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Course Handout (Lectures and Tutorials)

Intended for students of: 3rd-year. Level: bachelor's degree.

Elements of heat transfer
Courses and solved problems

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FOREWORD

The content of this handout is intended for third-year bachelor's degree students in Industrial Maintenance Engineering (mechanical engineering). It carries the fundamental notions and basic concepts in the field of heat transfer. With the help of this basic handout, undergraduate students should be able to comprehend and analyze heat transfer problems that they will probably come across in the field. The material covers principles, materials, and applications. It takes a methodical approach to discussing heat transfer problems.

This handout is broken up into four sections. A broad introduction to the study of thermal systems, the integral-volume energy conservation equation, and the heat flux vector (including the contributions from several mechanisms) is provided in Chapter One. To facilitate heat transfer visualization, the idea of heat flow vector tracking is highlighted.

Conduction heat transfer is covered in chapter two. This makes it possible to investigate how temperature and heat flow rate vary over time and space in a heat transfer medium or through multimedia composites. The ideas of thermal nodes, heat flow rate, and thermal conduction resistance in connected.

The third chapter addresses natural and forced convection, which is an important heat transfer mechanism that is a heat exchanger between a solid and fluid in contact. We will express the basic concepts and relationship of heat transfer by convection.

In Chapter Four, the principles of radiation heat transmission between opaque, diffuse, and gray surfaces are examined, along with radiation heat transfer on the heat transfer medium's bounding surfaces.

Throughout the book, example issues are solved to demonstrate to students how to apply their knowledge of thermal engineering to solve difficulties and how to obtain additional understanding.

Each chapter concludes with some problems solved to demonstrate to students how to apply their knowledge of thermal engineering skills to solve difficulties and how to obtain additional understanding.

Content

Chapter 4 Heat transfer by radiation

Chapter 1

Basic concepts of heat transfer

Chapter 1: Basic concepts of heat transfer

1.1. Heat transfer mechanisms

The word "heat transfer" refers to a field of study where the main focus is on the specifics or mechanisms involved in the transmission of energy in the form of heat. The energy that is transferred due to a temperature differential is known as heat. The transfer of heat occurs from hotter locations to colder ones. Different kinds of heat transfer mechanisms are commonly referred to as modes. Heat transport occurs primarily by conduction, radiation, and convection. Heat treatment of steel forgings and power plant waste heat dissipation are two common industrial applications of heat transfer.

1.2 Conduction

Conduction is the process by which heat moves from one zone of a higher temperature to another zone of a lower temperature as a result of a temperature gradient without the substance itself appearing to move. Energy is transferred from more energetic molecules to those with a lower energy level during the conduction process, which occurs at the molecular level. The heat flux, or the rate of heat transfer per unit area normal to the direction of heat flow, q" is directly proportional to the temperature gradient at the macroscopic level.

$$q''_x = -\lambda \frac{dT}{dx} \qquad (1.1)$$

where the proportionality constant, λ, is a material feature and a transport property called heat conductivity. Heat is transmitted in the direction of decreasing temperature, which results in the minus sign. The one-dimensional version of Fourier's law of heat conduction is found in Eq. (1.1). Since the heat flux is a vector quantity, we may express Fourier's law—also known as the conduction rate equation—in a more generic way as follows:

$$q'' = -\lambda \nabla T \qquad (1.2)$$

where T is the scalar temperature field and Δ is the three-dimensional del operator. It is evident from Eq. (1.2) that the heat flux vector q" truly depicts a thermal energy current flowing in the direction of the steepest temperature gradient. One can apply Eq. (1.1) directly to a one-dimensional heat flow in the x direction in the plane wall depicted in **Fig. 1.1**, leading to integration.

$$q_x'' = \lambda \frac{T_1 - T_2}{L} \tag{1.3}$$

In this case, Ax is the wall thickness, T_1 and T_2 are the wall-face temperatures, and the thermal conductivity is taken to be constant. Keep in mind that q = q"*A, where q is the rate at which heat moves through area A. Equation (1.3) can be expressed as follows:

$$q_x = \frac{kA}{L}(T_1 - T_2) = \frac{T_1 - T_2}{L/_{kA}} \tag{1.4}$$

$$q_x = \frac{T_1 - T_2}{R_{th}} = \frac{thermal\ potential\ difference}{thermal\ resistance} \tag{1.5}$$

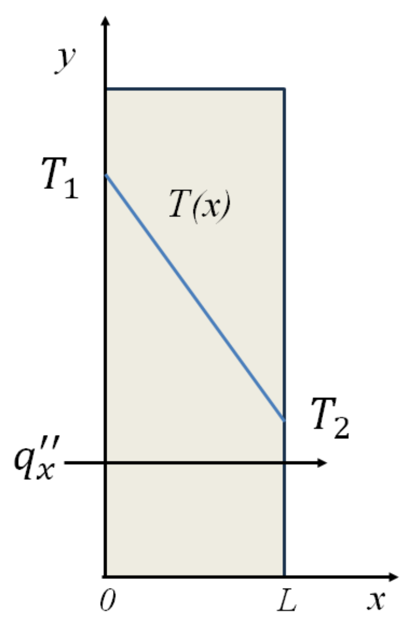


Figure 1.1: One-dimension heat transfer by conduction

1.3. Radiation

A body emits electromagnetic radiation due to its temperature and internal energy expenditure, which is also known as thermal radiation or radiation. Hence, thermal radiation shares the same characteristics as radio waves, x-rays, and visible light; the only differences are in their generating sources and wavelengths. The visible spectrum is defined as the range of electromagnetic radiation that the eye is sensitive to, which is between 0.39 and 0.78 µm. The wavelength ranges of radio waves and x-rays are 1x103 to 2x1010 µm and 1×10-5 to 2x10-2 µm, respectively, while the majority of thermal radiation falls within the range of 0.1 to l00 µm. Thermal radiation is released by all heated solids and liquids as well as certain gases. Radiation transfers energy without a material medium, whereas conduction transfers energy through one. A vacuum is the ideal environment for radiation transfer to occur. Macroscopic thermal radiation calculations are based on the Stefan-Boltzmann law, which expresses the relationship between the energy flux released by a blackbody, or ideal radiator, and the fourth power of the absolute temperature:

$$e_b = \sigma T^4 \qquad (1.6)$$

The Stefan-Boltzmann constant, denoted by τ, has a value of 5.669×10-8 W/ ($m^2.K^4$). Since engineering surfaces generally don't function as the best radiators, the following law is amended to apply to real surfaces.

$$e_b = \varepsilon \sigma T^4 \qquad (1.7)$$

The surface's emissivity, denoted by the term ε, ranges from 0 to 1. The net heat exchange that results from the radiation exchange between two blackbodies is then proportional to the difference in T^4. The net heat exchange from body 1 to body 2 is given by the following if the first body "sees" only body 2 (see **Fig. 1.2**):

$$q = \sigma A_1 (T_1^4 - T_2^4) \qquad (1.8)$$

When only a portion of the energy from body 1 is captured by body 2 due to the geometric arrangement

$$q = \sigma A_1 F_{1-2}(T_1^4 - T_2^4) \quad (1.9)$$

where F_{1-2}, also known as a view factor or shape factor, is the percentage of energy that body 2 intercepts as it leaves body 1.

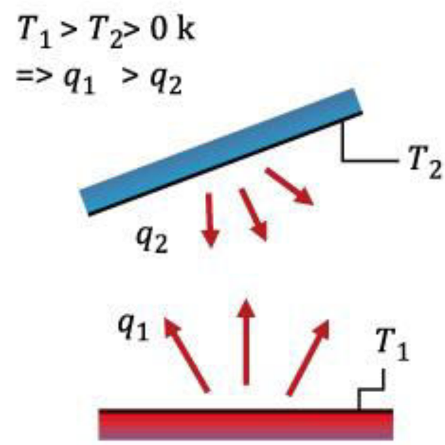


Figure 1.2: Thermal radiation between two surfaces.

The electromagnetic spectrum. Thermal radiation occurs in range of the electromagnetic spectrum of energy emission. Accordingly, it exhibits the same wavelike properties as light or waves. Each quantum of radiant energy has a wavelength, λ, and a frequency, v, associated with it.

The full electromagnetic spectrum includes an enormous range of energy-bearing waves, of which heat is only a small part. Table 1.1 lists the various forms over a range of wavelengths that spans 17 orders of magnitude. Only the tiniest "window" exists in this spectrum through which we can see the world around us. Heat radiation, whose main component is usually the spectrum of infrared radiation, passes through the much larger window- about three orders of magnitude in λ and v.

Black bodies. The model for the perfect thermal radiator is a so-called black body. This is a body which absorbs all energy that reaches it and reflects nothing. The term can be a little confusing, since such bodies emit energy. Thus, if we possessed infrared vision, a black body would glow with “color” appropriate to its temperature. Of course, perfect radiators are “black” in the sense that they absorb all visible light (and all other radiation) that reaches them.

Tabel 1.1: Various Forms of Thermal Radiation over Wavelengths

Type of Radiation	Wavelength Range (meters)	Wavelength Range (micrometers)	Typical Source
Ultraviolet (UV)	10^{-8} to 4×10^{-7}	0.01 to 0.4	Very hot objects (e.g., the Sun)
Visible Light	4×10^{-7} to 7×10^{-7}	0.4 to 0.7	Incandescent objects, like light bulbs
Near Infrared (NIR)	7×10^{-7} to 2.5×10^{-6}	0.7 to 2.5	Warm objects (e.g., human body)
Mid Infrared (MIR)	2.5×10^{-6} to 25×10^{-6}	2.5 to 25	Hot objects (e.g., furnaces)
Far Infrared (FIR)	25×10^{-6} to 1×10^{-4}	25 to 100	Cooler objects (e.g., planets)
Microwaves	1×10^{-4} to 1×10^{-1}	100 to 1,000,000	Extremely cold objects, cosmic background radiation

Notes:

• Ultraviolet Radiation (UV): Emitted by very hot objects like the Sun, often associated with temperatures above 3000 K.

• Visible Light: The range humans can see, with objects like the Sun or light bulbs emitting significant visible light.

• Infrared Radiation (IR): Emitted by cooler objects, like the human body or a stove.

• Microwaves: Typically emitted by extremely cold objects, such as in cosmic background radiation.

It is necessary to have an experimental method for making a perfectly black body. The conventional device for approaching this ideal is called by the German term hohlraum, which literally means “hollow space”.

What are the important features of a thermally black body. First consider a distinction between heat and infrared radiation. Infrared radiation refers to a particular range of wavelengths, while heat refers to the whole range of radiant energy flowing from one body to another. Suppose that a radiant heat flux, q, falls upon a translucent plate that is not black, as shown in **Fig. 1.3** a fraction, α, of the total incident anergy, called absorptance, is absorbed in the body; a fraction, ρ, called the reflectance, is reflected from it; and a fraction, τ, called the transmittance, passes through. Thus $\alpha + \rho + \tau = 1$

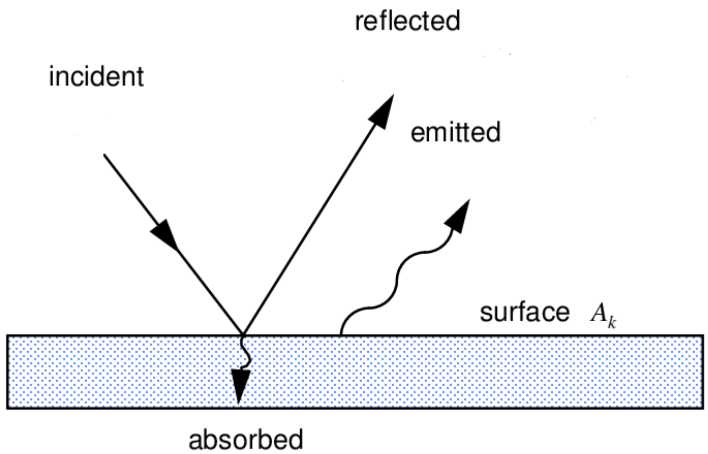


Figure 1.3: The distribution of radiation heat flux incident on a translucent plate

1.4. Convection

Convection is the process of transferring heat from a bounding surface to a fluid in motion or from the heat transfer across a flow plane inside the moving fluid. It is occasionally distinguished as a different mechanism of heat transmission. Forced convection is the term used when fluid motion is created by a pump, blower, fan, or other comparable apparatus. Free or natural convection is the process that takes place when fluid motion results from the density difference caused by temperature difference.

When the heat transfer process in these situations is examined in detail, it becomes clear that while the fluid's bulk motion contributes to heat transfer, conduction is the primary heat transfer mechanism—that is, energy is transferred as heat within the flowing fluid. More precisely, internal energy rather than heat is being condemned.

On the other hand, latent heat exchange is also present in some convection systems. The fluid's phase transition from its liquid to vapor phases is typically linked to this latent heat exchange. The two unusual circumstances are condensation and boiling.

Heat Transfer Coefficient: It is convenient to add a heat transfer coefficient h, which is described by Eq. (1.12), also known as Newton's rule of cooling, in convective processes involving heat transfer from a boundary surface exposed to a relatively low-velocity fluid stream:

$$q'' = h(T_w - T_f) \qquad (1.10)$$

Here T_w is the surface temperature and T_f is a characteristic fluid temperature.

Surfaces with unbounded convection, including plates, tubes, bodies of rotation, etc., submerged in a huge body of fluid are commonly defined as h in Eq. (1.10) with T_f representing the fluid's temperature far from the surface, which is commonly denoted as T_∞. (**Fig. 1.4**).

As q=A*q ", from Eq. **(1.10)** the thermal resistance in convection heat transfers is given by

$$q = hA(T_w - T_f) = (T_w - T_f)/\frac{1}{hA}\text{, with } R_{th} = \frac{1}{hA} \qquad (1.11)$$

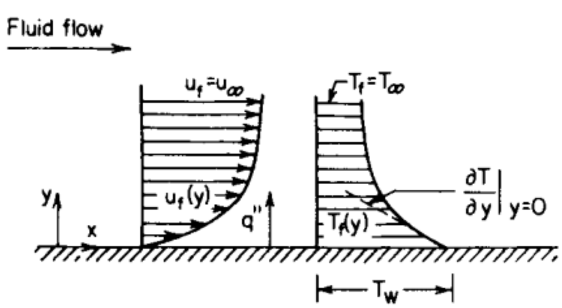


Figure 1.4: Velocity and temperature distribution in flow over a flat plate.

1.5. Combined Heat Transfer Mechanisms

In actuality, heat transmission usually happens via two parallel pathways. **Fig. 1.5** provides a typical example. In this instance, convection and radiation work together to remove the heat that is transferred through the plate from the plate surface. In this instance, an energy balance is provided.

$$q = q_{conv} + q_{rad}$$

$$-\lambda A \left.\frac{dT}{dx}\right|_w = hA(T_w - T_\infty) + \sigma A \varepsilon (T_w^4 - T_a^4) \qquad (1.12)$$

Where T_a is the temperature of the surroundings, λ is the thermal conductivity of the solid plate, and ε is the emissivity of the plate.

Radiation is frequently used in conjunction with other heat transfer methods in applications, and solving these issues can frequently be made easier by assigning a thermal resistance R_{th} to radiation. R_{th} has a definition akin to that of convection and conduction thermal resistance. For the example in Fig. 1.3, if the heat transfer by radiation is expressed as follows:

$$q = \frac{T_w - T_a}{R_{th}} \qquad (1.13)$$

The resistance is given by

$$R_{th} = \frac{T_w - T_a}{\sigma A \varepsilon (T_w^4 - T_a^4)} \qquad (1.14)$$

Also, a heat transfer coefficient h_r can be defined for radiation:

$$h_r = \frac{1}{R_{th} A} = \frac{\sigma \varepsilon (T_w^4 - T_a^4)}{T_w - T_a} = \sigma \varepsilon (T_w + T_a)(T_w^2 + T_a^2) \qquad (1.15)$$

Instead of making the heat rate proportional to the difference between two temperatures to the fourth power, we have linearized the radiation rate equation in this instance. It should be noted that although the convective heat transfer coefficient h has a relatively modest temperature dependence, h_r has a strong temperature dependence.

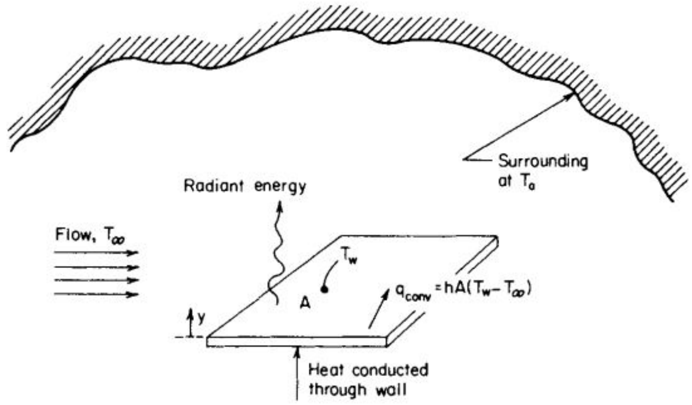


Figure 1.5: Combination of conduction, convection and radiation heat.

1.6. Conclusion

This chapter provides a comprehensive introduction of the fundamental mechanisms of heat transfer: conduction, convection, and radiation. Each mechanism operates through distinct physical processes and has unique characteristics. In practical applications, heat transfer often involves a combination of these mechanisms. For instance, a surface may lose heat through both convection and radiation simultaneously. Understanding these mechanisms and their interactions is crucial for designing efficient thermal systems. The chapter also introduces key concepts such as thermal resistance, which simplifies the analysis of heat transfer problems by treating heat flow analogously to electrical current in a circuit. Overall, this chapter lays the groundwork for understanding how heat is transferred in various contexts, providing the necessary tools to analyze and design systems where heat transfer plays a critical role.

Chapter 2

Heat transfer by conduction

2.1. Introduction

The purpose of conduction analysis is to determine the temperature distribution within a medium as a function of boundary circumstances. Using Fourier's law, the heat flux distribution may be calculated given the temperature distribution.

2.1.1. Fourier's law

In Section 1.1, we introduced Fourier's law, Eq. (1.1), which connects the heat flow (W/m^2) in the x-direction, per unit area perpendicular to the direction of transfer, to the product of thermal conductivity (W/m*K) and temperature gradient (dT/dx) in the x-direction.

$$q''_x = -k\frac{dT}{dx} \tag{2.1}$$

According to Fourier's law, heat flux is directional. **Figure 2.1-a** depicts the one-dimensional link between the coordinate system, heat flow direction, and temperature gradient.

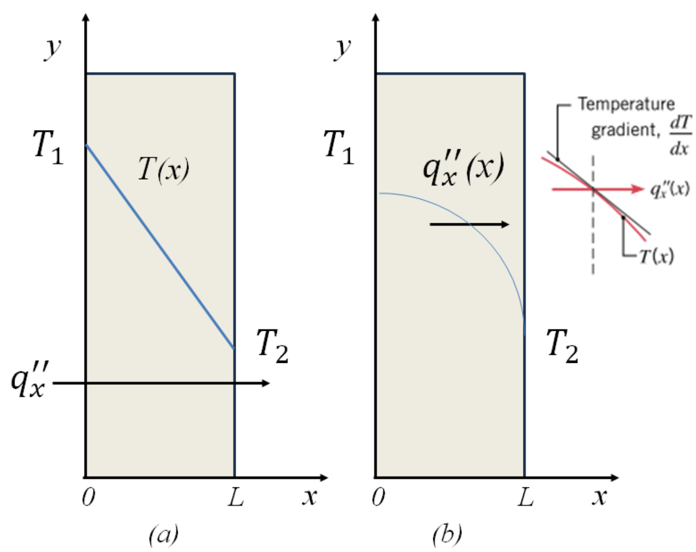


Figure 2.1: The relationship between heat flux, temperature gradient, and coordinate system. One-dimensional temperature distributions include (a) linear with constant heat flux and (b) nonlinear with varying heat flux.

If the temperature distribution is linear, the gradient is constant, and so the heat flux is a constant, q_x''. is independent of x. When the temperature distribution is not linear with the x-coordinate, as illustrated in **Fig. 2.1-b**, the gradient is no longer constant, and the heat flux becomes a function of the x-coordinate.

➢ heat flux distributions

Consider the object in Figure 2.2 that is undergoing two-dimensional conduction. Take note of the isothermal line near the object's middle. The heat flux (,q-n-$''$), a vector quantity, is in the direction normal to the isotherm. A temperature gradient in the n-direction sustains the heat flux, which can be represented in terms of x- and y-direction components.

$$q_n'' = q_x'' + q_y'' \qquad (2.2)$$

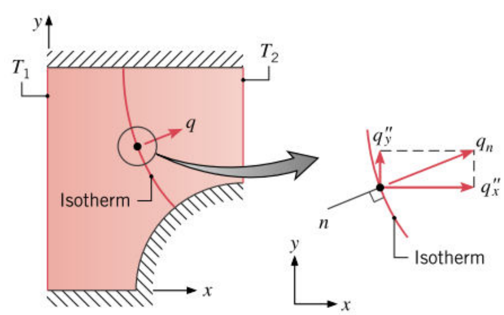


Figure 2.2: Heat flux vector normal to an isotherm in a two-dimensional coordinate system.

2.2. Heat Equation

We now explore how to determine the temperature distribution in a medium due to boundary conditions. We will calculate the temperature distribution T(x, t) associated with one-dimensional (Cartesian coordinate) heat transfer in a stationary, homogeneous medium with uniform volumetric energy generation (W/m^3). We will establish a differential system (element), identify key energy processes, introduce rate equations, and apply energy conservation. The result is a differential equation whose solution for the specified boundary and initial circumstances gives the temperature distribution in the medium.

In the above example of one-dimensional, transient conduction with volumetric energy generation, the heat equation is:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q} = \rho c \frac{\partial T}{\partial t} \qquad (2.3)$$

With $\dot{q} = \frac{\dot{E}}{V}$

Where the temperature is a function of the x coordinates and time T (x, t). In this book, we will simply show remedies for temporary situations. We will, nevertheless, develop solutions for the steady-state form of the heat diffusion equations for these conditions. Volumetric energy generation occurs under steady-state conditions.

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q} = 0 \qquad (2.4)$$

steady-state conditions, without volumetric energy generation, where the temperature depends exclusively on the x coordinate, T. (x).

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = 0 \qquad (2.5)$$

2.2.1 Boundary and Initial Conditions

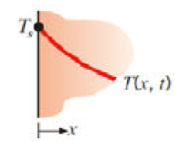
To calculate the temperature distribution in a medium, solve the appropriate form of the heat equation. The answer, however, is determined by the physical conditions at the medium's boundaries and, if the problem is time-dependent, by the conditions in the medium at some beginning point in time. Several common border conditions can be easily represented mathematically. Because the heat equation, Eq. 2.3, is second order in spatial coordinates, two boundary conditions are required to characterize the system. Because the equation is first order in time, only one condition, known as the beginning condition, needs to be given.

Table 2.1 summarizes the three types of boundary conditions most typically seen in heat transfer. For a one-dimensional system, the requirements are defined at the surface with x = 0. Heat transmission occurs in the positive x-direction with the temperature distribution, which may be time dependent (indicated as T) (x, t).

Table 2.1: Boundary Conditions for the Heat Equation at the Surface (x=0)

1. Constant surface temperature

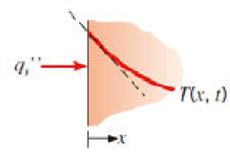
$$T(0, t) = T_s \qquad (2.31)$$



2. Constant surface heat flux

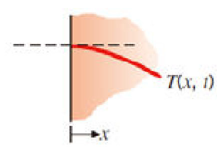
(a) Finite heat flux

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = q_s'' \qquad (2.32)$$



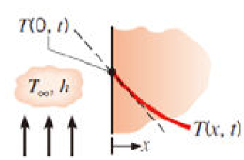
(b) Adiabatic or insulated surface

$$\frac{\partial T}{\partial x}\bigg|_{x=0} = 0 \qquad (2.33)$$



3. Convection surface condition

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = h[T_\infty - T(0, t)] \qquad (2.34)$$



➢ **Dirichlet boundary condition**

The first variety has a constant surface temperature. This state is closely approximated, for example, when the surface is subjected to exceptionally high convection coefficients. Such conditions arise from boiling or condensation, and in both cases, the surface remains at the temperature of the phase transition process.

➢ **Neumann boundary condition**

The second type is constant surface heat flux. Fourier's law describes the relationship between heat flux and surface temperature gradient. This situation could be achieved by applying a thin-film or patch electric heater to the surface or irradiating it with a heat lamp.

A specific case of this requirement is the completely insulated, or adiabatic, surface with a zero gradient. If the temperature distribution is asymmetric, a surface corresponding to the highest or lowest temperature may also be an adiabatic surface.

The third type of surface condition is convection. This situation corresponds to the presence of convection heating (or cooling) at the surface and is determined by the surface energy balance.

2.2.2 The Plane Wall

Temperature is only a function of the x coordinate in one-dimensional conduction in a flat wall under steady-state conditions, and heat transmission happens in this direction. In **Figure 2.3**, a plane wall separates two fluids with differing temperatures. Heat is transferred by convection from the hot fluid at $T_{\infty,1}$ to one surface of the wall at $T_{s,1}$, by conduction through the wall, and by convection from the opposite surface of the wall at $T_{s,2}$ to the cold fluid at $T_{\infty,2}$. We begin by looking at the circumstances within the wall. We first determine the temperature distribution, which allows us to calculate the conduction heat transfer rate.

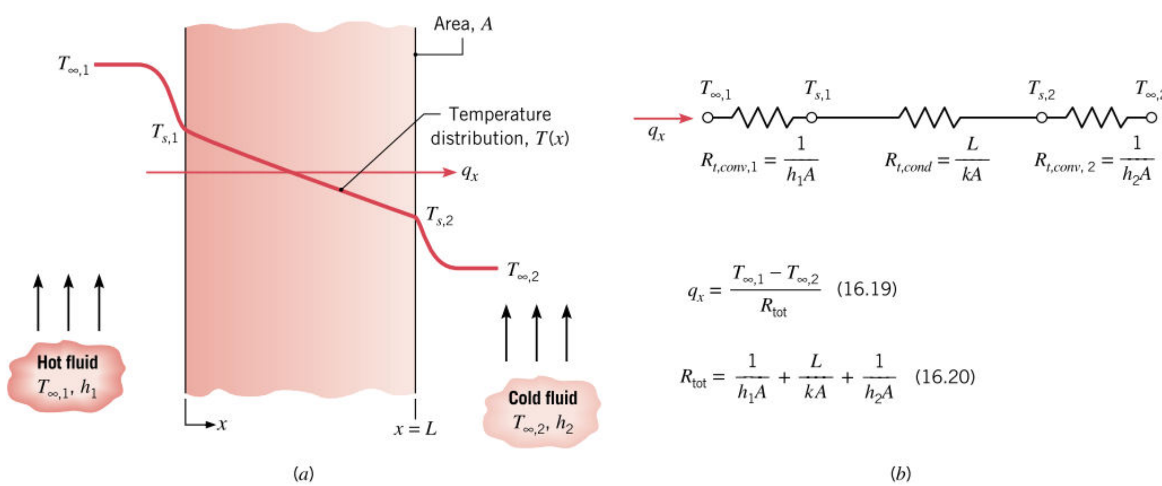


Figure 2.3: heat transfer across a plane wall. (a) Temperature distribution. (b) Equivalent thermal circuit.

➢ Temperature Distribution

The temperature distribution in the wall can be calculated by solving the heat equation with the appropriate boundary conditions. For steady-state conditions with no energy generation within the wall, the heat equation should be written as Eq (2.5).

$$\frac{\partial^2 T}{\partial x^2} = 0 \rightarrow \frac{d^2 T}{dx^2} = 0$$

The equation may be integrated twice to obtain the general solution

$$T(x) = C_1 x + C_2 \qquad (2.6)$$

To obtain the constants of integration, C_1 and C_2 , boundary conditions must be introduced.

We choose to apply conditions of the first kind at x=0 and x=L, in which case $T(0) = T_{s,1}$ and $T(L) = T_{s,2}$.

Applying the condition at x=0 to the general solution, it follows that $C_2 = T_{s,1}$.

Similarly, at x=L, $T_{s,2} = C_1 L + C_2 = C_1 L + T_{s,1}$.

In which case $C_1 = \frac{T_{s,2} - T_{s,1}}{L}$

Substituting into the general solution, the temperature distribution becomes

$$T(x) = \frac{T_{s,2} - T_{s,1}}{L} x + T_{s,1} \qquad (2.7)$$

This conclusion clearly shows that, for one-dimensional, steady-state conduction in a flat wall with no energy generation and constant thermal conductivity, the temperature varies linearly with x.

Now that we have the temperature distribution, we can use Fourier's law, Eq. (2.1), to calculate the conduction heat flux (W/m^2). That is.

$$q''_x = -k\frac{dT}{dx} = \frac{k}{L}(T_{s,1} - T_{s,2}) \qquad (2.8)$$

For the plane wall, $\boldsymbol{A}$ is the area of the wall normal to the direction of heat transfer, which is a constant independent of x, so that the conduction heat rate (W) equals

$$q_x = q''_x A = \frac{k}{L} A(T_{s,1} - T_{s,2}) \tag{2.9}$$

Eq. **(2.8)** and **(2.9)** indicate that both the heat flux and the heat rate are constants, independent of x.

➢ Thermal Resistance and Thermal Circuits

At this time, we should observe that Eq. 2.9 suggests an extremely essential concept. There is an analogy between the conduction of heat and electrical current. Thermal resistance, like electrical resistance, can be connected with heat conduction. Eq. (2.9) shows that the thermal resistance for conduction in a planar wall is:

$$R_{t,cond} = \frac{T_{s,1} - T_{s,2}}{q_x} = \frac{L}{kA} \tag{2.10}$$

Thermal resistance can also relate to heat transfer via convection at a surface. From Newton's law of cooling

$$q = hA(T_s - T_\infty) \tag{2.11}$$

The thermal resistance for convection from a surface is

$$R_{t,conv} = \frac{T_s - T_\infty}{q} = \frac{1}{hA} \tag{2.12}$$

Circuit representations are an effective tool for conceptualizing and quantifying heat transfer problems. Figure shows the analogous thermal circuit for the plane wall under convection surface conditions (2.3-b). The circuit is made up of resistance elements and nodes, which represent surface or fluid temperatures. The heat transfer rate can be calculated by considering the network's elements and nodes separately or in combination. As qx remains constant across the network, it follows that

$$q_x = \frac{T_{\infty,1} - T_{s,1}}{1/h_1 A} = \frac{T_{s,1} - T_{s,2}}{L/kA} = \frac{T_{s,2} - T_{\infty,2}}{1/h_2 A} \tag{2.13}$$

The heat transfer rate can be defined in terms of the overall temperature difference, $T_{\infty,1}$, $T_{\infty,2}$, and the total thermal resistance, R_{tot},

$$q_x = \frac{T_{\infty,1} - T_{\infty,2}}{R_{tot}} \qquad (2.14)$$

Because conduction and convection resistances are in series and can be added, the total thermal resistance is

$$R_{tot} = R_{t,conv,1} + R_{t,cond} + R_{t,conv,2} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \qquad (2.15)$$

Another resistance may be applicable if a surface is exposed to broad, isothermal surroundings (Sec. 1.4). In particular, radiation exchange between the surface and its surroundings may be crucial, and the rate can be calculated using Eq (2.13). Thus, a thermal resistance to radiation can be defined as

$$R_{t,rad} = \frac{T_s - T_{sur}}{q_{rad}} = \frac{1}{h_{rad} A} \qquad (2.16)$$

Where h_{rad} is the linearized radiation coefficient calculated from Eq (1.15). Surface radiation and convection resistances work in parallel, and if $T_{\infty}=T_{sur}$, they can be combined to form a single, effective surface resistance.

2.2.3 Composite Wall

Equivalent thermal circuits can also be used in more complex systems like composite walls. Because of the layers of different materials, such walls can have a variety of series and parallel thermal resistances. Consider the series composite wall in Figure 2.4. The one-dimensional heat transfer rate for this system can be represented as:

$$q_x = \frac{T_{\infty,1} - T_{\infty,3}}{R_{tot}} \qquad (2.17)$$

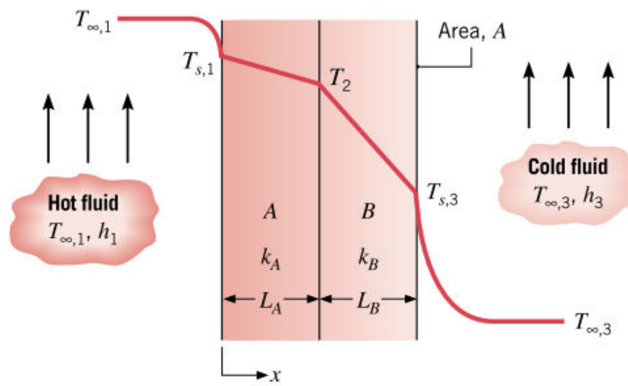


Figure 2.4.(a): Series-composite wall with convection on both surfaces.

Where $T_{\infty,1}$, $T_{\infty,3}$ is the overall temperature difference and R_{tot} includes all thermal resistances. Hence

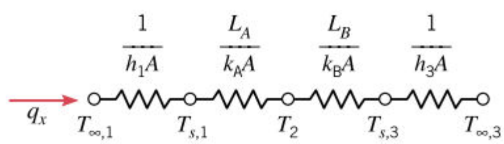


Figure 2.4.(b): equivalent thermal circuit of series-composite wall.

$$q_x = \frac{T_{\infty,1} - T_{\infty,3}}{[(1/h_1A) + (L_A/k_1A) + (L_B/k_BA) + (1/h_3A)]} \tag{2.18}$$

Alternatively, the heat transfer rate may be calculated using the temperature differential and resistance associated with each element. For example:

$$q_x = \frac{T_{\infty,1} - T_{s,1}}{(1/h_1A)} = \frac{T_{s,1} - T_2}{(L_A/k_1A)} = \frac{T_2 - T_{s,3}}{(L_B/k_BA)} = \frac{T_{s,3} - T_{\infty,3}}{(1/h_3A)} \tag{2.19}$$

In composite systems, it is typically useful to work with an overall heat transfer coefficient, U, which is specified by an equation similar to Newton's law of cooling. Accordingly

$$q_x = UA\Delta T \tag{2.20}$$

Where ΔT is the overall temperature difference. The overall heat transfer coefficient is related to the total thermal resistance, and from Eqs. (**2.17** and **2.20**) we see that $UA=1/R_{tot}$.

$$U = \frac{1}{R_{tot}A} = \frac{1}{[(1/h_1)+(L_A/k_1)+(L_B/k_B)+(1/h_3)]} \qquad (2.21)$$

Composite walls can also be identified by series-parallel layouts, with the heat rate dictated by a network of thermal resistances in series and parallel patterns.

2.2.4 Cylinder body

Cylindrical and spherical systems frequently experience temperature gradients exclusively in the radial direction, and can thus be considered as one-dimensional. As seen with the plane wall, such systems may be evaluated using the heat equation to determine temperature distribution and heat rate. In this part, we have avoided the associated derivations and presented the conclusions that are utilized to construct comparable thermal circuits for radial systems.

A popular design is the hollow cylinder, whose inner and outer surfaces are exposed to fluids of varying temperatures (Fig. 2.5). For steady-state conditions without energy generation, the temperature distribution in the radial (cylindrical) coordinate system is:

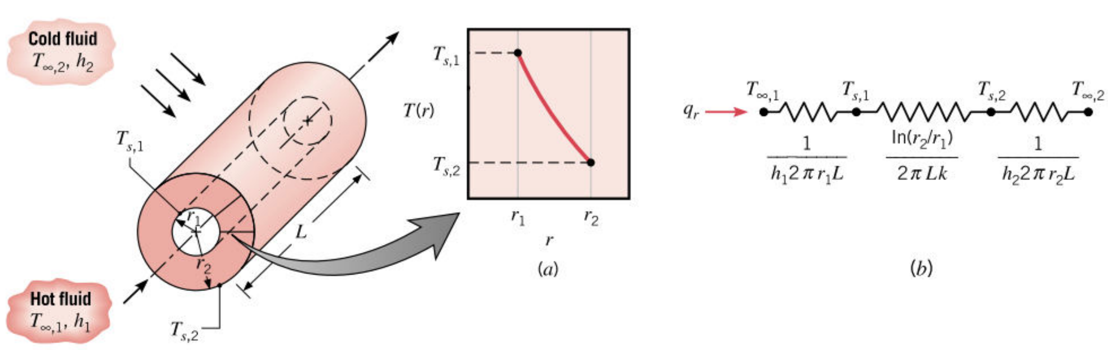


Figure 2.5: Hollow cylinder with convection surface conditions. (a) Logarithmic temperature distribution. (b) Equivalent thermal circuit.

To represent heat propagation in the cylinder, we write that, out of the quantity of heat (***dQ₁***) which penetrates during the period (***dt***) in the slice studied, a portion (***dQ***) is utilized to heat by (***dT***) the mass of the elementary volume (***dV=2πrdrdz***), while the other part (***dQ_2***) is transported to the neighboring slice.

We have:

$$dQ_1 = -k2\pi r dz \frac{\partial T}{\partial r} dt \qquad (2.22)$$

$$dQ = \rho c 2\pi dr dz dT = \rho c 2\pi dr dz \frac{dT}{dt} dt \qquad (2.23)$$

$$dQ_2 = -k2\pi(r + dr)dz \frac{\partial}{\partial r}\left(T + \frac{\partial T}{\partial r} dr\right) dt = -k2\pi r \left(\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} dr\right) dzdt - k2\pi dr \left(\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} dr\right) dzdt \qquad (2.24)$$

If we define the equivalence as (***dQ_1=dQ+dQ_2***), we can see that the last term of (***dQ_2***) closes an indefinitely tiny higher order (***dr_2***), which is inconsequential in comparison to the previous terms. Following the reductions, we obtain:

$$\rho c r \frac{dT}{dt} = k\left(r \frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r}\right) \qquad (2.25)$$

Finally

$$\frac{dT}{dt} = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \qquad (2.26)$$

This is the equation that governs the propagation of heat from the axis in a cylindrical body.

When the steady state is reached, T no longer depends on time t but only on r, and we have ($\partial T/\partial t=0$) and Eq. **(2.26)** takes the form:

$$\frac{d^2 T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0 \qquad (2.27)$$

To integrate this equation, we put ($\partial T/\partial r=u$), and it becomes:

$\frac{du}{dr} + \frac{u}{r} = 0$ or $\frac{du}{u} + \frac{dr}{r} = 0$

Its integration gives: $\ln u + lu\, r = lnA$ (A is a constant).

In this equation we draw: ur=A, hence by replacing ($\partial T/\partial r=u$) we will have $r\frac{dT}{dr} = A$ then $dT = A\frac{dr}{r}$ and

$$T = Aln(r) + B \qquad (2.28)$$

B is a new constant.

This last connection shows that when heat propagates in a solid cylinder from its axis, the temperature changes as a function of distance from the axis according to a logarithmic equation. The boundary conditions must determine the constants A, B.

The heat flux per unit length of cylinder is given by:

$$q_r = -k2\pi r\frac{dT}{dr} = -k2\pi r\frac{A}{r} = -2\pi Ak$$

In terms of boundary conditions, consider the classic instance that we shall study: a cylindrical tube. Case in which the tube's inner and outside temperature surfaces are specified. Let ($\boldsymbol{T_1}$) and ($\boldsymbol{T_2}$) be the temperatures of the tube's internal and external walls, and ΔT be the difference between them.

We will assume $T_1>T_2$

We have now: for $r=r_1$: $T=T_1=ALn(r_1)+B$

for $r=r_2$: $T=T_2=ALn(r_2)+B$

We then have, to determine A and B, the system of two equations, the resolution of which gives:

$$A = \frac{T_1 - T_2}{lnr_1 - lnr_2} = \frac{\Delta T}{ln(r_1/r_2)}$$

$$B = \frac{-T_1 lnr_2 + T_2 lnr_1}{lnr_1 - lnr_2} = \frac{-T_1 lnr_2 + T_2 lnr_1}{\ln{(r_1/r_2)}}$$

By substituting A and B into the general equation (2.28), we obtain the explicit expression of T as a function of r. This expression can be put in a particular form which is often interesting for calculations.

Indeed, let's write B in the form: $B=T_1-A\ln(r_1)=T_2-A\ln(r_2)$

The relation (2.28) is then written:

$$T(r) = \frac{T_1-T_2}{\ln{(r_1/r_2)}}\ln\left(\frac{r}{r_2}\right) + T_2 \qquad (2.29)$$

$$q_r = -k2\pi r\frac{dT}{dr} = -k2\pi r\frac{\Delta T}{ln(r_1/r_2)}\frac{1}{r} = 2k\pi\frac{\Delta T}{ln(r_2/r_1)} \qquad (2.30)$$

2.2.5 Spherical body

Similarly, and probably more so than for the cylinder, the propagation of heat in a enormous spherical body from its center does not match to a real-world scenario.

The main need for studying heat transmission in a spherical media is for the thermal insulation of certain systems, such as big spherical gas or liquid tanks and enormous spherical marine tanks.

We shall thus begin with the theoretical analysis of the propagation of heat in a homogeneous sphere constructed of a material of constant k and c and density rho, with the source in the center.

Isothermal surfaces are spherical surfaces concentric with the given material sphere. Let us consider, in this sphere, the volume between two neighboring spheres, of radius (***r*** and ***r+dr***) respectively (**fig. 2.6**)

Using the same rationale as for the cylinder, we can say that the amount of heat (dQ_1) that penetrates through its interior surface into the volume under consideration over a time interval dt is equal to the sum:

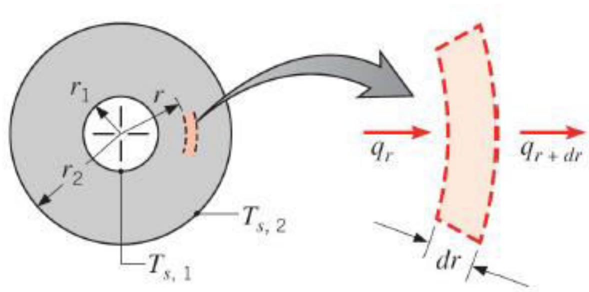


Figure 2.6: Isothermal surfaces of spherical body.

The amount of heat (***dQ***) utilized to heat the volume measured by (***dT***).

The amount of heat (***dQ₂***) that escapes the volume element via its exterior face.

We have now.

$$dQ_1 = -k4\pi r^2 \frac{\partial T}{\partial r} dt \qquad (2.31)$$

$$dQ = \rho c 4\pi r^2 dr dT = \rho c 4\pi r^2 dr \frac{\partial T}{\partial t} dt \qquad (2.32)$$

$$dQ_2 = -4\pi(r+dr)^2 \frac{\partial}{\partial r}\left(T + \frac{\partial T}{\partial r} dr\right) dt = -k4\pi(r^2 + 2rdr + dr^2)\left(\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} dr\right) dt$$
(2.32)

After expansion of the product of the second member of dQ2 and deletion of the infinitesimals of higher order and substitution in the expression: dQ1=dQ+dQ2 we find after reductions:

$$k\left(r\frac{\partial^2 T}{\partial r^2} + 2\frac{\partial T}{\partial r}\right) = \rho c r \frac{\partial T}{\partial t} \qquad (2.33)$$

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right) \qquad (2.34)$$

It is the equation that governs the propagation of heat in a massive spherical body from the center.

When the steady state is reached, we have: $\frac{\partial T}{\partial t} = 0$, and subsequently we have:

$$\left(\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r}\right)=0 \qquad (2.35)$$

To integrate we will pose as for the cylinder: $\frac{\partial T}{\partial r}=u$ and the equation then becomes:

$\frac{du}{dr}+2\frac{u}{r}=0$, or : $\frac{du}{u}=-2\frac{dr}{r}$

Integration is immediate and given: $lnu=-2lur+luA$, ***A*** denoting a constant. We then have: $lnur^2=luA$, and subsequently by replacing ***u*** by its value we find: $\frac{dT}{dr}=A/r^2$, hence $dT=A(dr/r^2)$ and finally we will have:

$$T(r)=-\left(\frac{A}{r}\right)+B \qquad (2.36)$$

B designating a new constant.

Thus, in the transmission of heat to the interior of a full sphere from its center, the temperature at a given place evolves as a function of the distance from that point to the center, according to a hyperbolic law.

The constants ***A*** and ***B*** are defined by the boundary conditions. In terms of heat flow, it has the following expression:

$$q_r=-kS\frac{dT}{dr}=-K4\pi r^2\frac{A}{r^2}=-4\pi kA \qquad (2.37)$$

Let us apply the previous results to the case of a hollow sphere with interior radii r_1 and exterior r_2 (figure). We consider the same boundary conditions as for the cylinder.

- Case where the interior and exterior surface temperatures of the wall are given

Let T_1 and T_2 respectively be the temperatures of its interior and exterior walls and $\Delta T=T_1-T_2$ their difference.

We will assume that $T_1>T_2$ and therefore: $\Delta T>0$

We therefore have: for $r=r_1$: $T=T_1=-\frac{A}{r_1}+B$ and for r=r1 : $T=T_2=-\frac{A}{r_2}+B$

From there we derive the values of $\boldsymbol{A}$ and $\boldsymbol{B}$: $A = -\Delta T \frac{r_1 * r_2}{r_2 - r_1}$ and $B = T_2 - \Delta T \frac{r_1 * r_2}{r_2(r_2 - r_1)}$

Substituting these values into (2.36) immediately gives the temperature at any instant at a point located at distance $\boldsymbol{r}$ from the center.

The heat flow is equal to:

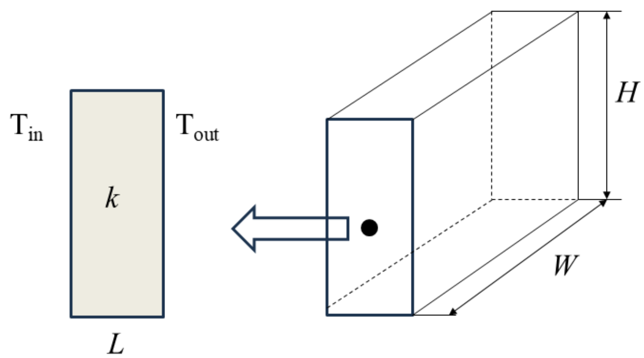
$$q_r = 4\pi k \frac{r_2 * r_1}{r_2 - r_1} \Delta T \qquad (2.38)$$

2.3. Conclusion

In summary, the second Chapter provides a comprehensive framework for analyzing heat conduction in various geometries. By combining Fourier's Law, the heat equation, and the concept of thermal resistance, the chapter equips readers with the tools to determine temperature distributions and heat transfer rates in practical engineering applications. The use of thermal circuits and the analogy to electrical resistance further simplifies the analysis, making it easier to handle complex systems like composite walls and radial geometries.

Problem 1

Problem Statement: The wall of an industrial microwave is constructed from refractory brick with a thickness of L=0.15 m and a thermal conductivity of k=1.7 W/m.K. Measurements taken in steady state reveal temperatures of T_{in} =1400 K and T_{out}=1150 K on the inner and outer surfaces, respectively. What is the heat transfer rate through a wall with dimensions of H=0.5 m by W=1.2 m?



Solution

To determine the heat transfer rate through the wall, we can use Fourier's law of heat conduction for steady-state conditions. Fourier's law states that the rate of heat transfer through a material is proportional to the negative gradient of temperatures and the area through which the heat is flowing.

The formula for heat conduction through a flat wall is:

$$\varphi = k * A * \frac{\Delta T}{\Delta x}$$

where:

φ: is the heat transfer rate (W),

k: is the thermal conductivity of the material (W/m·K),

A: is the area through which heat is transferred (m²),

T_{in}: is the temperature on the inner surface (K),

T_{out}: is the temperature on the outer surface (K), L: is the thickness of the material (m).

Given the values in the problem:

$k = 1.7$ W/m·K,

$T_{in} = 1400$ K,

$T_{out} = 1150$ K,

$d = 0.15$ m,

Area $A = 0.5 \text{ m} \times 1.2 \text{ m} = 0.6 \text{ m}^2$.

Let's plug these values into the formula:

Calculate the temperature difference:

$\Delta T = T_{in} - T_{out} = 1400 \text{ K} - 1150 \text{ K} = 250 \text{ K}$

Use Fourier's law to calculate the heat transfer rate:

$\varphi = k * A * \frac{(T_{in} - T_{out})}{L}$

Substituting the given values:

$$\varphi = 1.7 * 0.6 * \frac{250}{0.15} = 1700\ W$$

So, the heat transfer rate through the wall is 1700 W (or 1.7 kW).

Problem 2

Problem Statement: A single wall has a thickness (e), and it is assumed that heat is produced ($\dot{q}$ is constant) in its middle. The temperature is imposed on one face (x=0) and the heat flux (φ) on the other (x=e).

The following differential equation is given:

$$\frac{\partial^2 T}{\partial x^2} = -\frac{\dot{q}}{\lambda}$$

And the following boundary conditions: $T = T_o$ if $x=0$ and $\varphi = \varphi_1$ if $x=e$.

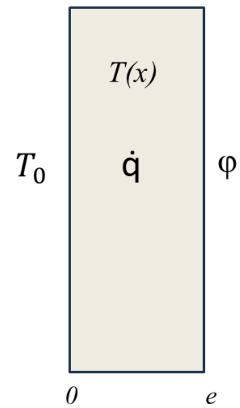
What is the temperature equation on the back face (at x=e), and provide the general form T=f(x).

For the following condition: at e=20mm,

$\Phi^1 = 100\frac{\mathrm{W}}{\mathrm{m}^2}, \dot{\mathrm{q}} = 15\frac{\mathrm{W}}{\mathrm{m}^2}, \mathrm{T}^0 = 100°\mathrm{C}, \lambda = 120\ \mathrm{W/m \cdot K}$

give the temperature equation T=f(x).

Determine the temperature at x=20mm.



Solution

1- The equation for the temperature of the rear face (at x = e), and its general form.

The equation for the temperature on the back face (at x = e), and its general form.
Integrating twice the equation: ($\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{\lambda} = 0$):

The first integral gives us:

$$\frac{\partial T}{\partial x} = -\frac{\dot{q}}{2\lambda}x + C_1$$

The second integral gives us:

$$T(x) = -\frac{\dot{q}}{2\lambda}x^2 + C_1 x + C_2$$

$at\ x = 0, T(0) = T_0 \Rightarrow C_2 = T_0.$

So, we have:

$$T(x) = -\frac{\dot{q}}{2\lambda}x^2 + C_1 x + T_0$$

With $\varphi = -\lambda\frac{dT}{dx} = \dot{q}x - C_1\lambda \Rightarrow q_1 = \dot{q}e - C_1\lambda$

So: $C_1 = \frac{\dot{q}e - \varphi_1}{\lambda}$

$$\Rightarrow T(e) = -\frac{\dot{q}}{2\lambda}e^2 + \frac{\dot{q}}{\lambda}e^2 - \frac{\varphi_1}{\lambda}e + T_0$$

So, the temperature equation is written as follows:

$$\Rightarrow T(e) = \frac{\dot{q}}{\lambda}e^2 - \frac{\varphi_1}{\lambda}e + T_0$$

The general form is thus:

$$\Rightarrow T(x) = \frac{\dot{q}}{\lambda}x^2 - \frac{\varphi_1}{\lambda}x + T_0$$

$\varphi_1 = 100\ W/m^2,\ \dot{q} = 15\ W/m^3,\ T_0 = 100°C,\ \lambda = 120\ W/m.K$

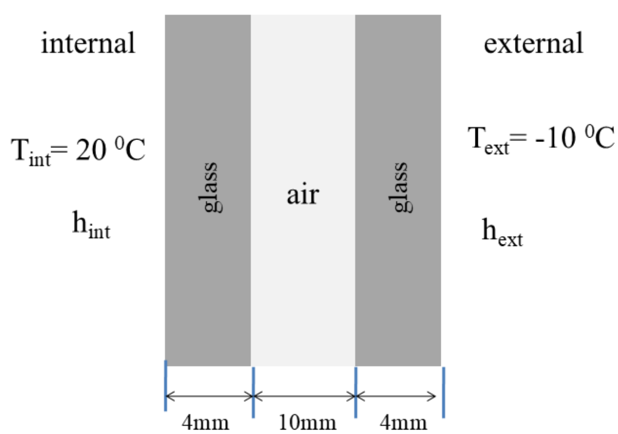
Under these conditions, the previous relation becomes:

$$\Rightarrow T(x) = 0{,}125\ x^2 - 0{,}833\ x + 100$$

T(20)=133,34 °C

Problem 3

Problem Statement: We have We have a double-glazed window separated by a 10mm thick layer of air (λ_a = 0.026 W/m·°C). The window is 0.8 m high and 1.5 m wide. The glass layers are 4 mm thick and have a (λ_g = 0.78 W/m·°C). Calculate the heat flux through the window if the temperature inside the house is 20°C and outside it is -10°C. The internal and external convection coefficients are given: h_1 = 10 W/m²·°C and h_2 = 40 W/m²·°C.



Solution

1- Define the problem and given data

Thickness of layer, e_{air} =10 mm = 0.01 m, Thermal conductivity of air, $\lambda_a = 0.026$ W/m·°C,

Thickness of each glass layer, $e_{glass} = 4$ mm = 0.004 m,

Thermal conductivity of glass, $\lambda_g = 0.78$ W/m·°C,

Dimensions of window: height = 0.8 m and width = 1.5 m,

Internal temperature, $T_{in} = 20$ °C,

External temperature, $T_{out} = -10$ °C,

Internal convection coefficient, $h_1 = 10$ W/m²·°C

External convection coefficient, $h_2 = 40$ W/m²·°C.

2- Calculate the thermal resistances

Inside convective resistance : $R_{conv,1} = \frac{1}{h_1.A} = \frac{1}{10*1.2} = 0.0833\ C/W$

Outside convective resistance : $R_{conv,2} = \frac{1}{h_2.A} = \frac{1}{40*1.2} = 0.0208\ C/W$

Conduction resistance through one glace layer: $R_{cond,g1} = \frac{e_g}{\lambda_g.A} = \frac{0.004}{0.78*1.2} = 0.0043\ C/W$

Conductive resistance (air layer) : $R_{cond,a} = \frac{e_a}{\lambda_a.A} = \frac{0.01}{0.026*1.2} = 0.32\ C/W$

Conduction resistance through one glace layer: $R_{cond,g2} = \frac{e_g}{\lambda_g.A} = \frac{0.004}{0.78*1.2} = 0.0043\ C/W$

Calculate the total thermal resistance:

$$R_{total} = R_{conv,1} + R_{cond,g1} + R_{cond,a} + R_{cond,g2} \approx 0.433 C/W$$

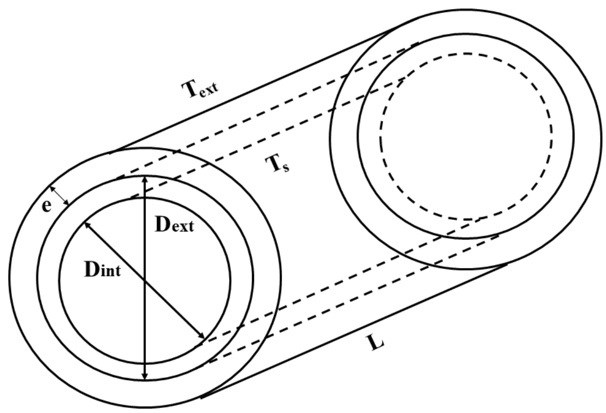
3- Calculate the heat flux q:

$$q = \frac{T_{in} - T_{out}}{R_{total}} = \frac{20 - (-10)}{0.433} \approx 69.3W$$

Therefore, the heat flux through the window is approximately 69.3W.

Problem 4

Problem Statement: Consider a cylindrical pipe with a length L=5 m and thermal conductivity λ_c = 2 (W/m°C), and inner diameter D_{in}= 10cm, and an outer diameter D_{ext}= 15cm, maintained at a constant and uniform temperature T_s= 100°C. This pipe is insulated with a layer of insulation of thickness e= 2cm and thermal conductivity λ_{iso} = 0.12 (W/m°C). The external surface of the insulation layer is maintained at a constant and uniform temperature T_{ext}= 15°C. Calculate the thermal losses to the outside.



Solution

Identify the given parameters:

Length of the pipe, L=5m,

Thermal conductivity of the pipe material, λ_c = 2 (W/m°C).

The inner diameter of the pipe, D_{in} = 10cm = 0.1m.

The outer diameter of the pipe, D_{out} = 15cm = 0.15m.

Temperature of the pipe surface, T_s= 100°C.

The thickness of the insulation layer, e = 2cm = 0.02 m.

Thermal conductivity of the insulation, λ_{ins} = 2 (W/mºC).

External Temperature, T_{ext}= 15 ºC.

Determine the outer diameter of the insulation layer:

$D_{ins} = D_{ext} + 2e= 0.15 + 2*0.02 = 0.19$ m

Calculate the thermal resistances:

Conduction resistance through the pipe wall:

$$R_{pipe} = \frac{ln(D_{ext}/D_{int})}{2\pi L\lambda_c} = \frac{ln(0.15/0.1)}{2*3.14*5*2} \approx 0.00645\ C/W$$

Conduction resistance through the insulation layer:

$$R_{ins} = \frac{ln(D_{ins}/D_{ext})}{2\pi L\lambda_{iso}} = \frac{ln(0.19/0.1\,5)}{2*3.14*5*0.12} \approx 0.0629\ C/W$$

Calculate the total thermal resistances

$$R_{total} = R_{ins} + R_{pipe} = 0.0629 + 0.00645 \approx 0.06935\ C/W$$

Calculate the thermal losses (or heat flux)

$Q = \frac{T_s - T_{ext}}{R_{total}} = \frac{100-15}{0.06935} \approx 1226\ W$

Therefore, the thermal losses to the outside are approximately 1226 W.

Chapter 3

Heat transfer by convection

Chapter 3: Heat transfer by convection

3.1. Introduction

We have covered heat transfer by conduction in the last chapter, and we will now turn our attention to heat transfer by convection.

3.1.1. Convection Principe

Convection is an important heat transfer mechanism in heat exchangers between a solid wall and the fluid in contact with it. In the first chapter, we expressed the quantity of heat transferred by convection between a solid wall and a fluid as follows:

$$dq = hdA(T_w - T_\infty) \qquad (3.1)$$

When written in this form, the convection equation appears relatively simple. In reality, this is not the case; the convective heat transfer coefficient ($\boldsymbol{h}$) is a complex function that depends on experimental conditions, specifically:

- Wall geometry, including shape and roughness.
- Fluid properties include density, viscosity, and specific heat. It should be noted that these quantities also depend on temperature.
- Fluid flow characteristics include velocity and flow regime (laminar or turbulent).
- Where the fluid's temperature is measured in the calculation.

Although this relation is commonly employed to calculate heat flux, it cannot explain how heat flows by convection. As a result, we will begin by providing a qualitative explanation of the heat transfer mechanism, followed by demonstrations of the methods used to calculate the convective heat transfer coefficient ($\boldsymbol{h}$).

3.1.2. Mechanism of Heat Transfer by Convection

Convection heat transfer is directly related to fluid movement. To investigate the heat flow process, it is required to first understand the fluid flow mechanism. In laminar flow, the fluid moves in separate layers that do not mix. The transverse flow is so weak that when the colorant is injected at any point in the fluid, it follows the stream with little diffusion. In contrast, turbulent flow distributes the colorant over a vast region a short distance downstream of the injection location. The many vortices inside the fluid mass help to promote mixing.

Thus, when a fluid flows in laminar motion along a surface with a temperature that differs from the fluid's, heat is transported only via molecular conduction, both inside the fluid and at the fluid-surface contact. In this situation, there are no vortices that move the energy stored in fluid particles across fluid streams. Heat is transferred between fluid layers at the microscopic level via molecular motion.

Fluids have a relatively low thermal conductivity, which means that energy transmission is similarly low. In contrast, turbulent flow alters the conduction mechanism and is aided by many vortices that transport fluid mass. When fluid particles mix with other particles, they behave as energy carriers.

Consequently, when turbulence increases, so does the quantity of heat that flows by convection, as the mixing of hot and cold fluids contributes greatly to energy transfer. Because mixing in turbulent flow is more significant than in laminar flow, heat exchange is far more efficient. As a result, the transmitted heat fluxes and heat transfer coefficients in turbulent flow are significantly higher than in laminar flow.

Important Note:

Furthermore, as we know from fluid mechanics, when a fluid flows along a solid surface, a boundary layer is formed, regardless of the type of flow. Here, we will concentrate on the turbulent boundary layer.

Within this layer, immediately adjacent to the wall, there is a very thin layer known as the laminar sublayer or laminar film. The buffer layer is the zone between the laminar sublayer and the turbulent boundary layer. **Figure 3.1** illustrates the flow structure in the turbulent boundary layer.

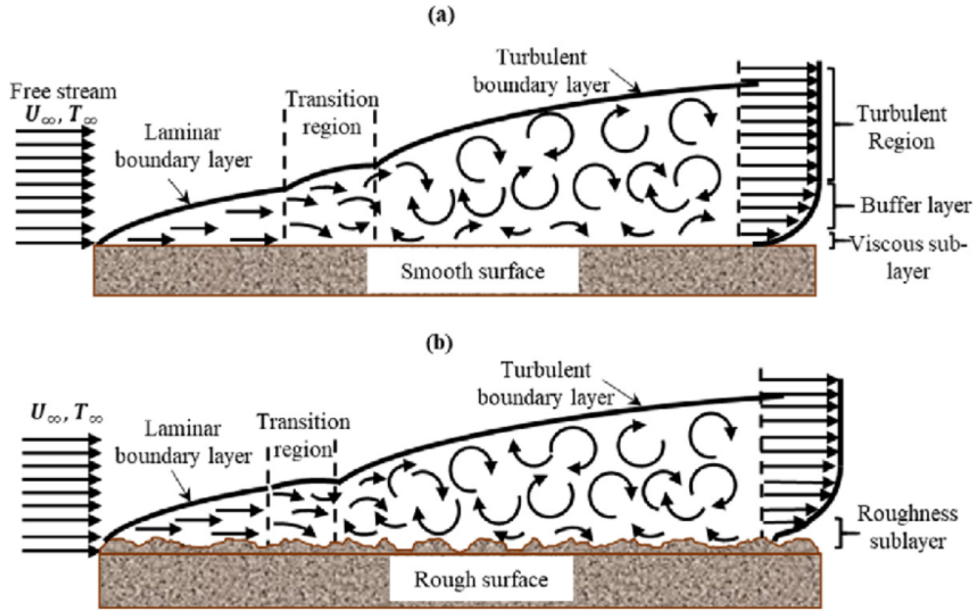


Figure 3.1: The flow structure in the turbulent boundary layer.

Heat flows only through conduction in the laminar sublayer next to the surface, where vortices are zero (no diffusivity). Against this initial zone, there is the buffer layer in which both conduction and convection contribute equally to the heat transfer process. Conduction is negligible compared to convection in the turbulent zone.

➢ Evaluation of the heat exchange coefficient by convection

* The major methods for calculating the heat exchanger coefficient (***h)*** include exact mathematical solutions to boundary layer equations.

* Approximate studies of the boundary layer.

* Comparison of heat and momentum transfer.

* Combination of dimensional analysis and experimental data.

- Exact mathematical analyses necessitate the simultaneous solution of equations describing fluid motion and energy transfer within the flowing fluid.

This method assumes that the physical mechanisms are well known enough to be described in mathematical language.

- An approximate examination of the boundary layer avoids a detailed mathematical description of flow within the boundary layer.

This method is relatively simple and provides solutions to situations that cannot be solved using exact mathematical methods.

Its solutions are satisfactory for practical applications.

- The comparison between heat and momentum transfer is a useful tool for evaluating turbulent flow.

- Dimensional analysis, which we will discuss in the next part.

3.2. Evaluation of the through dimensional analysis

3.2.1. Dimensional analysis Principle

Nusselt was the first to apply the dimensional analysis method to derive mathematical equations for convective heat transfer coefficients in both free and forced convection. Dimensional analysis is a mathematical method that is used to obtain equations governing an unknown physical phenomenon in terms of important parameters influencing that phenomenon. The influencing parameters are arranged into dimensionless groupings, resulting in fewer influencing parameters. Dimensional analysis for both free and forced convection includes the following steps:

- ❖ Identify all parameters and variables influencing the convective heat transfer coefficient.
- ❖ Writing influencing parameters using fundamental units of mass, length, time, and temperature.
- ❖ Using the idea of dimensional homogeneity develop mathematical formulas for convective heat transfer coefficient in fundamental units.
- ❖ Group all influencing parameters as non-dimensional numbers.

The dimensions of different parameters include length ***L***, time ***t***, mass ***M***, and temperature ***T***. The other quantities can be expressed in terms of these fundamental quantities. For example: velocity can be expressed in terms of ***L*** and ***t*** as ***V (L/t)*** (in meters per second).

Table 1: the dimensions of different physical parameters.

Parameter	Symbol	SI Units	Dimensions
Mass	M	Kg	M
Length	L	M	L
Time	T	Second	T
Temperature	θ	K	θ
Heat	Q	Joule	ML^2T^{-2}
Area	A	m^2	L^2
Volume	V	m^3	L^3
Velocity	U, V	m/s	LT^{-1}
Acceleration	A	m/s^2	LT^{-2}
Gravity Acceleration	G	m/s^2	LT^{-2}
Force or Resistance	F, R	N	MLT^{-2}
Density	ρ	kg/m^3	ML^{-3}
Dynamic viscosity	μ	Kg/(m.s)	$ML^{-1}T^{-1}$
Kinematic viscosity	ν	M^2/s	L^2T^{-1}
Energy, Work, Heat	E, W	Joule	ML^2T^{-2}
Convective heat transfer coefficient	H	$W/(m^2.deg)$	$MT\text{-}3\theta^{-1}$
Coefficient of volumetric expansion	β	Per deg	θ^{-1}
Specific heat	C_p, C_v	kJ/(kg.deg)	$L^2T^2\theta^{-1}$
Thermal conductivity	λ	W/(kg.deg)	$MLT^{-3}\theta^{-1}$
Thermal resistance per unit area	R_t	°C/W	$M^{-1}T^{-3}\theta^{-1}$
Thermal diffusivity	α	m^2/s	L^2T^{-1}

Methods of dimensional analysis

If you know how many variables influence the convective heat transfer coefficient, you can use Rayleigh's approach and Buckingham's π -theorem to create a mathematical formula that relates the variables to the convective heat transfer coefficient.

However, Rayleigh's approach will not be employed in dimensional analysis to determine the convective heat transfer coefficient for free and forced convection due to constraints that can be solved by using Buckingham's π -theorem method.

Buckingham's π -theorem method

In Rayleigh's method of dimensional analysis, the solution becomes more and more complex and laborious if the number of influencing variables becomes more than the fundamental units (M, L, t, and T) involved in the physical phenomenon. Buckingham's π-theorem method overcomes this limitation stating that if a physical phenomenon has 'n' variables (independent and dependent) and 'm' fundamental dimensions (M, L, T, and θ), the variables are arranged into (n-m) dimensionless terms called π-terms.

Buckingham's π-Theorem Method can be used to calculate heat transfer coefficients for both forced and free convection.

Dimensional Analysis for Forced Convection

Based on experience, it is concluded that the forced convection heat transfer coefficient is a function of variables listed in Table -2

Number	Variable/Parameter	Symbol	Dimensions
1	Fluid density	ρ	ML^{-3}
2	Dynamic viscosity of fluid	μ	$ML^{-1}T^{-1}$
3	Fluid velocity	v	LT^{-1}
4	Thermal conductivity of fluid	λ	$MLT^{-3}\theta^{-1}$
5	Specific heat of fluid	Cp	$L^{2}T^{-2}\theta^{-1}$
6	Characteristic length of heat area	D	L

So, the convective heat transfer coefficient is given as

$$h = f(\rho, \mu, V, \lambda, Cp, D) \qquad (3.2)$$

$$h = f(\rho, \mu, V, \lambda, Cp, D) \qquad (3.2)$$

$$f(h, \rho, \mu, V, \lambda, Cp, D) = 0 \qquad (3.3)$$

The dependent variable is the convective heat transfer coefficient ($\boldsymbol{h}$), while the residuals are independent variables.

Total number variables, n=7

Number of fundamental units, m=4

Buckingham's π-Theorem states that the difference between the total number of variables and the number of fundamental units equals the number of π-terms.

Number of π-terms = (n-m)= 7 – 4 = 3

These non-dimensional π-terms control the forced convective phenomenon and are expressed as

$$f(\pi_1, \pi_2, \pi_3) = 0 \qquad (3.4)$$

Each π-terms is written in terms of repeated variables and one additional variable. To choose repeated variables, use the following method.

- The number of repeated variables must be equal to the number of fundamental units involved in the physical phenomenon.
- The dependent variable shouldn't be chosen as the repeated variable.
- The repeated variables must be chosen in such a way that one of them has a geometric property, such as length, diameter, or height. The other repeating variable must contain a flow property like velocity or acceleration, while the third one must contain a fluid property like viscosity, density, specific heat, or specific weight.

- The chosen repeated variables must not form a dimensionless group.
- The selected repeated variables must have the same number of fundamental dimensions.
- No two selected repeated variables should have the same dimensions.

The following repeating variables are selected

1)- Dynamic viscosity, μ having fundamental dimensions $ML^{-1}T^{-1}$

2)- Thermal conductivity, λ having fundamental dimensions $MLT^{-3}\,\theta^{-1}$

3)- Fluid velocity, V having fundamental dimensions LT^{-1}

4)- Characteristic length, D having fundamental dimensions L

The π-term is written as:

$$\pi_1 = \mu^a, \lambda^b, V^c, D^d, h \tag{3.5}$$

Defining each term in the equation above in terms of fundamental dimensions

$$M^0L^0T^0\theta^0 = (ML^{-1}T^{-1})^a(MLT^{-3}\theta^{-1})^b(LT^{-1})^c(L)^d MT^{-3}\theta^{-1} \tag{3.6}$$

Comparing the powers of (***M***) we obtain

$$0 = a + b + 1 \Rightarrow a + b = -1 \tag{3.7}$$

Comparing the powers of (***L***) we obtain

$$0 = -a + b + c + d \tag{3.8}$$

Comparing the powers of (***T***) we obtain

$$0 = -a + 3b - c - 3 \tag{3.9}$$

Comparing the powers of ($\boldsymbol{\theta}$) we obtain

$$0 = -b - 1 \Rightarrow b = -1 \qquad (3.10)$$

Substituting the value of (***b***) from equation (3.10) in equation (3.7), we obtain (***a***=0)

Substituting the value of (***a***) in equation (3.9), we obtain (***c***=0)

Substituting the values of (***a***), (***b***) and (***c***) in equation (3.8), we obtain (***d***=0)

Substituting the values of (***a***), (***b***), (***c***) and (***d***) in equation (3.5), we obtain

$$\pi_1 = {}^{hD}/_{\lambda} = Nusselt\ Number \qquad (3.11)$$

The second π-term is presented as follows:

$$\pi_2 = \mu^a, \lambda^b, V^c, D^d, h \qquad (3.12)$$

Defining each term in the equation above in terms of fundamental dimensions

$$M^0 L^0 T^0 \theta^0 = (ML^{-1}T^{-1})^a (MLT^{-3}\theta^{-1})^b (LT^{-1})^c (L)^d ML^{-3} \qquad (3.13)$$

Comparing the powers of (***M***) we obtain

$$0 = a + b + 1 \ \Rightarrow a + b = -1 \qquad (3.14)$$

Comparing the powers of (***L***) we obtain

$$0 = -a + b + c + d - 3 \qquad (3.15)$$

Comparing the powers of (***T***) we obtain

$$0 = -a + 3b - c \qquad (3.16)$$

Comparing the powers of (***θ***) we obtain

$$0 = -b \Rightarrow b = 0 \qquad (3.17)$$

Substituting the value of (***b***) from equation (3.17) in equation (3.14), we obtain (***a***=-1)

Substituting the value of (***a***) and (***b***) in equation (3.16), we obtain (***c***=1)

Substituting the values of (***a***), (***b***), and (***c***) in equation (3.15), we obtain (***d***=1)

Substituting the values of (***a***), (***b***), (***c***) and (***d***) in equation (3.12), we obtain

$$\pi_2 = {}^{\rho hD}/_{\mu} = Reynolds\ Number \qquad (3.18)$$

The third π-term is presented as follows:

$$\pi_3 = \mu^a, \lambda^b, V^c, D^d, Cp \qquad (3.19)$$

Defining each term in the equation above in terms of fundamental dimensions

$$M^0 L^0 T^0 \theta^0 = (ML^{-1}T^{-1})^a (MLT^{-3}\theta^{-1})^b (LT^{-1})^c (L)^d L^2 T^{-2} \theta^{-1} \qquad (3.20)$$

Comparing the powers of (***M***) we obtain

$$0 = a + b \ \Rightarrow a + b = 0 \qquad (3.21)$$

Comparing the powers of (***L***) we obtain

$$0 = -a + b + c + d + 2 \qquad (3.22)$$

Comparing the powers of (***T***) we obtain

$$0 = -a + 3b - c - 2 \qquad (3.23)$$

Comparing the powers of (***θ***) we obtain

$$0 = -b - 1 \Rightarrow b = -1 \qquad (3.24)$$

Substituting the value of (***b***) from equation (3.24) in equation (3.21), we obtain (***a***=-1)

Substituting the value of (***a***) and (***b***) in equation (3.23), we obtain (***c***=0)

Substituting the values of (***a***), (***b***), and (***c***) in equation (3.22), we obtain (***d***=0)

Substituting the values of (***a***), (***b***), (***c***), and (***d***) in equation (3.20), we obtain

$$\pi_3 = \mu^1, \lambda^{-1}, V^0, D^0, Cp \qquad (3.21)$$

$$\pi_3 = {}^{\mu Cp}/_{\lambda} = Prandtl\ Number \qquad (3.22)$$

Substituting the values of π_1, π_2, π_3 into equation (3.4), we obtain

$$f\left({}^{hD}/_{\lambda}, {}^{\rho VD}/_{\mu}, {}^{\mu Cp}/_{\lambda}\right) = 0 \qquad (3.23)$$

$${}^{hD}/_{\lambda} = \varphi\left({}^{\rho VD}/_{\mu}, {}^{\mu Cp}/_{\lambda}\right) \qquad (3.24)$$

$$Nu = \varphi(Re, Pr) \qquad (3.25)$$

The above correlation is generally written as follows:

$$Nu = C(Re)^a(Pr)^b \qquad (3.26)$$

The constant ***C,*** as well as the exponents (***a***) and (***b***) are determined experimentally.

Dimensional Analysis for Free Convection

The convective heat transfer coefficient in the free convection heat transfer process is determined by the same parameters/variables as in forced convection, except for fluid velocity. It is because in free convection motion of fluid happens as a result of temperature differences in the density of different layers of fluid, whereas in forced convection motion of the fluid is generated by an external source. The fluid velocity in free convection depends on the following parameters;

1)- Temperature difference between solid surface and bulk fluid, ΔT

2)- Gravitational acceleration, g

3)- Coefficient of volumetric expansion of the fluid, β

The change in the volume due to temperature fluctuations can be written as follows:

$$dV = V_1\beta(T_2 - T_1) \qquad (3.27)$$

Where: dV: change in volume (m^3)

β: Coefficient of volumetric expansion of fluid ($m^3/ m^{3o}C$)

T_2: Final temperature (°C)

T_1: Initial temperature (°C)

Therefore, the free convection heat transfer coefficient is a function of variables given in Table 3.

Number	Variable/Parameter	Symbol	Dimensions
1	Fluid density	ρ	ML^{-3}
2	Dynamic viscosity of fluid	μ	$ML^{-1}T^{-1}$
3	Thermal conductivity of fluid	λ	$MLT^{-3}\theta^{-1}$
4	Specific heat of fluid	Cp	$L^2T^{-2}\theta^{-1}$
5	Characteristic length of heat area	D	L
6	Temperature difference between surface and bulk fluid	ΔT	θ
7	Coefficient of volumetric expansion	β	θ^{-1}
8	Acceleration due gravity	g	LT^{-2}

Therefore, the convective heat transfer coefficient is written as follows:

$$h = f(\rho, \mu, \lambda, Cp, D, \Delta T, \beta, g) \qquad (3.28)$$

In free convection, (ΔT, β, and g) is regarded as a single parameter because fluid particle velocity depends on these parameters. So, equation (3.28) can be presented as:

$$f\big(h, \rho, \mu, \lambda, Cp, D, (\Delta T \beta g)\big) = 0 \qquad (3.29)$$

The dependent variable is the convective heat transfer coefficient (h), while the remaining are independent variables.

Total number variables, n=7

Number of fundamental units, m=4

Buckingham's π-Theorem states that the difference between the total number of variables and the number of fundamental units equals the number of π-terms.

Number of π-terms = (n-m)= 7 – 4 = 3

These non-dimensional π-terms control the forced convective phenomenon and represent it as

$$f(\pi_1, \pi_2, \pi_3) = 0 \qquad (3.30)$$

Each π-term is written in terms of repeating variables and one additional variable. The following repeating variables are chosen.

1)- Dynamic viscosity, μ having fundamental dimensions $ML^{-1}T^{-1}$

2)- Thermal conductivity, λ having fundamental dimensions $MLT^{-3}\,\theta^{-1}$

3)- Fluid velocity, V having fundamental dimensions LT^{-1}

4)- Characteristic length, D having fundamental dimensions L

Each π-term is expressed as:

$$\pi_1 = \mu^a, \lambda^b, \rho^c, D^d, h \qquad (3.31)$$

Defining each term in the above equation in terms of fundamental dimensions

$$M^0L^0T^0\theta^0 = (ML^{-1}T^{-1})^a(MLT^{-3}\theta^{-1})^b(ML^{-3})^c(L)^dMT^{-3}\theta^{-1} \qquad (3.32)$$

Comparing the powers of (***M***) we obtain

$$0 = a + b + c + 1 \implies a + b + c = -1 \qquad (3.32)$$

Comparing the powers of (***L***) we obtain

$$0 = -a + b + c + d \qquad (3.33)$$

Comparing the powers of (***T***) we obtain

$$0 = -a + 3b - c - 3 \qquad (3.34)$$

Comparing the powers of (***θ***) we obtain

$$0 = -b - 1 \Longrightarrow b = -1 \qquad (3.35)$$

Substituting the value of (***b***) from equation (3.35) in equation (3.32), we obtain (***a***=0)

Substituting the value of (***a***) and (***b***) in equation (3.34), we obtain (***c***=0)

Substituting the values of (***a***), (***b***), and (***c***) in equation (3.33), we obtain (***d***=1)

Substituting the values of (***a***), (***b***), (***c***) and (***d***) in equation (3.31), we obtain

$$\pi_1 = \mu^a, \lambda^b, \rho^c, D^d, h \qquad (3.32)$$

$$\pi_1 = {}^{hD}\!/_{\lambda} = Nussel\ number \qquad (3.33)$$

The second π-term is presented as:

$$\pi_2 = \mu^a, \lambda^b, \rho^c, D^d, Cp \qquad (3.34)$$

Defining each term in the equation above in terms of fundamental dimensions

$$M^0 L^0 T^0 \theta^0 = (ML^{-1}T^{-1})^a (MLT^{-3}\theta^{-1})^b (ML^{-3})^c (L)^d L^2 T^{-2} \theta^{-1} \qquad (3.35)$$

Comparing the powers of (***M***) we obtain

$$0 = a + b + c \ \Longrightarrow a + b + c = 0 \qquad (3.36)$$

Comparing the powers of (***L***) we obtain

$$0 = -a + b - 3c + d + 2 \qquad (3.37)$$

Comparing the powers of (***T***) we obtain

$$0 = -a - 3b - 2 \qquad (3.38)$$

Comparing the powers of (***θ***) we obtain

$$0 = -b - 1 \Rightarrow b = -1 \qquad (3.39)$$

Substituting the value of (***b***) from equation (3.39) in equation (3.38), we obtain (***a***=1)

Substituting the value of (***a***) and (***b***) in equation (3.36), we obtain (***c***=0)

Substituting the values of (***a***), (***b***), and (***c***) in equation (3.37), we obtain (***d***=0)

Substituting the values of (***a***), (***b***), (***c***), and (***d***) in equation (3.34), we obtain

$$\pi_2 = {}^{\mu Cp}/_{\lambda} = Pr = Prandtl\ Number \qquad (3.40)$$

The third π-term is presented as:

$$\pi_3 = \mu^a, \lambda^b, \rho^c, D^d, \Delta\mathrm{T}\beta\mathrm{g} \qquad (3.41)$$

Defining each term in the equation above in terms of fundamental dimensions

$$M^0L^0T^0\theta^0 = (ML^{-1}T^{-1})^a(MLT^{-3}\theta^{-1})^b(ML^{-3})^c(L)^d(\theta^{-1}LT^{-2}\theta^1) \qquad (3.42)$$

$$M^0L^0T^0\theta^0 = (ML^{-1}T^{-1})^a(MLT^{-3}\theta^{-1})^b(ML^{-3})^c(L)^d(LT^{-2}) \qquad (3.43)$$

Comparing the powers of (***M***) we obtain

$$0 = a + b + c \ \Rightarrow a + b + c = 0 \qquad (3.44)$$

Comparing the powers of (***L***) we obtain

$$0 = -a + b - 3c + d + 1 \qquad (3.45)$$

Comparing the powers of (***T***) we obtain

$$0 = -a + 3b - 2 \qquad (3.46)$$

Comparing the powers of (***θ***) we obtain

$$0 = -b \Rightarrow b = 0 \qquad (3.47)$$

Substituting the value of (***b***) from equation (3.47) in equation (3.46), we obtain (***a***=-2)

Substituting the value of (***a***) and (***b***) in equation (3.44), we obtain (***c***=2)

Substituting the values of (***a***), (***b***), and (***c***) in equation (3.45), we obtain (***d***=3)

Substituting the values of (***a***), (***b***), (***c***), and (***d***) in equation (3.41), we obtain

$$\pi_3 = \mu^{-2}, \lambda^0, \rho^2, D^3, (\Delta T\beta g) \qquad (3.48)$$

$$\pi_3 = \rho^2 . D^3 . (\Delta T\beta g) / \mu^2 = D^3 . (\Delta T\beta g) / \upsilon^2 = Gr \qquad (3.49)$$

Substituting the values of π_1, π_2, and π_3 in equation (3.30), we obtain

$$f\left(\left(hD/\lambda\right), \left(\mu Cp/\lambda\right), \left(D^3 . (\Delta T\beta g)/\upsilon^2\right)\right) = 0 \qquad (3.50)$$

$$hD/\lambda = \varphi\left(\left(\mu Cp/\lambda\right), \left(D^3 . (\Delta T\beta g)/\upsilon^2\right)\right) \qquad (3.51)$$

$$Nu = \varphi(Pr, Gr) \qquad (3.52)$$

The above correlation is generally written as follows:

$$Nu = C(Pr)^a (Gr)^b \qquad (3.53)$$

The constant ***C***, as well as the exponents (***a***) and (***b***) are determined experimentally.

3.2.2. Conditions for applying convection formulas

The convection formulas produced in this chapter and subsequent chapters are usually the result of laboratory experiments research.

Their adaptation and application to real cases must be carefully considered: it is necessary to evaluate the operating conditions under which they were developed to apply them under the same conditions.

The fluid properties are very important, and the dimensionless groups are subject to specific conditions that must be respected.

If a local dimensionless number is calculated, it will be denoted by the subscript x. As an example, Re_x and Nu_x. If it is an average number, it will be written in the form: $\overline{Nu}$ or $\overline{h}$. If this number corresponds to the major dimension of a given geometric body, it will be denoted by a subscript. For example:

Nu_L: for a plate of length L.

Nu_D: for a cylinder of diameter D.

The use of theoretical formulas necessitates specifying the conditions under which fluid properties should be determined. Frequently, the fluid temperature is chosen, but this is not an absolute rule; sometimes formulas are provided for properties at the wall temperature T_W.

Sometimes, two reference temperatures are used: one for specific quantities, and another for the residual characteristics.

3.3. Natural convection

Principle: Natural convection (or free convection) occurs when the fluid moves only due to density differences produced by temperature differences.

3.3.1. Mechanism

Natural convection is a phenomenon that can be frequently observed. When the air comes into contact with a hot body, the temperature of the air increases, its density decreases.

The hot air then rises and removes heat from the body. It is replaced by cold air, which, when contact with the body, heats up, and the process repeats again, with cold air replacing heated air each time.

A similar phenomenon can be observed when a cold body is immersed in a hotter fluid, but the fluid moves in the opposite direction.

3.3.2. Application

A large number of devices use this mode of heat transfer: heating appliances, electrical lines, transformers, rectifiers, refrigerators...

Natural convection is also responsible for heat losses in pipes carrying hot fluids.

Natural convection is the dominant mechanism for heat flow from the motionless human body in a calm atmosphere.

3.3.3. Form of relationships

Fluid velocities in natural convection currents are typically low. The flow may be laminar or turbulent, depending on the distance from the leading edge, fluid characteristics, and temperature difference between the surface and the fluid. The intensity of mixing motion is usually lower in natural convection, and subsequently, heat exchange coefficients are smaller than in forced convection. So, in natural convection, instead of using the Reynolds number, another dimensionless group is preferred: the Grashof number (***Gr***), and the results are expressed by relationships of the form: ***Nu=f(Gr, Pr)***.

3.3.4. Grashof Number

The Grashof number is expressed by the following equation:

$$Gr = \frac{\beta g \rho^2 L^3 \Delta T}{\mu^2} = \frac{\beta g L^3 \Delta T}{\upsilon^2} \tag{3.54}$$

$\boldsymbol{\Delta T = T_s - T_\infty}$ is the temperature difference between the wall and the fluid, and $\boldsymbol{\Delta T}$ is also the temperature variation that causes the variation $\Delta\rho$ in density.

$\boldsymbol{L}$ = characteristic length of the wall ($\boldsymbol{L = D}$, if we have a cylinder, for example)

$\boldsymbol{\beta}$ = coefficient of fluid expansion at constant pressure.

We also write: $\beta = \frac{1}{\rho}\frac{\Delta\rho}{\Delta T}$

Some tables provide values of the term $\frac{\beta g \rho^2}{\mu^2}$ at different temperatures.

3.3.5. Scope of the Study

It is difficult to accurately evaluate the heat transfer coefficient via natural convection. The problem has been solved in the case of bodies with simple geometry, which we will investigate:

- Vertical flat plate,

- Vertical cylinder,

- Horizontal cylinder and sphere,

- Horizontal plate.

Note: We will examine formulas related to different cases of gases and liquids and leave out the case of liquid metals.

Each geometric shape is identified by a characteristic dimension such as its length ***L***, diameter ***D***, height ***H***, etc...

This dimension is represented in ***Nu*** and ***Gr*** by an index.

The average values of the Nusselt number for a given surface are denoted by a bar: $\overline{Nu}$.

All of the equations that will be investigated apply only to the temperature differences.

a)-Vertical flat plate

The plate is supposed to be infinitely thin and of infinite width. This assumption suggests that the vertical edges do not affect the study. Its temperature is considered to be uniform, and it is limited by two horizontal edges. When the plate is heated, a boundary layer forms on its surface, together with fluid streaks in motion (**Figure 3.2**).

The boundary layer created from the leading edge is initially laminar, but at a certain distance from this leading edge is initially laminar, but at a given distance from this leading edge (a distance dependent on the fluid characteristics and the temperature difference between the wall and the environment), ripples emerge, gradually amplifying as turbulence sets in. As the distance increases, the turbulence of the boundary layer increases, as shown in **Figure 3.2**.

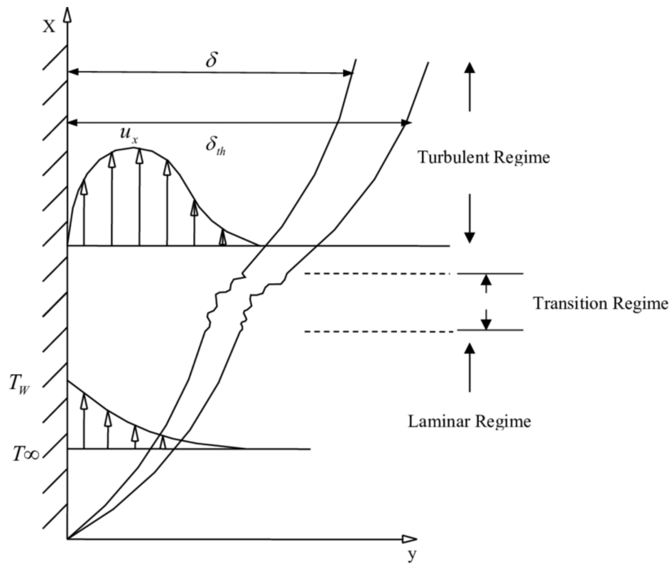


Figure 3.2: Velocity and temperature profiles for natural convection.

In the case of natural convection, the transition from the laminar zone to the turbulent zone is presented by the product ***Gr*Pr*** and the limit of 10^9. Schmidt and Beckman established the relationship for air with a Prandtl number equal to 0.75:

$$\frac{h_x x}{\lambda} = Nu_x = 0{,}360(Gr_x)^{1/4} \qquad (3.55)$$

Later, Eckert established the following relationship for fluids with a Prandtl number different other than 0.75 and for laminar flow:

$$Nu_x = 0{,}508\left[\frac{\Pr^2}{0{,}952+\Pr} \bullet Gr_x\right]^{1/4} \qquad (3.56)$$

The average Nusselt number is then equal to:

$$\overline{Nu} = 0{,}678\left[\frac{\Pr^2}{0{,}952+\Pr}\bullet Gr_L\right]^{1/4} \qquad (3.57)$$

Eckert and Jackson developed the following equation for natural convection in a turbulent regime on a vertical plate (***Gr*Pr > 10⁹***):

$$\overline{Nu} = 0{,}0246\left[\frac{\Pr^2}{1+0{,}494\Pr^{2/3}}\bullet Gr_L\right]^{2/5}\Pr^{7/15} \qquad (3.58)$$

However, over the years, it has been clear that the average Nusselt number was better represented in many situations by the following relationships:

$$\overline{Nu} = 0{,}59\left[Gr_f.\Pr_f\right]^{1/4} \text{ for } 10^4 \le Gr_f.\Pr_f \le 10^9 \qquad (3.59)$$

$$\overline{Nu} = 0{,}10\left[Gr_f.\Pr_f\right]^{1/3} \text{ for } 10^9 \le Gr_f.\Pr_f \le 10^{13} \qquad (3.60)$$

For (***Gr*Pr < 10⁴***), the subscript (***f***) denotes that the fluid properties are calculated at the fluid temperature, T_f. The length ***L*** of the plate is employed as the characteristic dimension in both the Nusselt number and the Grashof number.

Note:

- To calculate the Grashof number for an inclined plate, use (***cosα***) (where α is the angle formed by the plane of the plate and the vertical axis oriented upwards).

It is written as:

$$Gr_L = \frac{\beta g \rho^2 L^3 \Delta T \cos\alpha}{\mu^2} \qquad (3.61)$$

- The above relationships have been established for a surface with uniform and constant temperature. There are cases where constant heat flux is examined with a non-uniform surface temperature. In such cases, different relationships are indicated in the literature.

b)-Vertical Cylinder

In this condition, the heat transfer can be determined using the same relationships as for a vertical flat plate, with the condition that the thickness of the boundary layer doesn't exceed the diameter of the cylinder (for example, as with wires). The standard criterion for assimilating a vertical cylinder to a flat plate is as follows:

$\frac{D}{H} \geq \frac{35}{Gr_H^{1/4}}$, With ***D***: diameter of the cylinder and ***H***: height of the cylinder.

The height of the cylinder is the characteristic dimension used to calculate the Nusselt number (***Nu***) and Grashof number (***Gr***).

c) - Horizontal Cylinder and Sphere

- **Horizontal Cylinder:**

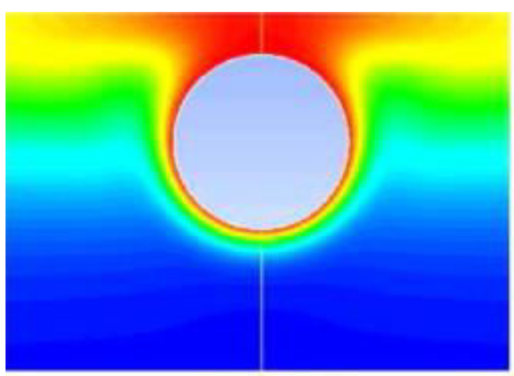


Figure 3.3 Fluid streams moving in free convection around a heated horizontal cylinder.

The recommended relationships are as follows:

$$\overline{Nu} = 0,53\left[Gr_f.\Pr_f\right]^{1/4} \text{ for } 10^4 \le Gr_f.\Pr_f \le 10^9 \qquad (3.62)$$

$$\overline{Nu} = 0,13\left[Gr_f.\Pr_f\right]^{1/3} \text{ for } 10^9 \le Gr_f.\Pr_f \le 10^{12} \qquad (3.63)$$

- **Sphere:**

The following relationships are recommended:

$$\overline{Nu} = 2 + 0,392 Gr_D^{0,25} \text{ for } 1 \le Gr_D.\Pr \le 10^5 \qquad (3.64)$$

$$\overline{Nu} = 2 + Gr_D^{0,5} \text{ for } Gr_D.\Pr \le 1 \qquad (3.65)$$

d) - Horizontal Flat Plate

The problem of the horizontal flat plate is one of the most difficult in natural convection. Both theoretical and experimental results are limited. The encountered difficulties are as follows:

- On the upward-flowing face, there is significant flow regime instability.
- Natural convection velocities near the downward-flowing face are extremely low, resulting in the flow regime being easily disrupted by parasitic movements.

The following relationships apply to diatomic gases and air:

- **Downward Flow:** This applies to the lower face of a hot plate and the upper face of a cold plate. The suggested relationship in the laminar region is as follows:

$$\overline{Nu} = 0,27\left[Gr_L.\Pr\right]^{1/4} \text{ for } 3.10^5 \le Gr_L.\Pr \le 3.10^{10} \qquad (3.66)$$

Note: No recommendation is provided for the turbulent region due to insufficient data.

- **Upward Flow:** This applies to the upper face of a hot plate or the lower face of a cold plate. The suggested relationship in the laminar region is as follows:

$$\overline{Nu} = 0{,}54\left[Gr_L.\mathrm{Pr}\right]^{1/4} \text{ for } 10^5 \le Gr_L.\mathrm{Pr} \le 2.10^7 \quad (3.67)$$

The recommended relationship in the turbulent region is as follows (The fluid characteristics are taken at temperature T_f.):

$$\overline{Nu} = 0{,}14\left[Gr_L.\mathrm{Pr}\right]^{1/3} \text{ for } 2.10^7 \le Gr_L.\mathrm{Pr} \le 3.10^{10} \quad (3.68)$$

These correlations have been developed for square plates of side length ***L***. For rectangular plates, ***L*** will be equal to the shorter side of the rectangle. For a plate shaped like a circular disc, ***L*** is replaced by ***0.9D***, where ***D*** is the disc's diameter.

3.4. Forced Convection

The study of forced convection is usually divided into two parts:

- Heat transfer by convection to and from a fluid flowing inside conduits.
- Forced convection in flow over the exterior surfaces of bodies such as cylinders, spheres, tubes, and tubular bundles.

Heating and cooling of fluids flowing inside or outside conduits is one of the most important industrial heat transfer processes. The dimensions of boilers, heat exchangers, and refrigeration or air conditioning units are largely determined by convection processes and the heat transfer coefficient, ***h***.

The heat transfer coefficient, h, can be determined using the Nusselt number (***Nu***), as previously demonstrated. But, in forced convection, Nu expressions assume the following form: **Nu = f(Re, Pr).**

The hydraulic diameter, D_h, where s = cross-sectional area and ***P*** = wetted perimeter. Is the characteristic length used in the Nusselt number and Reynolds number for flow inside conduits.

In the following paragraphs, we will study the several parameters that influence the expression: **Nu = f(Re, Pr).**

3.4.1. Choice of the fluid reference temperature

For flow over a flat surface, we have shown that the fluid temperature far from the heat source is usually constant, and it is customary to use this value as the fluid temperature, T_∞.

When heat is transferred to or from a fluid moving in a conduit, the fluid temperature does not reach a constant value but fluctuates with both the direction of fluid mass flow and the direction of heat flow.

As a result, the fluid reference temperature is often set to the average temperature of the fluid mass, T_m. The temperature of the mass middle between the inlet and outlet of the conduit is typically utilized as the reference temperature.

Note: Some formulas may need to be applied to calculate average temperatures between the surface and fluid temperatures. In any case, each formula must be accompanied by indications of the temperature used to determine fluid characteristics.

3.4.2. Effect of the Reynolds number

The Nusselt number for a given fluid is mostly determined by the flow conditions, which can be characterized using the Reynolds number, ***Re***.

Laminar flow within a conduit or along a plate does not mix hotter and cooler particles due to turbulent movement, and heat transfer occurs only through conduction.

As ***Re*** increases, some mixing occurs via turbulent eddies, which transport hot fluid into cooler regions and vice versa.

Turbulence significantly improves heat transfer, even on a very small scale, hence the heat transfer coefficient increases noticeably above **Re = 2100**.

Even in turbulent flow, the laminar boundary layer persists and controls heat flux due to its high thermal resistance.

The only effective way to increase heat transfer coefficients is thus to decrease the thermal resistance of the laminar sub-layer. This can be reached by increasing turbulence in the mainstream flow, i.e., by increasing the Reynolds number ***Re***.

3.4.3. Influence of the Prandtl number

As previously demonstrated, the Prandtl number, ***Pr***, is only determined by the fluid's characteristics. For a given Reynolds number, fluids with a high Prandtl number also have a high Nusselt number. For example, air has a Prandtl number close to 1.

The Prandtl numbers for ideal gases range between 0.6 and 0.9. Most oils have high Prandtl numbers due to their high viscosity and low thermal conductivity, $\boldsymbol{\lambda}$.

3.4.4. Influence of the Inlet section

When the conduit is short ($L/D_h < 50$), the inlet has a considerable effect on heat transfer.

Indeed, fluid mechanics has shown that when a fluid enters a conduit with a uniform velocity, a boundary layer forms along the wall.

The evolution of the thermal boundary layer in a fluid heated or cooled inside a conduit is qualitatively similar to that of the dynamic boundary layer.

As a result, it is observed that the local heat transfer coefficient varies significantly at the inlet: it is very important near the inlet and decreases as one moves along the conduit until the temperature and velocity profiles have reached their final form.

If the flow in the conduit is laminar, the effects of the inlet disappear at a distance equal to 50 times the diameter, whereas turbulent flow disappears at a distance equal to 10 times the diameter.

3.4.5. Variations in Physical Properties

When a fluid flowing in a pipeline is heated or cooled, its temperature and physical properties change significantly along the pipeline in any specific region. The fluctuation of viscosity with temperature in liquids is very important. In contrast, viscosity, thermal conductivity, and density of gases change considerably with temperature.

To account for temperature-dependent physical properties, the first method involves evaluating the attributes at an average fluid temperature defined as the average between T_f and T_s. Another method consists of evaluating all properties at T_m and then multiplying **f(Re, Pr)** by a corrective factor as we will demonstrate later. This method is preferable because its application is simpler.

3.4.6. Heat exchange coefficients for turbulent flow

Using the Reynolds analogy, we were able to obtain the relation:

$$Nu = 0{,}023Re^{0{,}8} \qquad (3.69)$$

This equation is in good agreement with experimental data on heat transfer in gases, with a Prandtl number close to 1 for smooth pipes for **10000 < Re < 120000**.

More complex analogies and $Pr \neq 1$ have exposed more correlations.

Dittus and **Boelter** (1930) proposed the following relationship:

$$Nu_D = 0{,}023Re_D^{0{,}8}Pr^n \qquad (3.70)$$

With: n=0.4 for fluid heating,

n=0.3 for fluid cooling

All fluid properties are evaluated at T_m, $Re > 10^4$, $0.7 < Pr < 100$, and $L/D_h > 60$.

Colburn (1933) proposed the relationship:

$$Nu_D = 0{,}023Re_D^{0{,}8}Pr^{0{,}4} \qquad (3.71)$$

With: Re_D and Pr evaluated at the film temperature T_f

$Re_D > 10^4$; $0.7 < Pr < 160$, and $L/D_h > 60$.

Sieder and **Tate** (1936) proposed the following relationships for fluids flowing in long pipes with corrective factors:

For gases: $Nu_D = 0{,}02Re^{0{,}8}.Pr^{1/3}\left[\frac{T_m}{T_s}\right]^n$ (3.72)

With: n=0.575 for gas heating, n=0.15 for gas cooling.

For liquids with high Prandtl numbers:

$$Nu_D = 0{,}027Re^{0{,}8}.Pr^{1/3}\left[\frac{\mu}{\mu_s}\right]^{0{,}14} \quad (3.73)$$

All fluid properties except $\boldsymbol{\mu_s}$ are evaluated at $\boldsymbol{T_m}$.

$Re_D > 10^4$, $0.7 < Pr < 17000$, $L/D_h > 60$.

Note: The inlet effect for gases and liquids flowing in a turbulent regime in short pipes (**$2 < L/D_h < 60$**) can be represented by the relationship:

For **$2 < L/D_h < 20$**, the relationship is:

$$\frac{h_L}{h} = 1 + \left[\frac{D_h}{L}\right]^{0{,}7} \quad (3.74)$$

For **$20 < L/D_h < 60$**, the relationship is:

$$\frac{h_L}{h} = 1 + \frac{6D_h}{L} \quad (3.75)$$

Where $\boldsymbol{h_L}$ is the average heat transfer coefficient per unit surface area for a pipe of length $\boldsymbol{L}$, and $\boldsymbol{h}$ is the heat transfer coefficient obtained from the various relationships presented above ($\boldsymbol{Nu_D}$).

3.4.7. Forced Convection for External Flows

There are several cases in practice when heat is transferred between a solid body and a fluid flowing outside of its surface. The cylinder and the sphere are the most common and interesting geometric shapes in engineering.

Determining the heat transfer coefficient per unit surface area around a cylinder or a sphere is a more difficult problem. Many experimenters have determined average coefficients of flow around cylinders and spheres. The results are summarized in the following correlation:

$$\frac{\bar{h}D}{\lambda_f} = Nu = C\left[\frac{V.D}{v_f}\right]^n Pr_f^{1/3} \qquad (3.76)$$

With: ***D*** = diameter of the circular cylinder immersed in a fluid flowing perpendicular to its axis at velocity ***V***.

Fluid properties are evaluated at T_f.

The constants ***C*** and n are given in the following table:

Re	C	n
0,4 to 4	0,989	0,330
4 to 40	0,911	0,385
40 to 4000	0,683	0,466
4000 to 40000	0,193	0,618
40000 to 400000	0,0266	0,805

This table provides the constants **C** and **n** for cylinders with circular cross-sections.

For spheres:

Several relationships have been proposed:

For various gas flows: (Mac Adams relationship), For **17 < Re_D < 70,000**.

$$\frac{\bar{h}D}{\lambda_f} = Nu = 0{,}37\left[\frac{V.D}{v_f}\right]^{0{,}6} \qquad (3.77)$$

For various liquid flows: (Kramers relationship), For **1 < Re_D < 2000**.

$$\frac{\bar{h}D}{\lambda_f}.Pr_f^{-0{,}3} = 0{,}97 + 0{,}37\left[\frac{V.D}{v_f}\right]^{0{,}5} \qquad (3.78)$$

For flows of water and oil: (**Vliet** and **Leppert**), For **1 < Re_D < 2.10^5**.

$$Nu.Pr^{-0,3}\left[\frac{\mu_s}{\mu}\right]^{0,25} = 1,2 + 0,53Re_D^{0,54} \quad (3.79)$$

All properties are evaluated at the fluid temperature far from the sphere, except μ_s which is evaluated at the surface temperature of the sphere.

Whitaker developed a unique equation applicable to both gases and liquids, which takes into account the previous equations:

$$Nu = 2 + \left(0,4Re_D^{1/2} + 0,06Re_D^{2/3}\right).Pr^{0,4}\left[\frac{\mu}{\mu_s}\right]^{1/4} \quad (3.80)$$

Valid for: **3.5 < Re_D < 8.10^4** and **0.7 < Pr < 380**, where fluid properties are evaluated far from the sphere.

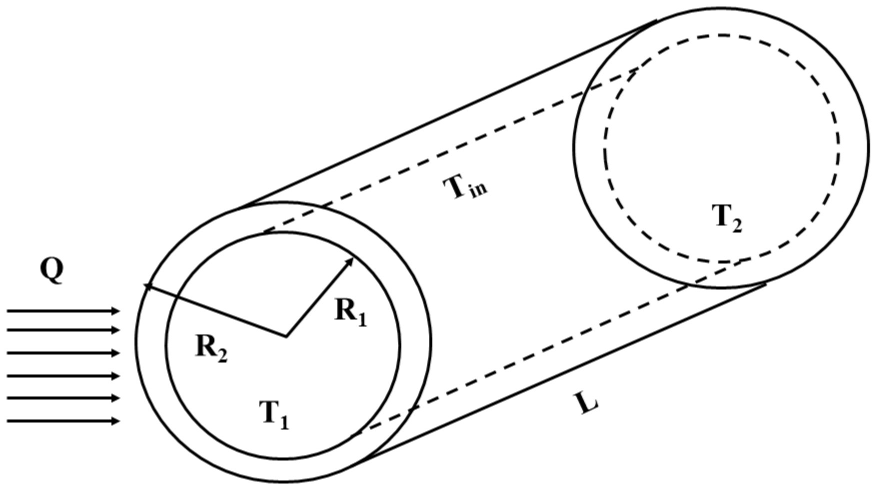
3.5. Conclusion

Chapter three provides a detailed theoretical and practical framework for analyzing and calculating convective heat transfer in various geometries and flow conditions. By introducing dimensionless groups and correlations, the chapter equips readers with the tools to predict heat transfer coefficients for both natural and forced convection. The emphasis on experimental validation and practical application ensures that the theoretical concepts can be effectively applied to real-world engineering problems, such as designing heat exchangers, cooling systems, and other thermal management devices.

Problem 1

Problem Statement: Consider a hollow cylindrical pipe with an inner radius (r_1 = 40) mm and an outer radius (r_2 = 80) mm, with a length (L = 1 m) and thermal conductivity (λ_c = 380 W/m°C). The inner surface of the pipe is maintained at a constant and uniform temperature (T_{in} = 120 °C). An incompressible fluid with a density ρ = 713 kg/m³, dynamic viscosity (μ= 1.1941kg·m⁻¹·s⁻¹), specific heat capacity (Cp = 2.371 kJ/kg°C), and thermal conductivity (λ_f = 0.029 W/m°C) flows inside this pipe with a volumetric flow rate (Q = 0.54 m³/s). The fluid enters at a temperature (T_1 = 30 °C) and exits at a temperature (T_2 = 60 °C). Let (h) denote the convective heat transfer coefficient between the fluid and the inner wall of the pipe, and (Pr = 20.6) denote the Prandtl number.

- Determine the thermal losses.



We give the following correlations:

$Nu=1,86(Re*Pr*De/L)^{0,3}$, Re<2300.

$Nu=0,0214(Re^{0,8} - 100)*Pr^{0,4}$, $10^4<Re<5*10^6$ et $0,5<Pr<1,5$

$Nu=0,027*Re^{0,8}*Pr^{0,3}$, liquide Re>2300

$Nu=0,023*Re^{0,8}*Pr^{0,4}$, Gaz Re>2300

Solution

To solve the problem of heat transfer by convection, we'll follow these steps:

1. Calculate the Reynolds number (Re) for the flow.
2. Determine the appropriate Nusselt number (Nu) correlation.
3. Calculate the thermal losses using the heat rate equation.

1- Calculate the Reynolds Number

The Reynolds number (Re) for the flow inside a cylindrical pipe is given by:

$$Re = \frac{\rho \vartheta D}{\mu}$$

First, calculate the fluid velocity using the volumetric flow rate Q: $v = \frac{Q}{A}$

Where A is the cross-sectional area of the pipe: $A = \pi r_1^2$

Given: $r_1 = 0.04$ m, $Q = 0.54\ m^3/s$,

$$A = 3.14 * (0.04)^2 = 0.005024\ m^2$$

Now, calculate the velocity: $v = \frac{0.54}{0.005024} = 107.5\ m/s$

Next, calculate the Reynolds number: $D = 2r_1 = 2*0.04 = 0.08$ m, $\mu = 1.1941 kg \cdot m^{-1} \cdot s^{-1}$.

$$Re = \frac{713 * 107.5 * 0.08}{1.1941} \approx 5118$$

2- Determine the appropriate Nusselt Number Correlation

Given that $Re \approx 5118$ and $Pr = 20.6$, we used the appropriate Nusselt number correlation for liquid with $Re > 2300$: $Nu = 0{,}027 * Re^{0{,}8} * Pr^{0{,}3}$.

$$Nu = 0.027 * (5118)^{0.8} * (20.6)^{0.3} = 146.45$$

3- Calculate the convective heat transfer coefficient.

The convective heat transfer coefficient (h) is given by:

$$h = \frac{Nu * \lambda_f}{D} = \frac{146.45 * 0.029}{0.08} \approx 53.08 (W/m^2 . C)$$

4- Determine the thermal losses

The thermal losses through the inner surface of the pipe are given by:

$$\varphi = h * A * (T_i - T_1) = h * 2\pi r_1 L * (T_i - T_1)$$

$$\varphi = 53.08 * (2 * 3.14 * 0.04 * 1) * (120 - 30)$$

$$\varphi = 1200W$$

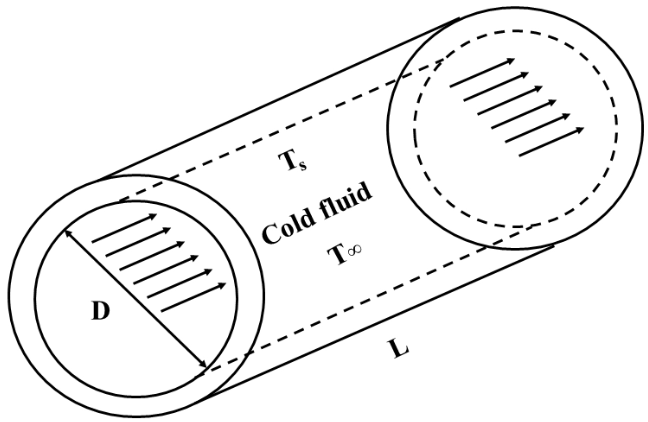
Problem 2

Problem Statement: In the objective of cooling a tube with a diameter of D = 12 cm, length L=1m, and a surface temperature T_S = 118 °C, the cooling fluid is air at a velocity v = 6 m/s and a temperature $T\infty$ = 22 °C. Calculate the heat flux exchanged (ϕ).

Data: ρ= 1.175 kg/m³, ν = 1.85 × 10⁻⁵ m²/s, λ = 0.029 W/m°C, Pr = 0.71

$Nu = 0.193 * Re^{0.618} * Pr^{0.33}$ for Re > 30000.

$Nu = 0.223 * Re^{0.4} * Pr^{0.33}$ for Re < 30000.



Solution

1- Calculate the Reynolds Number (Re)

The Reynolds number (Re) is a dimensionless quantity that determines the flow regime (laminar or turbulent) in the tube. It is calculated using the formula: Re = (ρ*v * D) / ν

Where:

- v is the velocity of the air = 6 m/s
- D is the diameter of the tube = 0.12 m (since 12 cm = 0.12 m)
- ν is the kinematic viscosity of air = 1.85×10^{-5} m²/s

Substituting the values:

$$Re = \frac{\rho \vartheta D}{\mu}$$

$$Re = \frac{1.175 \ * 6 * 0.12}{1.85 \ \times \ 10^{-5}} \approx 45729$$

2- Determine the Nusselt Number (Nu)

The Nusselt number (Nu) is a dimensionless number that relates the convective heat transfer to the conductive heat transfer. Depending on the Reynolds number, we use the appropriate correlation.

Since Re > 30000, we use the correlation: Nu = 0.193 * $Re^{0.618}$ * $Pr^{0.33}$

Substituting the values:

$$Nu = 0.193 * 45729^{0.618} * 0.71^{0.33}$$

Nu ≈ 130.76

2- Calculate the Convective Heat Transfer Coefficient (h)

The convective heat transfer coefficient (h) can be determined from the Nusselt number using the formula: h = (Nu * λ) / D

Where:

- λ is the thermal conductivity of air = 0.029 W/m°C
- D is the diameter of the tube = 0.12 m

Substitute the values:

$$h = \frac{Nu * \lambda_f}{D} = \frac{130.76 * 0.029}{0.12} \approx 31.6(W/m^2.C)$$

3- Calculate the Heat Flux Exchanged per Unit Length (ϕ)

The heat flux exchanged per unit length (ϕ) can be calculated using the formula:

$$\varphi = h * A * (T_i - T_1) = h * 2\pi r_1 L * (T_s - T_\infty)$$

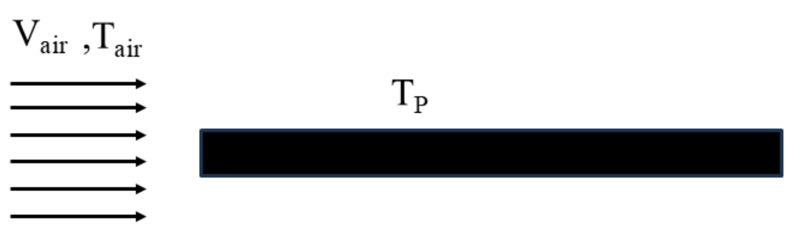
$$\varphi = 31.6 * 2 * 3.14 * 0.06 * 1 * (118 - 22)$$

Final Answer

The heat flux exchanged (ϕ) is approximately 1143 W.

Problem 3

Problem Statement: A thin plate with a length of L=3m and a width of w=1.5m is subjected to airflow with a velocity of V_{air}=2.0 m/s and a temperature of T_{air}=20°C, in the longitudinal direction. The temperature of the plate's surfaces is Tp= 84 °C.



It is required to calculate:

1- The convective heat transfer coefficient for Pr=0.71.
2- The heat flux transmitted by the plate to the air.

The thermophysical properties of air at 20ºC are:

$\rho = 1.175\,\frac{kg}{m^3}, \mu = 1.8 * \frac{10^{-5} kg}{m}.s, C_p = 1.006\,\frac{KJ}{kg}.C, et\ \lambda_f = 0.026\,\frac{W}{m}.C$

Solution

1- Calculate the convective heat transfer coefficient (h)

$$Re = \frac{\rho.V.L}{\nu} = \frac{1.175 * 2 * 3}{1.8 * 10^{-5}} = 3.91 * 10^5$$

Re<5.105 therefore the flow regime is laminar

We applied in this case the following equation

$Nu = 0.628 Re^{0.5} * Pr^{1/3}$

$$Nu = 0.628 * (3.91 * 10^5)^{0.5} * (0.71)^{\frac{1}{3}} = 350.32$$

$$Nu = \frac{hL}{\lambda} \Rightarrow h = \frac{\lambda.Nu}{L} = \frac{0.026 * 350.32}{3} = 3.03(W/m^2.C)$$

2- Calculate the heat flux transmitted by the plate to the air

$$\varphi = 2hA(T_w - T_a) = 2 * 3.03 * 4.5 * (84 - 20)$$

$$\varphi = 163.62\,W$$

Chapter 4

Heat transfer by radiation

4.1. Introduction

Radiation, or more correctly thermal radiation heat flux is the result of the emission of electromagnetic radiation by objects due to their temperature and the expense of its internal energy. This emitted radiation (photons) covers the entire electromagnetic spectrum. The radiation heat flux vector $\mathbf{q}_r$ is the average of the photon flux (moving at the speed of light c) taken over all the frequencies f (or wavelengths λ) and in the desired direction designated by the surface normal s_n. The photon flux changes as it passes through a material. On the macroscopic level, thermal radiation is calculated using the Stefan-Boltzmann law, which relates the energy heat flux emitted by an ideal radiator (or blackbody) to the fourth power of the absolute temperature:

$$e_b = \sigma T^4 \qquad (4.1)$$

the Stefan-Boltzmann constant (σ) has a value of 5.669*10^{-8} W/(m^2.K^4). Engineering surfaces in general, do not perform as ideal radiators, hence the following law is modified to read

$$e_b = \varepsilon\sigma T^4 \qquad (4.2)$$

The surface's emissivity (ε) ranges from 0 and 1. When two blackbodies exchange heat by radiation, the net heat exchanger is then proportional to the difference in T^4. If the first body "sees" only body 2, then the net heat exchanger between bodies 1 and 2 is given by

$$q = \sigma A_1(T_1^4 - T_2^4) \qquad (4.3)$$

Because of the shape arrangement, only a fraction of the energy leaving body 1 is intercepted by body 2,

$$q = \sigma A_1 F_{1-2}(T_1^4 - T_2^4) \qquad (4.4)$$

Where F_{1-2} (also known as a form factor or a view factor) is the fraction of energy leaving body 1 that is intercepted by body 2. If the bodies are not black, then the view factor F_{1-2}

must be replaced by a new factor $\mathcal{F}_{1\text{-}2}$ that depends on the emissivity ε of the surfaces involved and the shape view.

4.2. The heat exchanger problem

Figure 4.1 depicts two arbitrary surfaces emitting energy to one another. The net heat exchanger (Q_{net}) from the hotter surface (1) to the cooler surface (2) is dependent on the following factors:

- T_1 and T_2.
- areas A_1 and A_2.
- the shape, orientation, and spacing of (1) and (2).
- Surfaces' radiative characteristics.
- Additional surfaces in the environment can reflect radiation to each other.
- The substance between (1) and (2) that absorbs, emits, or "reflects" radiation. (When the substance is air, we can generally neglect these effects.)

4.3. Some definitions

4.3.1. Emissive power

Remember that emissions occur from any surface with a finite temperature. The concept of emissive power is developed to measure the rate of radiation emitted per unit surface area. The spectral emissive power E_λ (W/m^2.µm) is the rate at which radiation of wavelength λ is emitted in all directions into the hemispheric space from a surface, per unit surface area and unit wavelength interval $d\lambda$ about λ (fig. 4.1).

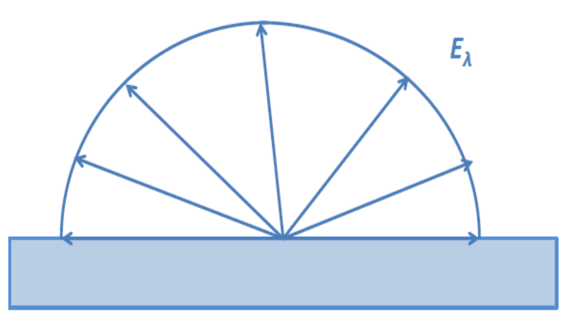


Figure 4.1: The spectral emissive power $\boldsymbol{E_\lambda}$

The total emissive power $\boldsymbol{E}$ (W/m^2) represents the rate at which radiation is emitted per unit area in all possible directions and wavelengths. as shown in Fig. 4.2.

$$E = \int_0^\infty E_\lambda(\lambda)d\lambda \qquad (4.5)$$

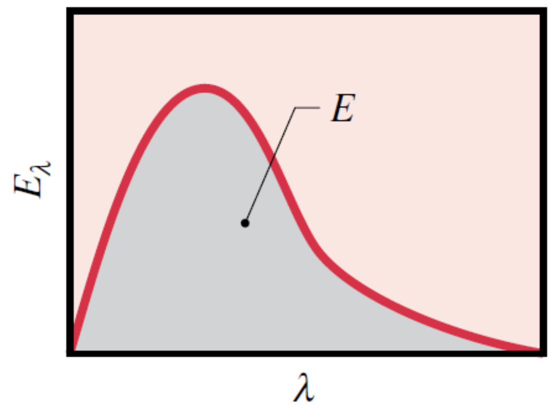


Figure 4.2: The total emissive power $\boldsymbol{E}$

4.3.2. Irradiation

The preceding method can be used for incident radiation. Such radiation may be caused by emission and reflection on other surfaces, as well as by the surrounding environment and radiation sources like lamps. The incident radiation represents a radiative flux known as irradiation, which includes radiation that comes from all directions.

The spectral irradiation $\boldsymbol{G_\lambda}$ (W/m^2.µm) is defined as the rate at which radiation of wavelength $\boldsymbol{\lambda}$ is incident on a surface, per unit area and unit wavelength interval $\boldsymbol{d\lambda}$ about $\boldsymbol{\lambda}$.

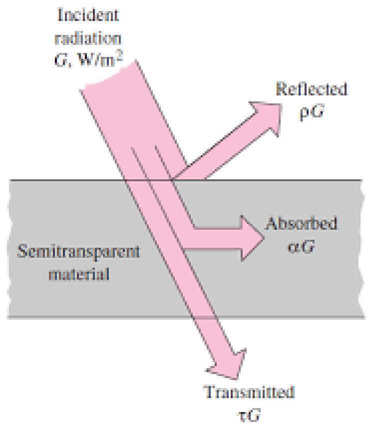


Figure 4.3: The spectral irradiation G_λ

In the total irradiation G (W/m^2) is the rate of radiation incident per unit area from all directions and wavelengths. That means:

$$G = \int_0^\infty G_\lambda(\lambda)d\lambda \qquad (4.6)$$

$G_\lambda(\lambda)$ is represented as spectral distribution, as shown in fig. 4.3.

4.3.3. Radiosity

The third radiative flux of interest, known as radiosity, accounts for the radiant energy that leaves a surface. The radiosity differs from the emissive power because this radiation includes both reflected and direct emission (**Fig. 4.4**).

The spectral radiosity J_λ (W/m^2.µm) represents the rate at which radiation of wavelength λ leaves a unit area of the surface, per unit wavelength interval $d\lambda$ about λ

$$J_\lambda = E_\lambda + G_{\lambda,ref} \qquad (4.7)$$

The spectral emissive power (E_λ) represents the direct emission component, while $G_{\lambda,ref}$ represents the reflected fraction of the spectral irradiation G_λ. The integral of the spectral variables can be used to get the total radiosity J (W/m^2) of the entire spectrum.

$$J = \int_0^\infty J_\lambda d\lambda = \int_0^\infty (E_\lambda + G_{\lambda,ref})d\lambda \qquad (4.8)$$

Alternatively, in terms of total emissive power and reflected portion of total irradiation.

$$J = E + G_{ref} \qquad (4.9)$$

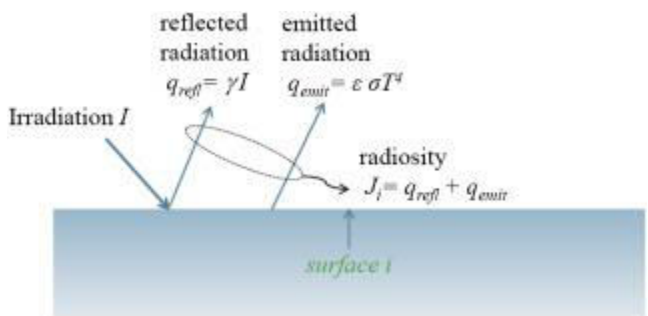


Figure 4.4: total emissive power and reflected portion of total irradiation

4.3.4. Surface energy balances with radiation processes

Following the methods of Sec. 1.2, it is critical to identify two types of surface energy balances that will be relevant in radiation processes.

We are interested in performing energy balances on spectrally selective, single surfaces that are emitted and irradiated. As we will see shortly, the characteristics that affect irradiation emission and absorption are usually different. According to eq. 16, the surface energy balance shown in Fig. 4.6 has the following form:

$$q''_{rad,net} = E - G_{abs} \qquad (4.10a)$$

Where $\boldsymbol{q''_{rad,net}}$ is the net radiation flux leaving the surface, $\boldsymbol{E}$ is the total emissive power of the surface, and G_{abs} is the absorbed part of the total irradiation $\boldsymbol{G}$. The net radiative flux $\boldsymbol{q''_{rad,net}}$ differ from the radiosity J, which simply represents the radiant flux leaving the surface.

The surface energy balance can be expressed alternatively in terms of total radiosity and total irradiation. For the surface in Figure 4.4.

$$q''_{rad,net} = J - G \qquad (4.10b)$$

Where $q''_{rad,net}$ is the net radiation leaving the surface by radiation, J is the surface's total radiosity, and G is the total irradiation. Eq. 4.10b states that radiosity is the sum of the emitted and reflected irradiation leaving the surface. It is not always convenient to calculate the radiosity, thus eq. 4.10a may be more beneficial.

4.3.5. Emittance

A real body at temperature T does not emit with the black body emissive power $e_b = \sigma T^4$ but rather with a fraction, ε, of e_b. The monochromatic emissive power, $e_\lambda(T)$, is always lower for a real body than the black body value provided by Planck's law. Thus, we define either the monochromatic emittance, ε_λ:

$$\varepsilon_\lambda = \frac{\varepsilon_\lambda(\lambda,T)}{\varepsilon_{\lambda b}(\lambda,T)} \qquad (4.11)$$

Alternatively, ε represents the total emittance:

$$\varepsilon = \frac{e(T)}{e_b(T)} = \frac{\int_0^\infty e_\lambda(\lambda,T)d\lambda}{\sigma T^4} = \frac{\int_0^\infty \varepsilon_\lambda e_\lambda(\lambda,T)d\lambda}{\sigma T^4} \qquad (4.12)$$

For real bodies, both ε and ε_λ are superior to zero and less than one; for black bodies, $\varepsilon = \varepsilon_\lambda = 1$

The emittance is fully determined by the surface properties of the specific body and its temperature. It functions independently of the body's environment.

Although most common materials and coatings' emittance vary with wavelength in the thermal range, we can use total emittance to write the emissive power as if the body is gray, without having to integrate over wavelength.

$$e(T) = \varepsilon\sigma T^4 \qquad (4.13)$$

4.3.6. The Planck distribution

Planck established the spectral distribution of blackbody radiation, which is generally recognized. This is the form:

$$\lambda_{\lambda,b}(\lambda,T) = \frac{C_1}{\lambda^5[exp(C_2/\lambda T)-1]} \qquad (4.14)$$

The first and second radiation constants are $C_1 = 2\pi h c_0^2 = 3.742 * 10^8 W.\mu m^4/m^2$ and $C_2 = \left(\frac{hc_0}{k}\right) = 1.439 * 10^4 \mu m^4.K$, where T is the absolute temperature of the blackbody. $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$ are determined using the universal constants, $\boldsymbol{h}$, $\boldsymbol{k}$, and $\boldsymbol{c_o}$, which are the Planck constant, the Boltzmann constant, and the speed of light in a vacuum, respectively.

Eq. (4.14), often known as the Planck spectral distribution, is shown in **Fig. 4.5** for various temperatures. Several key elements should be highlighted:

- The emitted radiation changes with wavelength.
- At any wavelength, the intensity of emitted radiation increases with increasing temperature.
- Temperature affects the spectral region where radiation is concentrated, with higher temperatures resulting in greater radiation-specific wavelengths.

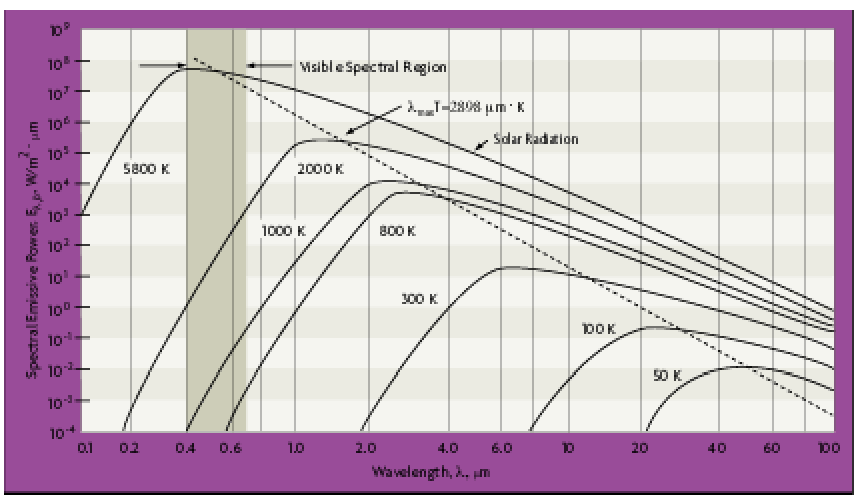


Figure 4.5: Two-dimensional representation of the spectrum of a blackbody at different temperatures.

4.3.7. Wien's displacement law

Figure 4.5 shows that the blackbody spectral distribution has a maximum, and the wavelength $\boldsymbol{\lambda_{max}}$ depends on temperature. To determine the nature of this dependence, differentiate equation 4.8 concerning $\boldsymbol{\lambda}$ and set the result equal to zero. In so doing, we get:

$$\lambda_{max}T = C_3 \qquad (4.15)$$

The third radiation constant is C_3=2897.8 μm.K. Eq. 4.15 is known as Wien's displacement law.

4.4. The Stefan-Boltzmann law

Substituting the Planck distribution, Eq. 4.8 into Eq.4.5 the total emissive power of a blackbody, Eb may be expressed as

$$E_b = \int_0^\infty E_{\lambda,b} d\lambda = \int_0^\infty \frac{C_1}{\lambda^5[exp(C_2/\lambda T)-1]} d\lambda \qquad (4.16)$$

The result obtained by calculating the integration is called the Stefan-Boltzmann law, with the form is

$$E_b = \sigma T^4 \qquad (4.17)$$

The Stefan-Boltzmann constant, which depends on C_1 and C_2, has the numerical value $\sigma=5.670\times10^{-8}$ $W/m^2.K^4$. this simple but significant law allows for the determining the amount of radiation emitted in all directions and across all wavelengths depending on the blackbody temperature.

4.4.1. Blackbody radiation

The blackbody is the standard for determining the behavior of all real radiating materials. It contains well-defined quantities that are solidly based on theory and experimentation. These important properties will be discussed in detail because they are essential for understanding the radiative transfer phenomenon.

4.4.2. General Definitions and Characteristics

The most crucial attributes of a blackbody are as follows:

- A blackbody is a surface or volume that completely absorbs incident radiation. This contains radiation of all wavelengths and from all directions.
- The blackbody emits radiation optimally across all wavelengths and directions.
- A blackbody's radiation increases monotonically with absolute temperature at all wavelengths.
- Radiation within an isothermal enclosure with blackbody boundaries is isotropic, meaning uniform in all directions.

With such characteristics, the blackbody is seen to be a useful reference for comparing the characteristics of physical materials. All physical materials will reflect part of the incident radiant energy; hence they are not ideal absorbers. Because they do not absorb as much as a perfect blackbody, they must emit less in order to maintain thermal equilibrium with their environment. A real surface thus emits less than a blackbody (at all wavelengths and directions).

Blackbody Intensity and Emissive Power. The intensity emitted by a black surface is independent of T and ϕ, indicating isotropy. This fact gives another useful reference point for comparing the behavior of real surfaces.

the Planck distribution of blackbody intensity describes the spectral intensity of a blackbody.

$$I_{\lambda b} = \frac{2C_1}{n^2\lambda^5(\exp(C_2/n\lambda T)-1)} \qquad (4.18)$$

T is the absolute temperature (in K) and C_1 and C_2 are constants with values

$$C_1 = hc_0^2 = 0.59552.10^8 \ W\mu m^4/m^2 \qquad (4.19)$$

$$C_2 = hc_0/k = 14.388.10^8 \ \mu mK \qquad (4.20)$$

With $h=6.626075.10^{-34}$ Js (Planck constant), $k=1.380658.10^{-23}$ J/K (Boltzmann constant), and $c_0=2.99792458.10^{8}$ m/s (the speed of light in vacuum).

4.5. Kirchoff's law

4.5.1. The problem of predicting α

Equation (4.12) demonstrates that a surface's total emittance, ε, is governed only by the physical properties and temperature. The overall absorptance (α) of a surface is determined by both radiation source and its own characteristics. This occurs because the surface may absorb certain wavelengths more efficiently than others. Thus, the total absorptance will be determined by how the incoming radiation is dispersed in wavelength. This distribution is in turn determined by the temperature and physical qualities of the surface or surfaces from which radiation is absorbed.

The overall absorptance (α) is determined by the physical properties and temperatures of all bodies participating in the heat exchange process. Kirchoff's law is a restriction-based expression for determining (α).

Kirchoff's law describes the relationship between the monochromatic and directional emittance, hence the monochromatic and directional absorptance for a surface that is in thermodynamic equilibrium with its environment.

Exact form of Kirchhoff's law

$$\varepsilon_\lambda(T,\theta,\phi) = \alpha_\lambda(T,\theta,\phi) \qquad (4.21a)$$

Kirchhoff's law states that a body in thermodynamic equilibrium emits the same amount energy as it absorbs in each direction and wavelength.

For a diffuse body, the emittance and absorptance are independent on the angles, and Kirchhoff's law becomes:

$$\varepsilon_\lambda(T) = \alpha_\lambda(T) \qquad (4.21b)$$

If the body is gray, Kirchhoff's law is further simplified

$$\varepsilon(T) = \alpha(T) \qquad (4.21c)$$

4.5.2. Total absorptance during radiant exchanger

Lat us focus our attention on diffuse surfaces, so that eq. 4.21b is the appropriate form of Kirchhoff's law. Consider two plates as illustrated in Fig. 4.6.

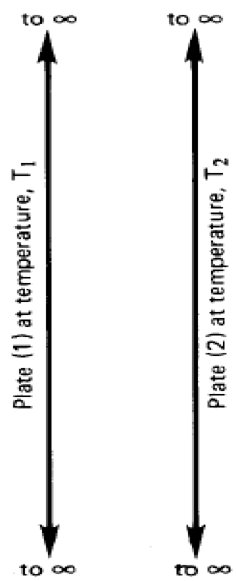


Figure 4.6: Heat transfer between two infinite parallel plates.

Let the plate at T_1 be non-black and that at T_2 be black. Then net heat transfer from plate 1 to plate 2 is the difference between what plate 1 emits and what it absorbs. Since all the radiation reaching plate 1 originates from a black source at T_2, we may write:

$$q_{net} = \int_0^{\infty} \varepsilon_{\lambda 1}(T_1)\, e_{\lambda b}(T_1) d\lambda - \int_0^{\infty} \alpha_{\lambda 1}(T_1)\, e_{\lambda b}(T_2) d\lambda \qquad (4.22)$$

Equation 4.12 allows us to express the first integral in terms of total emittance, as $\varepsilon_1 \sigma T_1^4$. We define the total absorptance, $\alpha_1(T_1, T_2)$, as the second integral divided by σT_2^4. Hence,

$$q_{net} = \varepsilon_1(T_1)\sigma T_1^4 - \alpha_1(T_1, T_2)\sigma T_2^4 \qquad (4.23)$$

We find that the total absorptance depends on T_2 and T_1.

4.5.3. Gray body approximation

If plate 1 is painted white and plate 2 is a temperature close to plate 1 (say T_1= 400 K and T_2= 300 K), then the incoming radiation from plate 2 will have a wavelength distribution comparable to plate 1. We might be very comfortable approximating $\varepsilon_1=\alpha_1$. The net heat flux between the plates can be represented very simply

$$q_{net} = \varepsilon_1 \sigma T_1^4 - \alpha_1(T_1, T_2)\sigma T_2^4 \cong \varepsilon_1 \sigma T_1^4 - \varepsilon_1 \sigma T_2^4$$

$$q_{net} = \varepsilon_1 \sigma (T_1^4 - T_2^4) \qquad (4.24)$$

In fact, we are approximating plate 1 as a gray body.

4.5.4. Radiation heat exchanger between two finite black bodies

Let us now return to the purely geometric problem of calculating the view factor $F_{1\text{-}2}$. Although the evaluation of $F_{1\text{-}2}$ is utilized to calculate of heat exchange among diffuse, nonblack bodies, it is the only Stefan-Boltzmann law that we need for black bodies.

Some obvious results: Figure depicts three elementary cases in which the value of $F_{1\text{-}2}$ is evident using only the definition:

$F_{1\text{-}2} \equiv$ fraction of field of view of (1) occupied by (2).

When the surfaces are each isothermal and diffuse, this corresponds to

$F_{1\text{-}2} \equiv$ fraction of energy leaving (1) that reaches (2)

A second apparent effect of the view factor is that all of the energy that leaves a body (1) reaches something else. Thus, the conservation of energy demands

$$1 = F_{1-1} + F_{1-2} + F_{1-3} + \cdots + F_{1-n} \qquad (4.25)$$

Where (2), (3), . . ., (n) are all of the bodies in the neighborhood of (1).

View factor reciprocity: until now, we have referred to the net radiation from black surface (1) to black surface (2) as $\boldsymbol{Q_{net}}$. Let us refine our notation a bit, and call this $\boldsymbol{Q_{net1\text{-}2}}$:

$$Q_{net_{1-2}} = \sigma A_1 F_{1-2}(T_1^4 - T_2^4) \qquad (4.26)$$

Similarly, the net radiation between (2) and (1) is

$$Q_{net_{2-1}} = \sigma A_2 F_{2-1}(T_2^4 - T_1^4) \qquad (4.27)$$

Of course, Q_{net1-2} equals - $Q_{net\,2-1}$. If follows that

$$\sigma A_1 F_{1-2}(T_1^4 - T_2^4) = -\sigma A_2 F_{2-1}(T_2^4 - T_1^4)$$

$$A_1 F_{1-2} = A_2 F_{2-1}$$

This conclusion, known as view factor reciprocity, is very important in calculations.

4.5.5. Calculation of the black-body view factor

Consider two elements, dA_1 and dA_2, of larger black bodies (1) and (2), as illustrated in **Fig 4.7**. Body (1) and body (2) are both isothermal. Since element dA_2 subtends a solid angle dW_1, we use the following equation:

$$dQ_{outgoing} = (idw)(cos\theta dA) = \begin{cases} radiant\ energy\ from\ dA \\ that\ is\ intercepted\ by\ dA_a \end{cases} \qquad (4.28)$$

$$dQ_{1\ to\ 2} = (i_1 dw_1)(cos\beta_1 dA_1) \text{ and } i_1 = {\sigma T_1^4}/{\pi}$$

It should be noted that because black bodies radiate diffusely, (i_1) does not vary with angle; also, because these bodies are isothermal, it does not vary with position. The element of solid angle is defined as:

$$dw_1 = \frac{cos\beta_2 dA_2}{S^2}$$

Where (s) is the distance from (1) to (2) and (cos β_2) enters because dA_2 is not essentially normal to (s). thus,

$$dQ_{1\ to\ 2} = \frac{\sigma T_1^4}{\pi}\left(\frac{cos\beta_1 cos\beta_2 dA_1 dA_2}{s^2}\right)$$

By the same token,

$$dQ_{2\ to\ 1} = \frac{\sigma T_2^4}{\pi}\left(\frac{cos\beta_2 cos\beta_1 dA_2 dA_1}{s^2}\right)$$

Then

$$Q_{net_{1-2}} = \sigma(T_1^4 - T_2^4)\int_{A_1}\int_{A_2}\frac{cos\beta_2 cos\beta_1}{\pi s^2}dA_1 dA_2 \qquad (4.29)$$

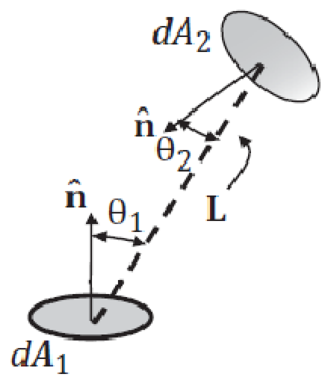


Figure 4.7: Two large blackbodies dA_1 and dA_2) participating in their net heat transfer.

The view factor F_{1-2} and F_{2-1} are immediately obtainable from eq. (4.29). If we compare this result with $Q_{net_{1-2}} = \sigma A_1 F_{1-2}(T_1^4 - T_2^4)$, we get

$$F_{1-2} = \frac{1}{A_1}\int_{A_1}\int_{A_2}\frac{cos\beta_2 cos\beta_1}{\pi s^2}dA_1 dA_2 \qquad (4.30)$$

From the inherent symmetry of the problem, we can also write

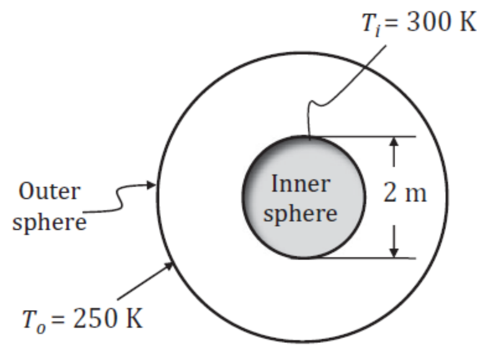
$$F_{2-1} = \frac{1}{A_2}\int_{A_2}\int_{A_1}\frac{cos\beta_2 cos\beta_1}{\pi s^2}dA_2 dA_1 \qquad (4.31)$$

4.6. Conclusion

Chapter four presents a comprehensive understanding of thermal radiation, beginning with the fundamental principles of blackbody radiation and progressing to the examination of real surfaces. The chapter introduces fundamental concepts such as emissivity, Kirchhoff's law, and view factors, giving readers the skills they need to analyze and calculate radiative heat transfer in various engineering applications. The emphasis on blackbody radiation and the gray body approximation simplifies complex radiative heat transfer problems, making them more applicable in practical scenarios such as heat exchangers, thermal insulation, and energy systems. Overall, the chapter provides a good foundation for understanding the role of radiation in heat transfer and its importance in thermal engineering.

Problem 1

Problem Statement: For an enclosed sphere what is the heat transfer from the inner sphere to outer sphere if the heat transfer surface have blackbody characteristics?



Solution

From basic geometry, we know the area of a sphere is $s = 4\pi r_i^2$. If the diametre of the inner sphere is D_{in}=2.0 m the the radius r_{in}= 1 m.

View factor summation rules for an enclosure give Fi-o=1. Thus, an equation for the net heat transfer from the inner sphere to the outer sphere is

$$q_{io} = \sigma A_1 F_{i-o}(T_i^4 - T_o^4)$$

$$q_{io} = 4 * 3.14 * 5.67 * 10^{-8} * 1 * [300^4 - 250^4]$$

$$q_{io} \approx 3kW$$

Problem 2

Problem Statement: A cylindrical system, 1 m long and 3 cm in diameter, is heated and positioned in a vacuum furnace with interior walls at 900 K. Current is passed through the rod, and its surface is maintained at 1100 K. Calculate the power supplied to the heating rod if its surface has an emissivity of 0.8.

Solution

For steady state conditions, the electric power supplied to rod equals the radiant heat loss from it. Further, since the walls of the furnace completely enclose the heating rod, all the radiant energy emitted by the surface of the rod is intercepted the furnace walls.

We use the Stefan-Boltzmann law for radiative heat transfer:

$$q_{r-w} = \epsilon\sigma A_r (T_r^4 - T_w^4)$$

Where:

- Power supplied to the rod (in watts),
- Emissivity of the rod (0.8),
- Stefan-Boltzmann constant ($5.67\times10{-}8 W/m^2K^4$),
- Surface area of the rod (A= πdL),
- Surface temperature of the rod (1100 K),
- Temperature of the furnace walls (900 K).

$$q_{r-w} = 0.8 * 5.67 * 10^{-8} * 3.14 * 0.03 * 1[1100^4 - 900^4]$$

The power supplied to the heating rod is approximately **3444 W** or **3.44 kW**.

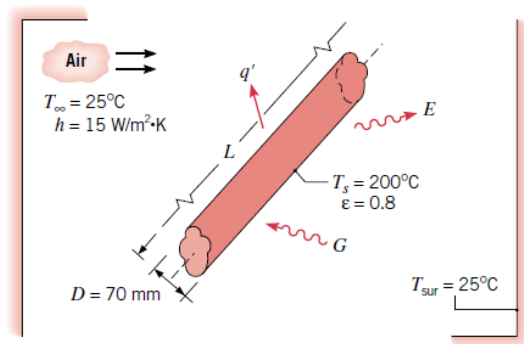
Problem 3

Problem Statement: An uninsulated staem pipe passes through a large room in which the air at 25 ºC. The outside diameter of the pipe is 70 mm, and its surface temperature and emissivity are 200 ºC and 0.8, respectively. What are the surface emissive power and irradiation?

If the coefficient associated with free convection heat transfer from the surface to the air is 15 $W/m^2.K$ and the surface is gray, what is the rate heat transfer from the surface per length of pipe?

Solution

For steady state conditions, radiation exchange between the pipe and the room is between a small surface and large, isothermal surroundings. The surface is diffuse-gray; that is, the emissivity and absorptivity are equal.



The surface emissive power may be evaluated by the equation

$$E = \varepsilon\sigma T_s^4 = 0.8 * 5.67 * 10^{-8} * 473^4 = 2270\ W/m^2$$

While the irradiation corresponds to

$$G = \sigma T_{sur}^4 = 5.67 * 10^{-8} * 298^4 = 447\ W/m^2$$

Heat transfer from the pipe is by convection to the room air and by radiation exchange with the walls. Hence, $q = q_{conv} + q_{rad}$

$$q = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{sur}^4)$$

$$q = h(\pi DL)(T_s - T_\infty) + \varepsilon(\pi DL)\sigma(T_s^4 - T_{sur}^4)$$

$$q' = \frac{q}{L} = h(\pi D)(T_s - T_\infty) + \varepsilon(\pi D)\sigma(T_s^4 - T_{sur}^4)$$

$$q' = 15 * 3.14 * 0.07 * (200 - 25) + 0.8 * 3.14 * 0.07 * 5.67 * 10^{-8} * (473^4 - 298^4)$$

$$q' = 577 + 421 = 988\ W/m$$

General conclusion

General Conclusion

The chapters on heat transfer covering conduction, convection, and radiation provide a detailed and comprehensive understanding of how thermal energy is transferred in various systems, each governed by distinct mechanisms and principles. Conduction involves the transfer of heat through solids or stationary fluids due to temperature gradients, driven by molecular interactions and described by Fourier's Law, which relates heat flux to the temperature gradient and the material's thermal conductivity. This mode of heat transfer is critical in applications like thermal insulation, electronic cooling, and heat exchangers, where heat must be efficiently conducted through solid materials or across interfaces. Convection, on the other hand, deals with heat transfer between a solid surface and a moving fluid, influenced by fluid properties. Convection can be natural, driven by buoyancy forces due to temperature differences, or forced, induced by external means like pumps or fans. The heat transfer coefficient (h) is a key parameter in convection, and dimensionless numbers such as the Nusselt number (Nu)**,** Reynolds number (Re), and Prandtl number (Pr) are used to predict and analyze convective heat transfer in various geometries and flow conditions. Radiation involves the transfer of heat through electromagnetic waves, requiring no medium, and is governed by the Stefan-Boltzmann law, which states that the energy radiated by a body is proportional to the fourth power of its absolute temperature. Real surfaces are characterized by emissivity (ε), which quantifies their ability to emit radiation compared to a blackbody, and view factors (F), which determine the fraction of radiation exchanged between surfaces. Radiation is particularly significant in high-temperature applications like furnaces, solar energy systems, and thermal insulation.

Thermophysical proporieties tables

Table 1. Thermophysical Properties of Common Materials at Room Temperature

Material	State	Density (ρ) [kg/m³]	Specific Heat (Cp) [J/kg·K]	Thermal Conductivity (k) [W/m·K]	Thermal Diffusivity (α) [m²/s]
Water	Liquid	1000	4186	0.6	1.4×10^{-7}
Air	Gas	1.225	1005	0.026	2.2×10^{-5}
Aluminum	Solid	2700	900	237	9.7×10^{-5}
Copper	Solid	8960	385	401	1.1×10^{-4}
Iron	Solid	7870	449	80	2.3×10^{-5}
Steel (Carbon)	Solid	7850	502	50	1.2×10^{-5}
Glass	Solid	2500	840	1.4	6.7×10^{-7}
Ice	Solid	920	2100	2.2	1.1×10^{-6}
Ethanol	Liquid	789	2440	0.17	8.8×10^{-8}
Hydrogen	Gas	0.0899	14300	0.18	1.4×10^{-4}
Helium	Gas	0.1786	5193	0.15	1.6×10^{-4}
Mercury	Liquid	13534	140	8.3	4.4×10^{-6}
Silicon	Solid	2330	705	148	9.0×10^{-5}
Polyethylene	Solid	950	2300	0.33	1.5×10^{-7}

Thermophysical Properties of Water (0°C to 100°C)

Temperature (°C)	Density (ρ, kg/m³)	Specific Heat (Cp, J/kg·K)	Thermal Conductivity (k, W/m·K)	Dynamic Viscosity (μ, Pa·s)	Thermal Expansion (α, 1/K)
0	999.8	4217	0.569	0.001792	-0.000063
10	999.7	4192	0.580	0.001307	0.000088
20	998.2	4182	0.598	0.001002	0.000207
30	995.7	4178	0.615	0.000798	0.000303
40	992.2	4178	0.630	0.000653	0.000385
50	988.1	4181	0.643	0.000547	0.000457
60	983.2	4185	0.654	0.000467	0.000522
70	977.8	4191	0.663	0.000406	0.000583
80	971.8	4198	0.670	0.000355	0.000642
90	965.3	4206	0.675	0.000315	0.000700
100	958.4	4215	0.679	0.000282	0.000750

Thermophysical Properties of Air (0°C to 100°C)

Temperature (°C)	Density (ρ, kg/m³)	Specific Heat (Cp, J/kg·K)	Thermal Conductivity (k, W/m·K)	Dynamic Viscosity (μ, Pa·s)	Kinematic Viscosity (ν, m²/s)
0	1.292	1005	0.0242	0.0000172	0.0000133
10	1.246	1005	0.0249	0.0000178	0.0000142
20	1.204	1005	0.0257	0.0000183	0.0000151
30	1.164	1005	0.0265	0.0000187	0.0000160

40	1.127	1005	0.0273	0.0000191	0.0000169
50	1.092	1007	0.0280	0.0000195	0.0000178
60	1.060	1007	0.0287	0.0000199	0.0000187
70	1.029	1007	0.0294	0.0000203	0.0000197
80	1.000	1007	0.0301	0.0000207	0.0000207
90	0.972	1007	0.0308	0.0000211	0.0000217
100	0.946	1007	0.0314	0.0000215	0.0000227

Thermophysical Properties of Aluminum (0°C to 1000°C)

Temperature (°C)	Density (ρ, kg/m³)	Specific Heat (Cp, J/kg·K)	Thermal Conductivity (k, W/m·K)	Thermal Expansion (α, 1/K)
0	2700	900	237	0.000023
100	2698	913	237	0.000023
200	2695	937	236	0.000024
300	2692	963	235	0.000025
400	2688	990	233	0.000026
500	2683	1018	230	0.000027
600	2678	1047	227	0.000028
700	2672	1077	223	0.000029
800	2665	1108	218	0.000030
900	2658	1140	213	0.000031
1000	2650	1173	207	0.000032

Thermophysical Properties of Copper (0°C to 1000°C)

Temperature (°C)	Density (ρ, kg/m³)	Specific Heat (Cp, J/kg·K)	Thermal Conductivity (k, W/m·K)	Thermal Expansion (α, 1/K)
0	8960	385	401	0.000016
100	8940	390	398	0.000017
200	8920	395	394	0.000018
300	8900	400	389	0.000019
400	8880	405	384	0.000020
500	8860	410	379	0.000021
600	8840	415	373	0.000022
700	8820	420	367	0.000023
800	8800	425	360	0.000024
900	8780	430	353	0.000025
1000	8760	435	345	0.000026

Thermophysical Properties of Iron (0°C to 1000°C)

Temperature (°C)	Density (ρ, kg/m³)	Specific Heat (Cp, J/kg·K)	Thermal Conductivity (k, W/m·K)	Thermal Expansion (α, 1/K)
0	7870	450	80.2	0.000012
100	7860	480	76.5	0.000013
200	7850	510	72.8	0.000014
300	7840	540	69.1	0.000015
400	7830	570	65.4	0.000016
500	7820	600	61.7	0.000017
600	7810	630	58.0	0.000018
700	7800	660	54.3	0.000019
800	7790	690	50.6	0.000020
900	7780	720	46.9	0.000021
1000	7770	750	43.2	0.000022

Thermophysical Properties of Ethanol (0°C to 100°C)

Temperature (°C)	Density (ρ, kg/m³)	Specific Heat (Cp, J/kg·K)	Thermal Conductivity (k, W/m·K)	Dynamic Viscosity (μ, Pa·s)	Thermal Expansion (α, 1/K)
0	806	2260	0.180	0.00177	0.00075
10	800	2300	0.178	0.00145	0.00080
20	789	2350	0.175	0.00120	0.00085
30	781	2400	0.172	0.00100	0.00090
40	772	2450	0.169	0.00085	0.00095
50	763	2500	0.166	0.00072	0.00100
60	754	2550	0.163	0.00062	0.00105
70	745	2600	0.160	0.00054	0.00110
80	736	2650	0.157	0.00047	0.00115
90	727	2700	0.154	0.00041	0.00120
100	718	2750	0.151	0.00036	0.00125

Thermophysical Properties of Hydrogen (0°C to 100°C)

Temperature (°C)	Density (ρ, kg/m³)	Specific Heat (Cp, J/kg·K)	Thermal Conductivity (k, W/m·K)	Dynamic Viscosity (μ, Pa·s)	Kinematic Viscosity (ν, m²/s)
0	0.0899	14300	0.182	0.0000084	0.000093
10	0.0858	14350	0.187	0.0000087	0.000101
20	0.0820	14400	0.192	0.0000090	0.000110
30	0.0785	14450	0.197	0.0000093	0.000119
40	0.0752	14500	0.202	0.0000096	0.000128
50	0.0721	14550	0.207	0.0000099	0.000137
60	0.0692	14600	0.212	0.0000102	0.000147
70	0.0665	14650	0.217	0.0000105	0.000158

80	0.0639	14700	0.222	0.0000108	0.000169
90	0.0615	14750	0.227	0.0000111	0.000181
100	0.0592	14800	0.232	0.0000114	0.000193

Thermophysical Properties of Helium (0°C to 100°C)

Temperature (°C)	Density (kg/m³)	Viscosity (μPa·s)	Thermal Conductivity (W/m·K)	Specific Heat (J/kg·K)	Thermal Expansion (1/K)
0	0.1786	19.0	0.1513	5193	0.00366
10	0.1720	19.3	0.1552	5201	0.00367
20	0.1664	19.6	0.1592	5209	0.00368
30	0.1610	19.9	0.1632	5217	0.00369
40	0.1559	20.2	0.1673	5225	0.00370
50	0.1510	20.5	0.1714	5233	0.00371
60	0.1463	20.8	0.1756	5241	0.00372
70	0.1418	21.1	0.1798	5249	0.00373
80	0.1375	21.4	0.1841	5257	0.00374
90	0.1333	21.7	0.1884	5265	0.00375
100	0.1293	22.0	0.1928	5273	0.00376

Thermophysical Properties of Mercury at Different Temperatures

Temperature (°C)	Density (kg/m³)	Viscosity (μPa·s)	Thermal Conductivity (W/m·K)	Specific Heat (J/kg·K)	Thermal Expansion (1/K)
0	13595	1.526	8.34	139.5	0.00018
10	13550	1.480	8.40	140.0	0.00018
20	13505	1.437	8.46	140.5	0.00018
30	13460	1.396	8.52	141.0	0.00018
40	13415	1.357	8.58	141.5	0.00018
50	13370	1.320	8.64	142.0	0.00018
60	13325	1.285	8.70	142.5	0.00018
70	13280	1.251	8.76	143.0	0.00018
80	13235	1.218	8.82	143.5	0.00018
90	13190	1.187	8.88	144.0	0.00018
100	13145	1.157	8.94	144.5	0.00018

Thermophysical Properties of Oil at Different Temperatures

Temperature (°C)	Density (kg/m³)	Viscosity (μPa·s)	Thermal Conductivity (W/m·K)	Specific Heat (J/kg·K)	Thermal Expansion (1/K)
0	900	250	0.13	1900	0.0007
10	890	150	0.135	1920	0.00072
20	880	100	0.140	1940	0.00074
30	870	70	0.145	1960	0.00076
40	860	50	0.150	1980	0.00078

50	850	35	0.155	2000	0.00080
60	840	25	0.160	2020	0.00082
70	830	18	0.165	2040	0.00084
80	820	12	0.170	2060	0.00086
90	810	8	0.175	2080	0.00088
100	800	5	0.180	2100	0.00090

Thermophysical Properties of Natural Gas at Different Temperatures

Temperature (°C)	Density (kg/m^3)	Viscosity (µPa·s)	Thermal Conductivity (W/m·K)	Specific Heat (J/kg·K)	Thermal Expansion (1/K)
0	1.34	11.0	0.024	2200	0.0034
10	1.29	11.2	0.025	2210	0.0035
20	1.25	11.4	0.026	2220	0.0036
30	1.20	11.6	0.027	2230	0.0037
40	1.16	11.8	0.028	2240	0.0038
50	1.12	12.0	0.029	2250	0.0039
60	1.08	12.2	0.030	2260	0.0040
70	1.05	12.4	0.031	2270	0.0041
80	1.01	12.6	0.032	2280	0.0042
90	0.98	12.8	0.033	2290	0.0043
100	0.95	13.0	0.034	2300	0.0044

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