenon under consideration; in other words, it is a calibration operation.

An optimization problem is often represented as follows:

Minimize : $f(x)$, $x_i^L < x_i < x_i^U$, i =1,2,……,Npar (2.1)

With *f(x)* is the objective function to minimize (Figure. 2.1a and Figure. 2.1d), *Npar* is the number of parameters to be identified, $x = [x_1, x_2, \ldots, x_{npar}]^T$is the vector of *Npar* dimensional parameters. The values of the x_i parameters are located in the search space, limited by the lower bounds x_i^L and upper bounds x_i^U(Figure. 2.1b).

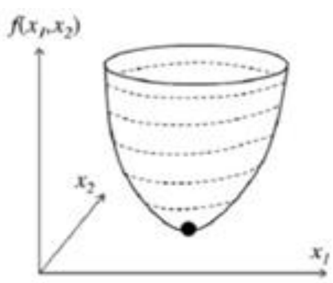
By surveying the literature, it was found that the terms used in the field of optimization are not well unified, but in general, the classification of optimization problems can be carried out according to:

- The number of objectives , when it comes to optimize a single function *f(x)*, in this case we talk about mono -objective optimization, but when *n* functions $f_j(x)$ *(j=1.2,.. n)* are to be optimized, we are talking about multi-objective optimization, such as maximizing water availability while minimizing the cost of pumping;
- The number of constraints; sometimes an optimization problem is subject to constraints related to the analyzed problem (Figure. 2.1c), these constraints are written according to the parameters to be identified in the form of inequalities or equalities;
- The mathematical behavior and the landscape of the objective function, possibility of local optima (Figure.2.1d), as well as the type of parameters (real, integer,.. etc.);
- The type of the employed optimization technique, because optimization problems are complexes and require appropriate resolution methods called optimization techniques or algorithms.

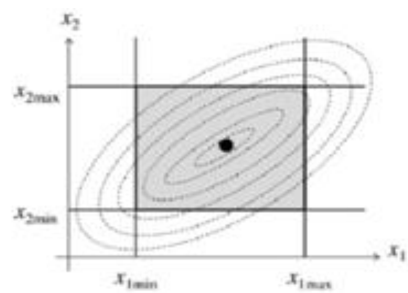
Table 2.1 below, summarizes the classification of optimization problems.

Table 2.1 *Classification of optimization problems.*

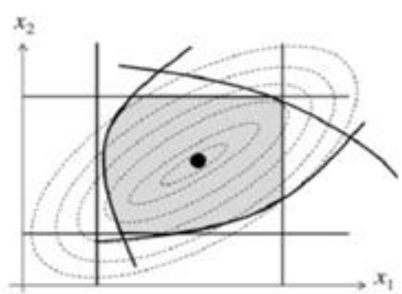
Optimization	Objective	• Mono-objective • Multi-objective
	Constraints	• Constrained • Unconstrained
	Landscape of *f(x)*	• Unimodal • Multimodal
	Behavior	• Linear • Non-linear
	Variables	• Discretes • Continues
	Techniques	• Deterministics • Stochastics



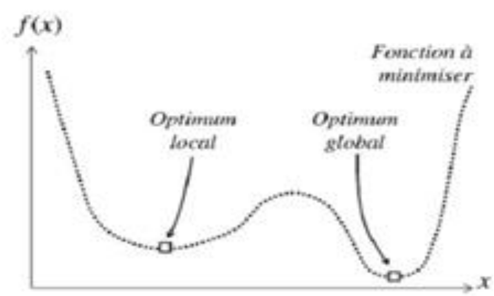
(a) Objective function



(b) Unconstrained research



(c) Search space with constraints



(d) Local and global optimum

Figure 2.1*: Schematization of an optimization problem (Bruyneel and al. 2010).*

An optimization process often combines an optimization technique with a numerical simulation model (Figure.2.2). The optimization technique generates the *x* parameter vector, and injects it into the simulation model in an iterative way in order to minimize a given objective function. It is an operation that transfers information from observed state variables to estimated parameters. The objective function represents the differences between the state variables calculated by the simulation model and those observed. It is expressed according to the parameters to be identified. The objective function can take several forms depending on the nature of the problem being addressed, such as the sum of squared errors (SSE) (Eq. 2.2).

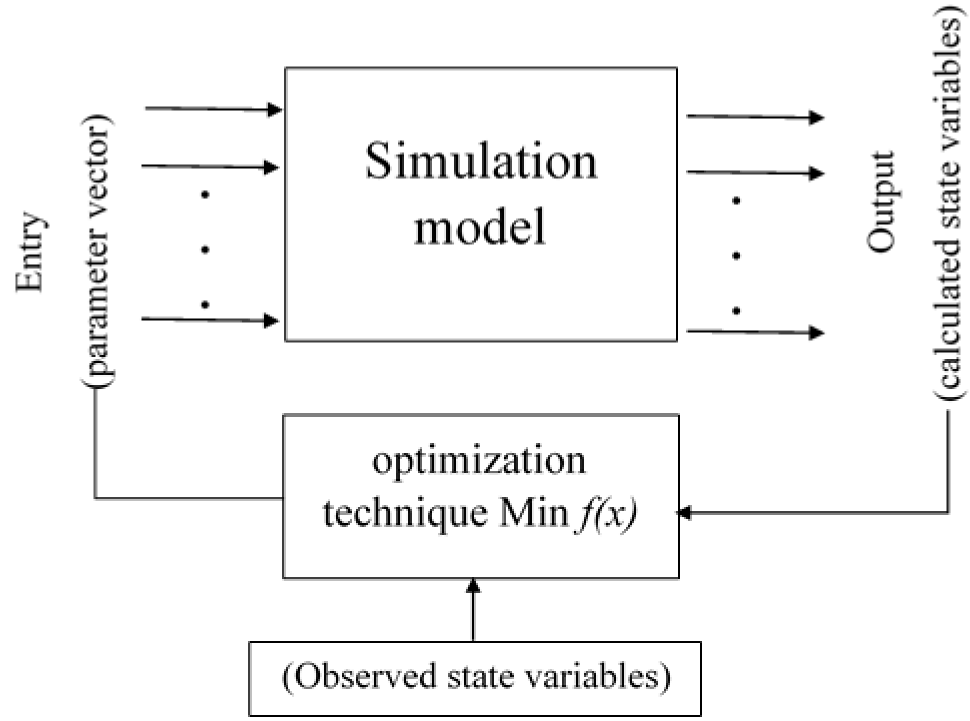


Figure 2.2: *Optimization Process.*

With *N* is the number of observed points, y_i^o are the observed variables, y_i^c are the variables computed by the simulation model. Depending on the inverse problem complexity and on the availability of data, the problem optimization process can be simple, slightly difficult, moderately difficult or very difficult (Sun and Sun 2015). There is no general rule that dictates the amount of data (observed variables) required for the parameter identification process, but the more complex the model is, the more data are needed.

2.2 Metaheuristics

Optimization techniques available in literature can be classified as deterministic and stochastic methods. Since they are gradient-based approaches, deterministic methods require the computation of the derivatives of the objective function being optimized, so the optimized function should be differentiable. Furthermore, they can easily get trapped into local optima due to their deterministic nature, which is something undesirable for most applications. Stochastic methods are tools of choice when it comes to solve complex problems like those encountered in groundwater engineering, which are highly non linear and multimodal.

Stochastic algorithms are also called metaheuristics, they belong to computational intelligence and most of them are nature-inspired.

Mirjalili et al. (2014) summarized the advantages of stochastic techniques in the following points:

- First, metaheuristics are simple. They have been mostly inspired by simple concepts. The inspiration is typically related to physical phenomena, animals' behaviors, or evolutionary concepts. The simplicity allows computer scientists to simulate different natural concepts, propose new meta-heuristics, hybridize two or more meta-heuristics, or improve the current meta-heuristics. Moreover, the simplicity assists other scientists to learn metaheuristics quickly and apply them to their problems.
- Second, flexibility refers to the applicability of meta-heuristics to different problems without any special changes in the structure of the algorithm. Meta-heuristics are readily applicable to different problems since they mostly assume problems as black boxes. In other words, only the input(s) and output(s) of a system are important for a meta-heuristic.
- Third, the majority of meta-heuristics have derivation-free mechanisms. In contrast to gradient-based optimization approaches, meta-heuristics optimize problems stochastically. The optimization process starts with random solution(s), and there is no need to calculate the derivative of search spaces to find the optimum. This makes meta-heuristics highly suitable for real problems with expensive or unknown derivative information.

Finally, meta-heuristics have superior abilities to avoid local optima compared to conventional optimization techniques. This is due to the stochastic nature of meta-heuristics which allow them to avoid stagnation in local solutions and search the entire search space extensively. The search space of real problems is usually unknown and very complex with a massive number of local optima, so meta-heuristics are good options for optimizing these challenging real problems.

2.3 Grey Wolf Optimizer (GWO)

In this work, we employed the Grey Wolf Optimizer (GWO) which is a metaheuristic algorithm inspired by the hunting behavior and social hierarchy observed in packs of grey wolves. GWO has ben proposed by Mirjalili et al. in 2014.

2.3.1 Inspiration

Grey wolf (Canis lupus) belongs to Canidae family. Grey wolves are considered as apex predators, meaning that they are at the top of the food chain. Grey wolves mostly prefer to live in a pack. The group size is 5–12 on average. Of particular interest is that they have a very strict social dominant hierarchy as shown in Figure. 2.3.

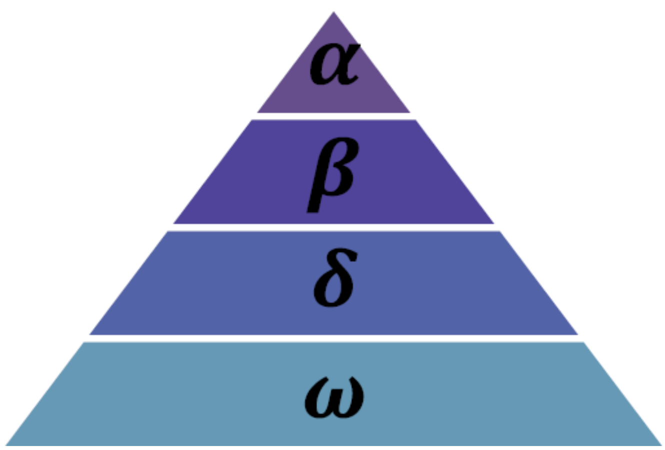


Figure. 2.3 *Hierarchy of grey wolf (dominance decreases from top down).*

The leaders are a male and a female, called alphas. The alpha is mostly responsible for making decisions about hunting, sleeping place, time to wake, and so on. The alpha’s decisions are dictated to the pack. However, some kind of democratic behavior has also been observed, in which an alpha follows the other wolves in the pack. In gatherings, the entire pack acknowledges the alpha by holding their tails down. The alpha wolf is also called the dominant wolf since his/her orders should be followed by the pack. The alpha wolves are only allowed to mate in the pack. Interestingly, the alpha is not necessarily the strongest member of the pack but the best in terms of managing the pack. This shows that the organization and discipline of a pack is much more important than its strength.

The second level in the hierarchy of grey wolves is beta. The betas are subordinate wolves that help the alpha in decision-making or other pack activities. The beta wolf can be either male or female, and he/she is probably the best candidate to be the alpha in case one of the alpha wolves passes away or becomes very old. The beta wolf should respect the alpha, but commands the other lower-level wolves as well. It plays the role of an advisor to the alpha and discipline rfor the pack. The beta reinforces the alpha’s commands throughout the pack and gives feedback to the alpha.

The lowest ranking grey wolf is omega. The omega plays the role of scapegoat. Omega wolves always have to submit to all the other dominant wolves. They are the last wolves that are allowed to eat. This assists satisfying the entire pack and maintaining the dominance structure.

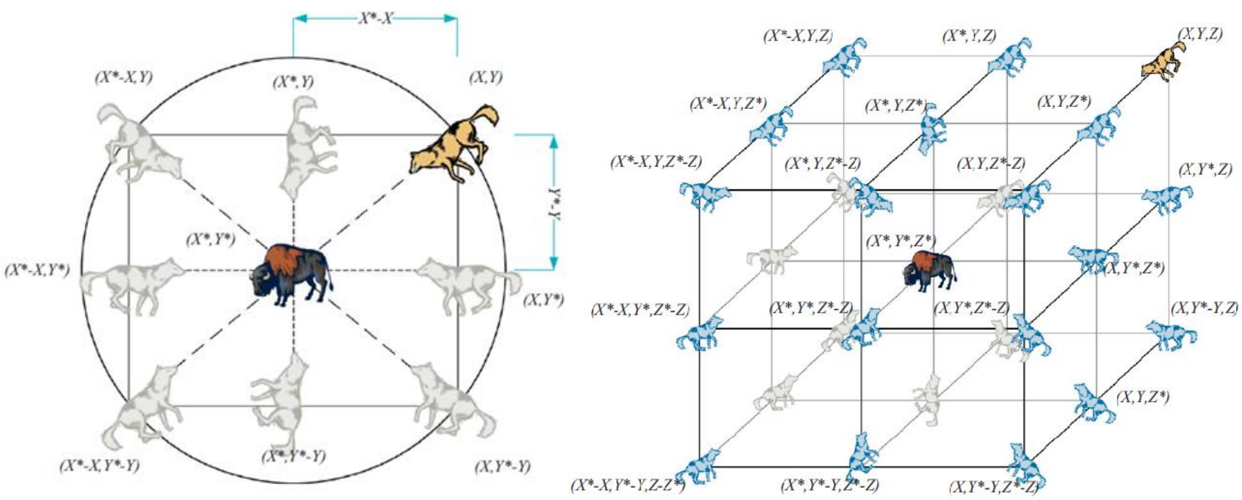
If a wolf is not an alpha, beta, or omega, he/she is called subordinate (or delta in some references). Delta wolves have to submit to alphas and betas, but they dominate the omega. Scouts, sentinels, elders, hunters, and caretakers belong to this category. Scouts are responsible for watching the boundaries of the territory and warning the pack in case of any danger. Sentinels protect and guarantee the safety of the pack. Elders are the experienced wolves who used to be alpha or beta. Hunters help the alphas and betas when hunting prey and providing food for the pack. Finally, the care takers are responsible for caring for the weak, ill, and wounded wolves in the pack.

In addition to the social hierarchy of wolves, group hunting is another interesting social behavior of grey wolves. The main phases of grey wolf hunting are as follows:

- Tracking, chasing, and approaching the prey;
- Pursuing, encircling, and harassing the prey until it stops moving;
- Attack towards the prey.

These steps are shown in Figure. 2.4

Figure. 2.4 *Hunting behavior of grey wolves: (A) chasing, approaching, and tracking prey (B–D)pursuiting, harassing, and encircling (E) stationary situation and attack..*



(a) (b)

Figure. 2.5 (a-b). *2D and 3D position vectors and their possible next locations.*

2.3.2 Mathematical model and algorithm

In this subsection the mathematical models of the social hierarchy, tracking, encircling, and attacking prey are provided. Then the GWO algorithm is outlined.

- **Social hierarchy**

In order to mathematically model the social hierarchy of wolves when designing GWO, we consider the fittest solution as the alpha (a). Consequently, the second and third best solutions are named beta (b) and delta (d) respectively. The rest of the candidate solutions are assumed to be omega (x). In the GWO algorithm the hunting (optimization) is guided by a, b, and d. The *x* wolves follow these three wolves.

- **Encircling prey**

As mentioned above, grey wolves encircle prey during the hunt.In order to mathematicallymodel encircling behavior the following equations are proposed:

$\vec{D} = \left|\vec{C}.\vec{X}_P(t) - \vec{X}(t)\right|$(2.2)

$$\vec{X}(t+1) = \vec{X}_P(t) - \vec{A} \cdot \vec{D} \quad (2.3)$$

where t indicates the current iteration, $\vec{A}$ and $\vec{C}$ are coefficient vectors,$\vec{X}_P$ is the position vector of the prey, and $\vec{X}$ indicates the position vector of a grey wolf.The vectors $\vec{A}$ and $\vec{C}$ are calculated as follows:

$$\vec{A} = 2\vec{a} \cdot \vec{r_1} - \vec{a} \quad (2.4)$$

$$\vec{c} = 2 \cdot \vec{r}_2 \quad (2.5)$$

where components of $\vec{a}$ are linearly decreased from 2 to 0 over the course of iterations and r_1, r_2 are random vectors in [0, 1].To see the effects of Eqs. (2.2) and (2.3), a two-dimensional position vector and some of the possible neighbors are illustrated in Figure. 2.5(a). As can be seen in this Figure, a grey wolf in the position of (X, Y) can update its position according to the position of the prey(X*, Y*). Different places around the best agent can be reached with respect to the current position by adjusting the value of $\vec{A}$ and $\vec{C}$vectors. For instance, (X*–X, Y*) can be reached by setting $\vec{A} = (1,0)$ and $\vec{c} = (1,1)$. The possible updated positions of a greywolf in 3D space are depicted in Figure. 2.5(b). Note that the random vectors r1 and r2 allow wolves to reach any position between the points illustrated in Figure (2.5). So a grey wolf can update its position inside the space around the prey in any random location by using Eqs. (2.2) and (2.3).
The same concept can be extended to a search space with n dimensions, and the grey wolves will move in hyper-cubes (or hyper-spheres) around the best solution obtained so far.

- **Hunting**

Grey wolves have the ability to recognize the location of prey and encircle them. The hunt is usually guided by the alpha. The beta and delta might also participate in hunting occasionally. However, in an abstract search space we have no idea about the location of the optimum (prey). In order to mathematically simulate the hunting behavior of grey wolves, we suppose that the alpha (best candidate solution) beta, and delta have better knowledge about the potential location of prey. Therefore, we save the first three best solutions obtained so far and oblige the other search agents (including the omegas) to update their positions according to the position of the best search agents.

The following formulas are proposed in this regard.

$$\vec{D}_\alpha = |\vec{C}_1 \cdot \vec{X}_\alpha - \vec{X}| , \vec{D}_\beta = |\vec{C}_2 \cdot \vec{X}_\beta - \vec{X}| , \vec{D}_\delta = |\vec{C}_3 \cdot \vec{X}_\delta - \vec{X}| \quad (2.6)$$

$$\vec{X}_1 = \vec{X}_\alpha - \vec{A}_1.(\vec{D}_\alpha) , \vec{X}_2 = \vec{X}_\beta - \vec{A}_2 \cdot (\vec{D}_\beta) , \vec{X}_3 = \vec{X}_\delta - \vec{A}_3 \cdot (\vec{D}_\delta) \quad (2.7)$$

$$\vec{X}(t+1) = \frac{\vec{X}_1 + X_2 + \vec{X}_3}{3} \quad (2.8)$$

Figure. 2.6 shows how a search agent updates its position according to alpha, beta, and delta in a 2D search space. It can be observed that the final position would be in a random place within a circle which is defined by the positions of alpha, beta, and delta in the search space. In other words alpha, beta, and delta estimate the position of the prey, and other wolves updates their positions randomly around the prey.

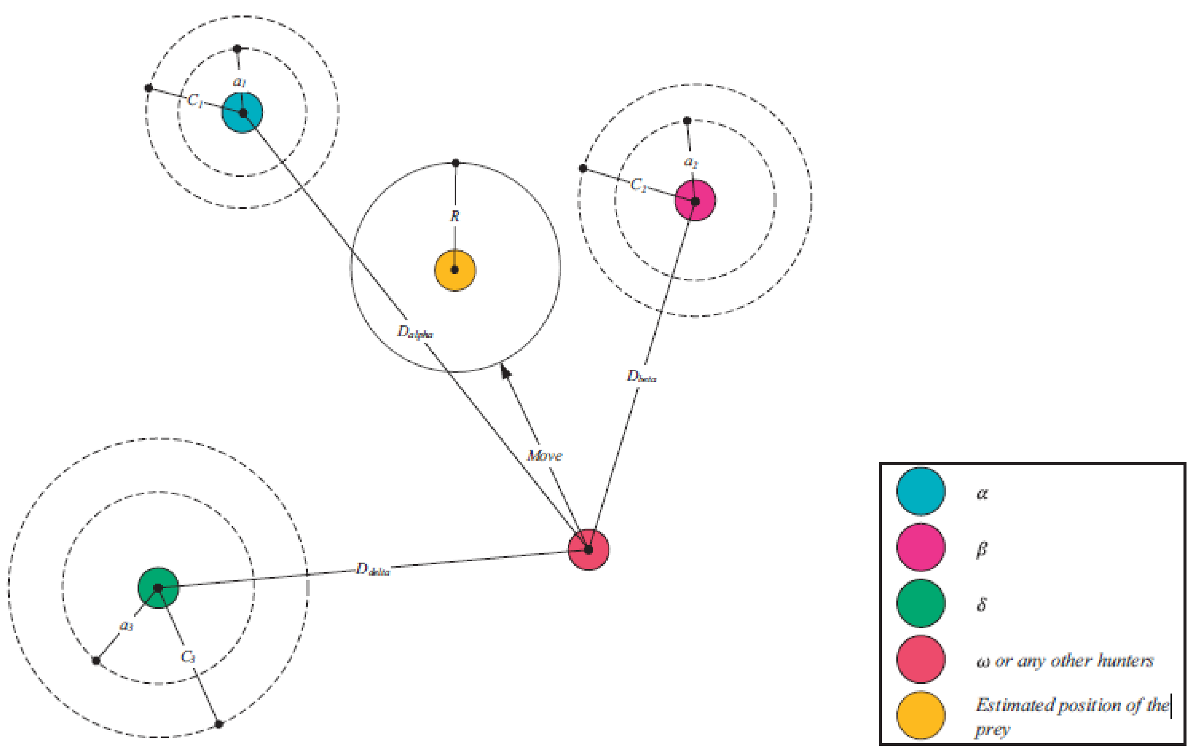


Figure. 2.6 *Position updating in GWO.*

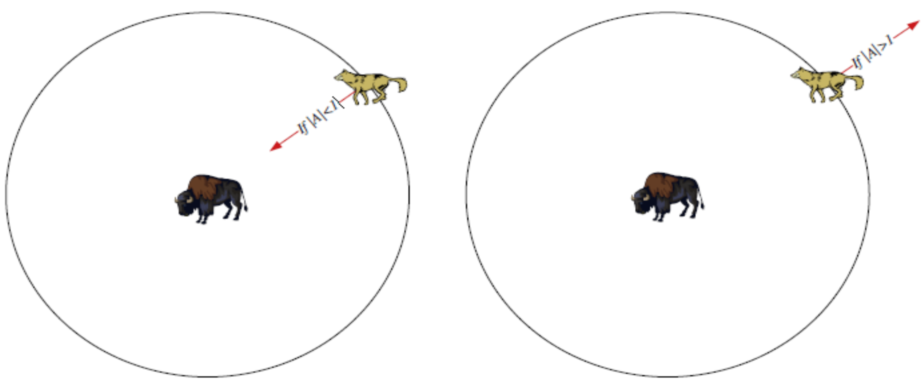
- **Attacking prey (exploitation)**

As mentioned above the grey wolves finish the hunt by attacking the prey when it stops moving. In order to mathematically model approaching the prey we decrease the value of $\vec{a}$. Note that the fluctuation range of $\vec{A}$ is also decreased by $\vec{a}$. In other words $\vec{A}$ is a random value in the interval [-2a, 2a] where a is decreased from 2 to 0 over the course of iterations. When random values of $\vec{A}$ are in[-1, 1], the next position of a search agent can be in any position between its current position and the position of the prey. Figure. 2.7(a) shows that $|A| < 1$ forces the wolves to attack towards the prey.

With the operators proposed so far, the GWO algorithm allows its search agents to update their position based on the location of the alpha, beta, and delta; and attack towards the prey. However, the GWO algorithm is prone to stagnation in local solutions with these operators. It is true that the encircling mechanism proposed shows exploration to some extent, but GWO needs more operators to emphasize exploration.

- **Search for prey (exploration)**

Grey wolves mostly search according to the position of the alpha, beta, and delta. They diverge from each other to search for prey and converge to attack prey. In order to mathematically model divergence, we utilize $\vec{A}$ with random values greater than 1 or less than -1 to oblige the search agent to diverge from the prey.This emphasizes exploration and allows the GWO algorithm to search globally. Figure. 2.7(b) also shows that $|A| > 1$ forces the grey wolves to diverge from the prey to hopefully find a fitter prey.



(a) *(b)*

Figure. 2.7 *Attacking prey versus searching for prey.*

Conclusion

We saw through this chapter that the Grey wolf optimizer (GWO) is one of recent metaheuristics swarm intelligence methods. It has been widely tailored for a wide variety of optimization problems due to its impressive characteristics over other swarm intelligence methods, it has very few parameters, and no derivation information is required in the initial search. Also it is simple, easy to use, flexible, scalable, and has a special capability to strike the right balance between the exploration and exploitation

3.1 Introduction

In this chapter, the results of interpretation of time-drawdown data are presented. The GWO presented in the previous chapter was coupled with the Theis drawdown solution when the aquifer is confined and with the Hantush and Jacob solution when the aquifer is leaky. The obtained results are compared with those obtained using ant colony optimization (Bateni et al. 2015) and with those obtained by manual curve matching (Kresic 1997).

3.2 Automatic interpretation

The problem of estimating aquifer parameters is an optimization problem, formulated as follows:

$$\text{Minimize } f(x_i) \text{ with : } x_i^L < x_i < x_i^U \quad ; \quad i = 1,2, \ldots, Npar \qquad (3.1)$$

With $f(x_i)$ is the objective function to be minimized, x_i^L et x_i^U are respectively the lower and upper limits of the search spaces, which must be large enough to include all the feasible parameter values (Table 3.1).

The Theis solution is a two-parameter model $x = [T \;\; S\,]^T$, while Hantush and Jacob's solution is a three-parameter $x = [T \;\; S \;\; B]^T$.

In this work, the objective function is formulated as a sum of squared errors (SSE) defined by:

$$f(x) = SSE = \sum_{i=1}^{N} (s^o(t_i) - s^c(t_i))^2 \qquad (3.2)$$

where N is the number of observed data points ,$s^o(t_i)$ et $s^c(t_i)$ are the observed and calculated drawdown at a given pumping time, respectively. The automatic interpretation flowchart is given in Figure 3.1.

***Table 3.1**: Search space.*

Hydraulic parameters of aquifer ($x_{i=1,2,\ldots,Npar}$)	x_i^L	x_i^U
$x_1 = T$ [m²/s]	10^{-10}	10^{-1}
$x_2 = S$ [-]	10^{-10}	10^{-1}
$x_3 = B$ [m]	10^{-10}	10^{4}

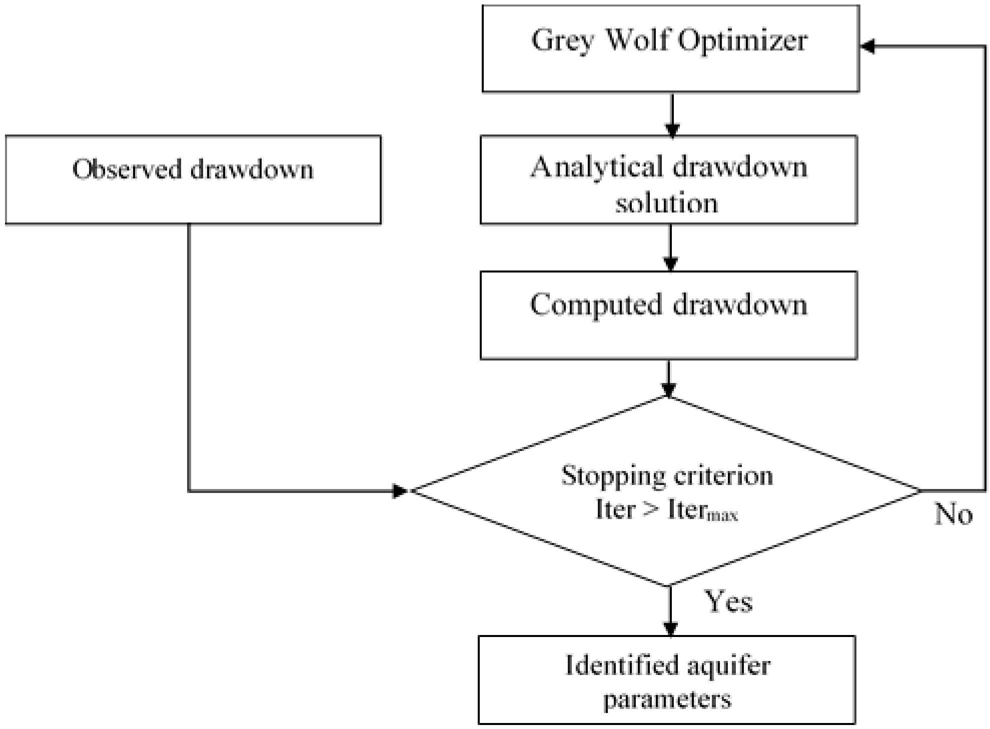


Figure 3.1*: Flowchart of Automatic Interpretation.*

3.3 Applications

Several datasets of time-drawdown data coming from confined and leaky aquifers were selected to test the ability of the proposed interpretation approach. The population size of the GWO was set equal to 60.The results are presented in the form of time-drawdown curves, tables with identified aquifer parameters and performance curves.

3.3.1 Interpretation by the Theis solution

Two datasets were interpreted by the Theis solution.

- **Datasets [T1] and [T2] :**

Time-drawdown datasets coming from pumping tests realized in confined aquifers are denoted T1 and T2.

T1 and T2 are time-drawdown datasets observed in two piezometers located at 25m and 75m, respectively from the pumping well. The pumping rate equals to 0.008m^3/s (Batu 1998).

Figure 3.2 compares the calculated time-drawdown curve using the estimated aquifer parameters with the observed data.

***Table 3.2**: Identified aquifer parameters, [Dataset: T1].*

Methods		Estimated parameters		Objective function
		T (m^2/*day*)	*S* ($\times 10^{-4}$)	*RMSE** ($\times 10^{-2}$)
T1 dataset				
Present work	GWO	234.49	1.56	4.43
Bateni et al. (2015)	ACO**	236	1.56	4.45

* $RMSE = \sqrt{SSE/n}$ is the root mean square error, used here just for comparison purpose; *n* is the size of the dataset.
**ACO is an optimization technique called ant colony optimization algorithm.

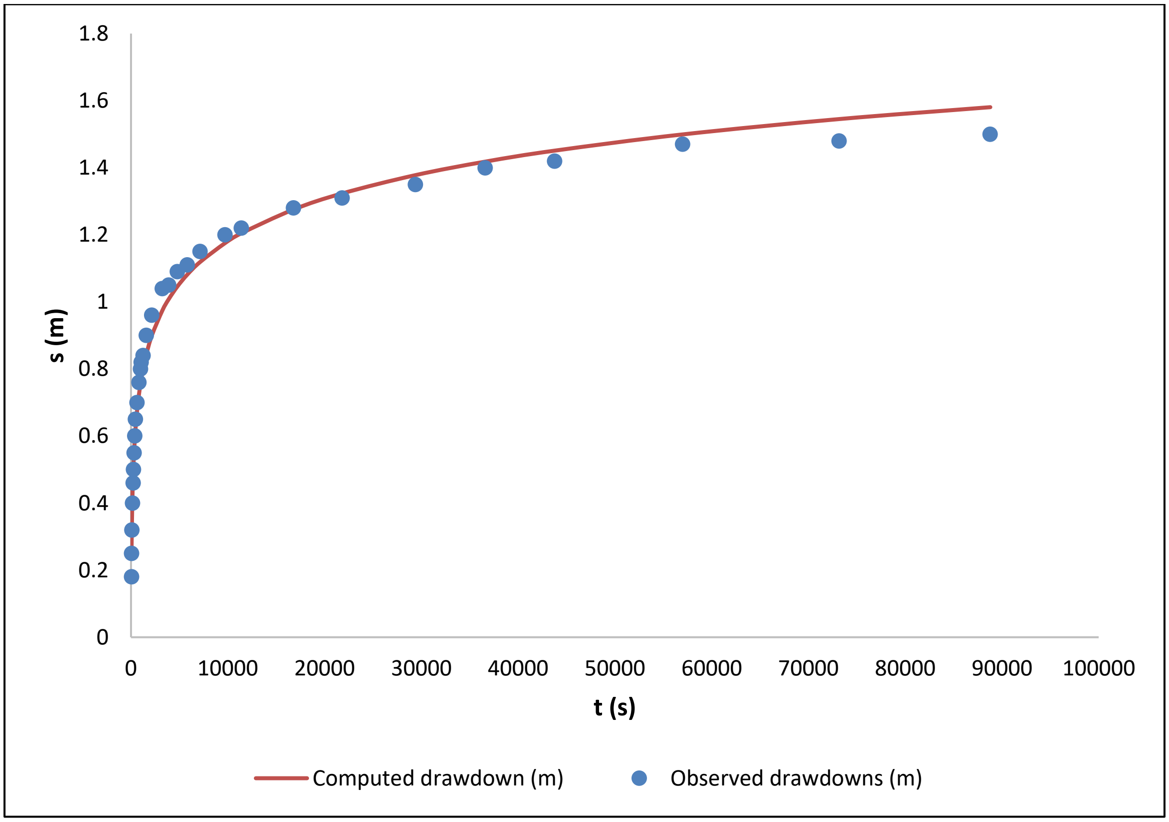


***Figure 3.2**: Comparison of observed and calculated values, [Dataset: T1].*

Table 3.3*: Identified aquifer parameters, [Dataset: T2].*

Methods		Estimated parameters		Objective function
		T (m^2*/day)*	*S* ($\times 10^{-4}$)	*RMSE** ($\times 10^{-2}$)
T2 dataset				
Present work	GWO	236.43	3.02	2.96
Bateni et al. (2015)	ACO**	246	2.87	3.98

* $RMSE = \sqrt{SSE / n}$ is the root mean square error, used here just for comparison purpose; *n* is the size of the dataset.

**ACO is an optimization technique called ant colony optimization algorithm.

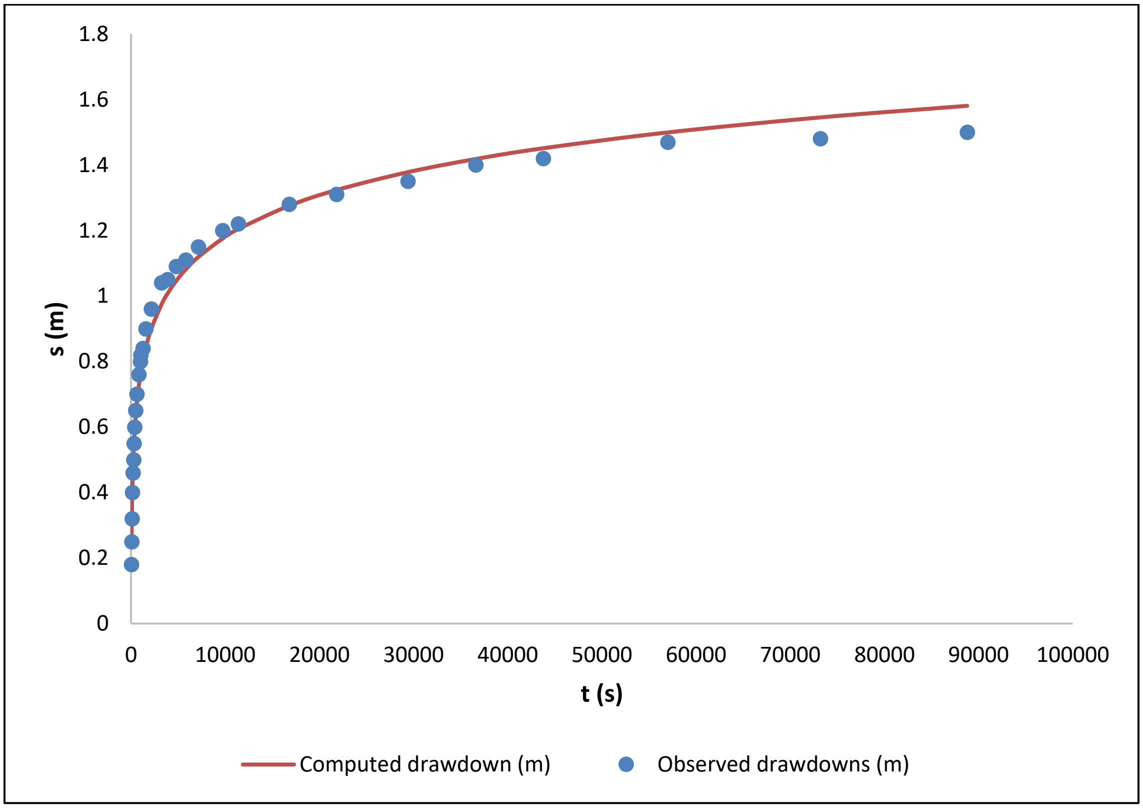


Figure 3.3*: Comparison of observed and calculated values, [Dataset: T2].*

From the time-drawdown figures of the datasets [T1 and T2], it is clear that the values of the computed drawdown correspond well to the values of observed drawdown.

The obtained results indicate that the proposed automatic interpretation procedure is accurate,

3.3.2 Interpretation by the Hantush solution

- **Dataset [H1]:**

The time-drawdown dataset coming from a pumping test realized in leaky aquifer is denoted (H1).

H1 is the time-drawdown dataset observed in a piezometer located at 128m from the pumping well. The pumping rate equals to $0.012 m^3/s$ (Kresic 1997).

Tableau 3.4: *Identified aquifer parameters, [Dataset: H1].*

Methods		Estimated parameters			Objective function
		T (m^2/day)	*S ($\times 10^{-5}$)*	*B(m)*	*SSE* ($\times 10^{-2}$)
Present work	GWO	149.40	1.36	416.24	7.71
Kresic (1997)	Curve matching	257.47	1.09	640.00	334.64

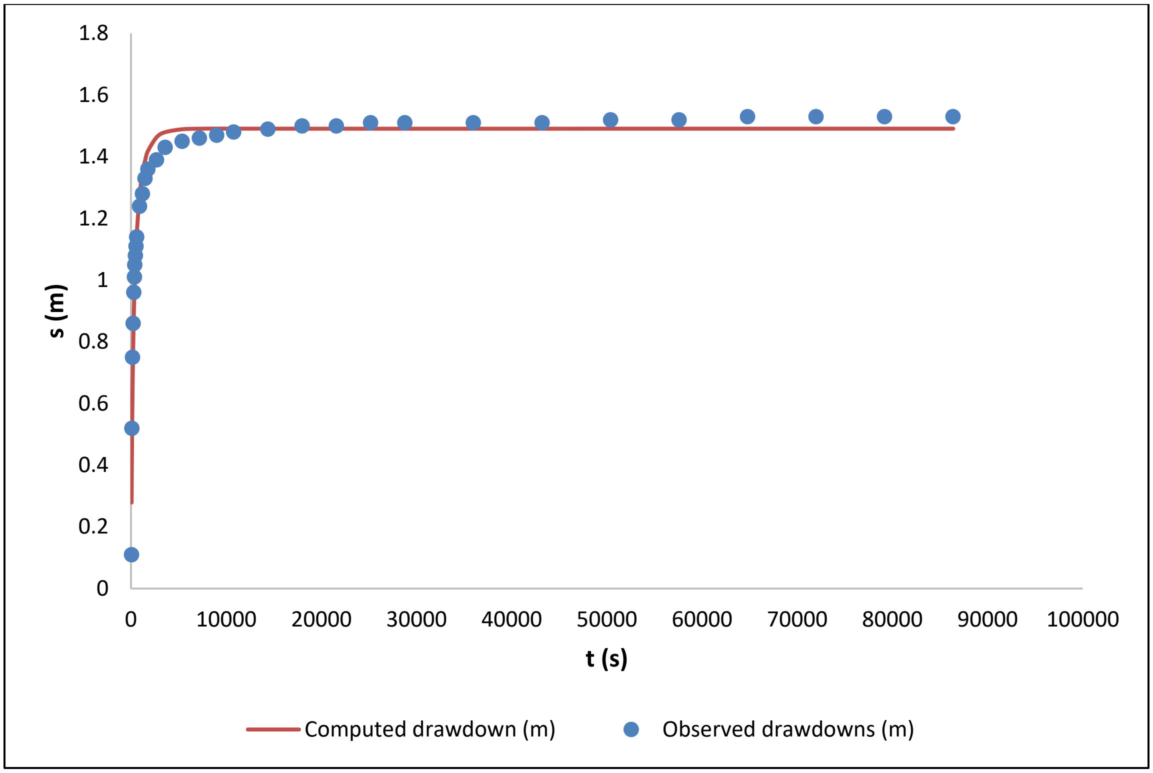


***Figure 3.4**: Comparison of observed and calculated values, [Dataset: H2].*

The results obtained by GWO for the H1 dataset are approximately 45 times more accurate than those obtained by a manual curve matching.

Based on the obtained results, we can say that the gray wolf optimizer algorithm (GWO) is reliable, so that as we've seen in our work, the results are accurate and the error rate is very small.

3.4 performance curves

A performance curve shows the convergence of the GWO towards the optimal solution. It presents the variation of the objective function according to iterations. The maximum number of iterations $Iter_{max}$ was set equal to 200 for the Theis solution, while it was set equal to 500 for the Hantush and Jacob solution. These maximum number of iterations were obtained by experimental tests, these values are considered sufficient for the automatic interpretation procedure to converge towards the optimal solution.

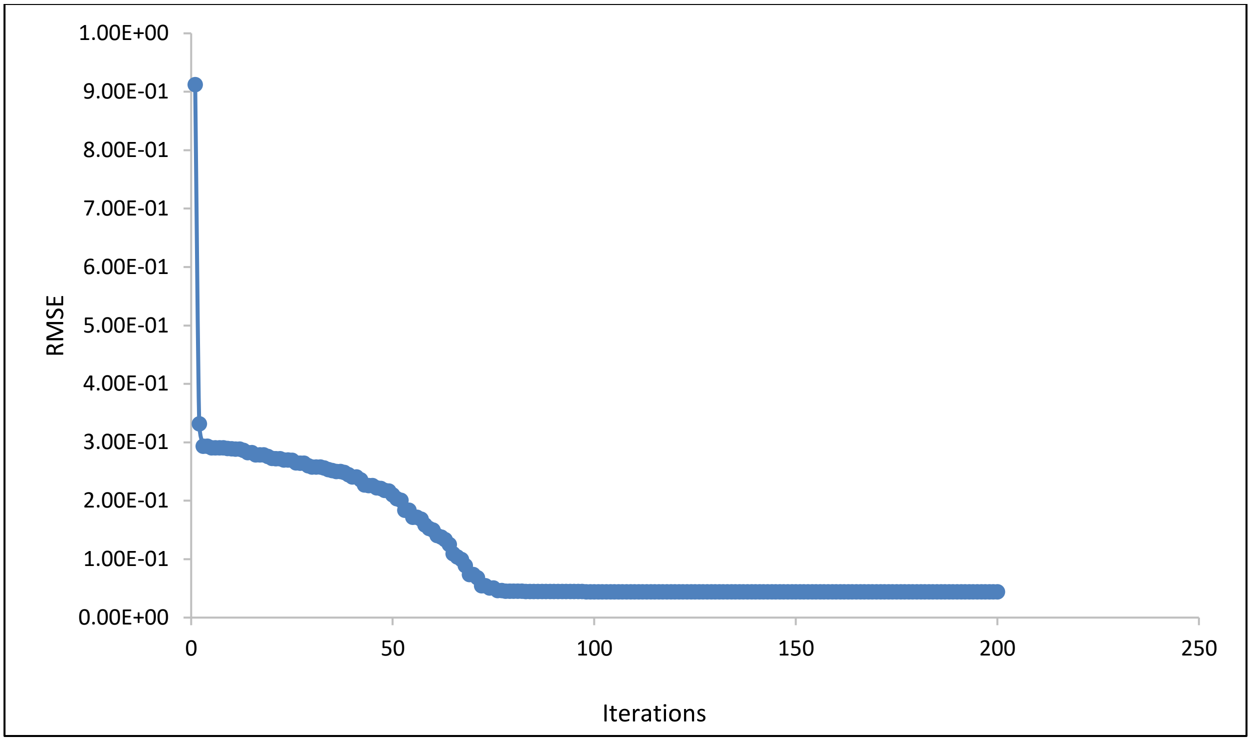


***Figure 3.5**: Best agent performance curve, [Dataset: T1].*

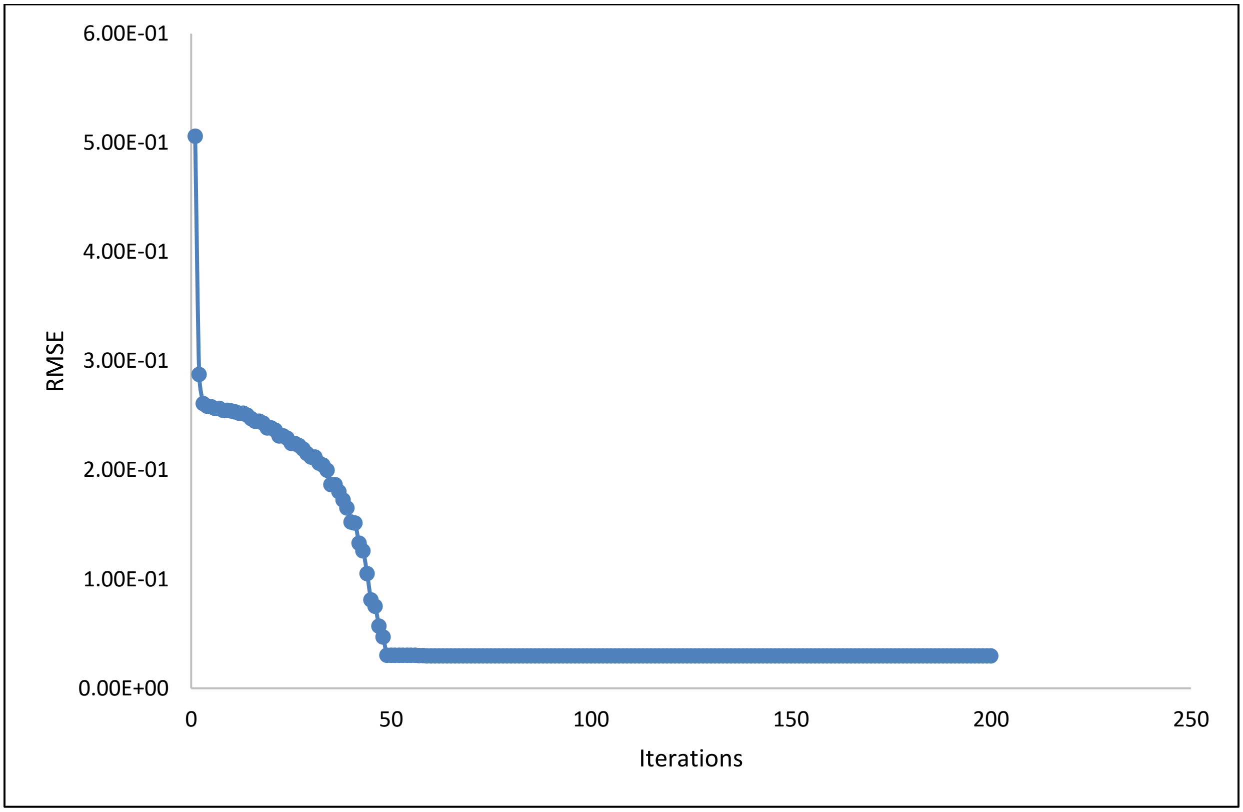


***Figure 3.6**: Best agent performance curve, [Dataset: T2].*

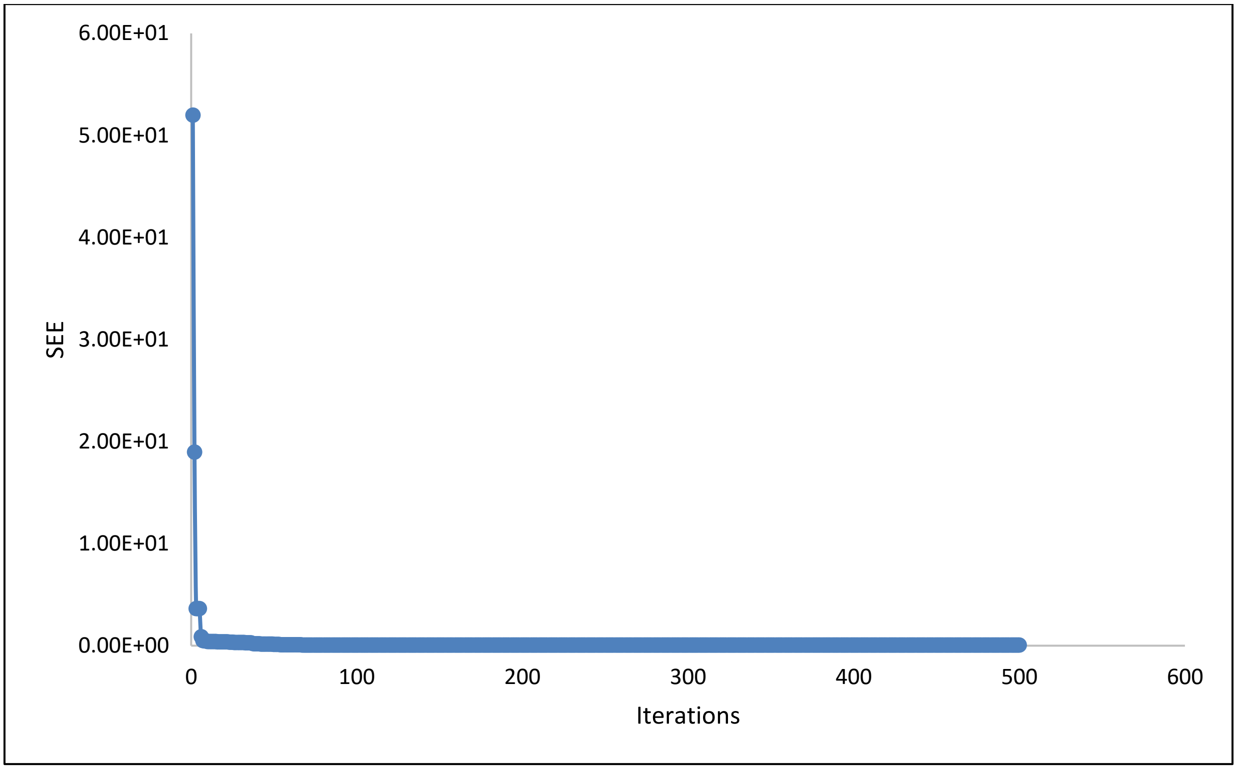


***Figure 3.7**: Best agent performance curve,, [Dataset: H1].*

The performance curves show that the value of the objective function decreases rapidly in the first iterations and then continues slowly their decent. The optimal solutions when Theis' solution is used were obtained at 98 and 103 iterations respectively for the T1 and T2 datasets. For the interpretation by the solution of Hantush and Jacob, the optimal solution was obtained at the 473 iteration for the H1 dataset.

Conclusion

The results presented in this chapter show the GWO ability to interpret data from pumping tests conducted in confined and leaky aquifers. Knowing that the quality of adjustment of analytical solutions to the observed data also depends on the ability of the analytical solutions to sustain the hydrogeological field conditions, as well as the quality of drawdown measurement.

General conclusion

The interpretation of data (time-drawdown) from pumping tests is the best to identify the hydraulic parameters of an aquifer. Today, interpretation using curves matching is still used, even if it depends on visual inspection of the operator, which gives it a subjective aspect.
In this work, the interpretation of time-drawdown curves was automated using a stochastic optimization algorithm. Standard curves that are graphic solutions such as W(u) and W (u,r/B) have been replaced by their equations Mathematics. Based on his experience and the appearance of the curve time-drawdown observed, the engineer chooses from the analytical solutions available the one that is most representative of the site studied. Since, apart from the ability of the optimization technique to feed analytical models with the best parameters (transmissivity, coefficient storage, ... etc.), the quality of interpretation also depends on the ability of the analytical solution adopted to support the hydrogeological conditions of the field. In this modest work, we automated the interpretation of time-drawdown data measured during the execution of pumping tests using a nature-inspired metaheuristic algorithm called Grey Wolf Optimizer (GWO). Two types of aquifer systems were considered, the confined and the leaky aquifers. The GWO was linked to the Theis drawdown solution when analyzing time-drawdown data coming from confined aquifers and to Hantush and Jacob solution for those coming from leaky aquifers. Here, the role of the GWO is to minimize the misfit between the observed and the computed time-drawdown data. The sum of squared errors (SSE) was used as an objective function to assess the interpretation accuracy. The proposed approach was tested on four time-drawdown datasets. As an iterative optimizer, the maximum number of iterations was fixed equal to 200 when analyzing confined aquifers datasets, and 500 when analyzing leaky aquifers datasets. The obtained results are very accurate compared to those obtained by using the classical curve matching method. The methodology developed in this work is encouraging, and proves to be a good tool for interpreting pumping tests. However, the robustness of the GWO should be evaluated in a future study.

References

Bateni SM, Mortazavi-Naeini M, Ataie-Ashtiani B, Jeng DS, Khanbilvardi R (2015) Evaluation of methods for estimating aquifer hydraulic parameters. Appl Soft Comput J 28:541–549

Batu V (1998) Aquifer Hydraulics. Wiley, New York

Halford KJ, Kuniansky EL (2002) Documentation of spreadsheets for the analysis of aquifer-test and slug-test data. USGS, Report 2002-197

Hantush MS, Jacob CE (1955) Non-steady radial flow in an infinite leaky aquifer. Trans Am Geophys. Union 36(1):95–100

Hunt B (1977) Calculation of The Leaky Aquifer Function. J Hydrol 33(1-2):179–183

Kresic N (1997) Quantitative Solutions in Hydrogeology and Groundwater Modeling, 1st edn. CRC Press, Boca Raton, FL, USA

Kresic N (2007) Hydrogeology and groundwater modeling. 2nd ed, CRC Press, Boca Raton

Mirjalili S, Mirjalili SM, Lewis A (2014) Grey Wolf Optimizer. Adv Eng Softw 69: 46–61

Theis CV (1935) The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage. Am Geophys Union Trans 16(2):519–524

Veling EJM, Maas C (2010) Hantush well function revisited. J Hydrol 393(3–4):381–388