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Observability and Observer Design for Switched Linear Systems

Application to Multicellular Converters

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To my late father

To my mother for her ongoing love and support

To my husband and my daughters

To all my family

To all who wish the best for me

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مُلَـــخَّص

أنظمة التبديل هي أنظمة تضم مجموعة من الديناميات المستمرة مدبرة بواسطة إشارة التبديل. شهدت هذه الفئة الهامة من الأنظمة الديناميكية الهجينة تطورات ملحوظة، سواء من الناحية النظرية أوالعملية، وبالتالي الكشف عن الفوائد المحتملة للعديد من التطبيقات في العالم الحقيقي. يمكن أن يمثل هذا النوع من الأنظمة مجموعة واسعة جدًا من الأنظمة، مثل: إلكترونيات الطاقة وأنظمة التحكم المتعددة والروبوتات والأنظمة الكهروميكانيكية والأنظمة ذات التوقفات. نتيجة لذلك، يعد تطوير أدوات لتحليل الخصائص الديناميكية مثل قابلية الملاحظة أمرًا ضروريًا قبل أي خطوة تصميم لعنصر التحكم أو التوليف.

تنقسم مساهمة هذه الرسالة إلى قسمين. يقترح الجزء الأول ، من ناحية، تحليل قابلية ملاحظة أنظمة التبديل ذات أنظمة فرعية غير قابلة للملاحظة بالمعنى الكلاسيكي. تم تطوير شرط ضروري وكافي. من ناحية أخرى، تم اقتراح إستراتيجية لتقدير الحالات المنفصلة والمستمرة لأنظمة التحويل. هذه المشكلة تم معالجتها من قبل اثنين من المراقبات الجبرية. الأول قدرالوضع النشط. والثاني، قدرالحالة المستمرة. تعتمد هذه الاستراتيجية على الأسلوب القوي لتقدير المشتقات الجبرية، مع مراعاة الملاحظة الجزئية للحالة المستمرة ، يقوم المراقب الجبري بإعادة بناء الحالات باستخدام قياس خروج الحالة وتقدير مشتقاتها. يوفر المراقب المختلط المقترح تقديراً أفضل وأسرع في الوقت الفعلي.

يكرس الجزء الثاني من هذه الأطروحة للتحقق من صحةالمنهجية النظرية المقترحة للمراقبة التوتر لمحول متعدد الخلايا. تؤكد نتائج محاكاة المحول المكون من خليتين وثلاث خلايا النتائج النظرية وتُظهر كفاءة وقوة المراقب المختلط.

الكلمـــات المفتاحيـــة:
الأنظمة التبديلة، التشغيل التلقائي الهجين ، إمكانية المراقبة ، المراقبة الهجينة ، الوضع النشط ، التمايز العددي ، النهج الجبري ، المحول متعدد الخلايا ، محول الطاقة.

Abstract

Switching systems are systems comprising a set of continuous dynamics orchestrated by a switching signal. This important class of hybrid dynamic systems have witnessed remarkable developments from theoretical point of view to practical insights with potential benefits to many real world applications such as: power electronics, multi-controller systems, robotics, electromechanical systems and systems with discontinuities. As a result, the development of tools allowing analyze the dynamic properties such as observability and observer design represents a challenging issue notably in the case where the subsystems(modes) are unobservable in classical sense.

The contribution of this thesis is divided into two parts. The first part provides, in one hand, the observability condition of the switched linear system .On the other hand, a strategy for the discrete and continuous states estimation for linear switched system including unobservable modes. This problem has been addressed using two algebraic observers. The first one estimated the actif mode. The second one, estimate the contiunous state . This strategy is based on the recent robust algebraic derivative estimation technique. Take into account the partial observability of the continuous state, the algebraic observer reconstructs the states using the output measurement and their derivatives estimation. The proposed observer provides better and fast estimation in real time.

The second part of this thesis, is dedicated to the validation of the theoretical approaches for the observation of floating voltages of the multicell converter. Simulation results of two and three-cells converter confirm the theoretical results and show the effectiveness and the robustness of the hybrid observer in spite of load and input variations.

Keywords:

Switched systems, Hybrid automaton, Observability, Hybrid observer, Active mode, Numerical differentiation, Algebraic approach, Multicellular converter, Power converter.

Résumé

Les systèmes à commutation sont des systèmes comprenant un ensemble de dynamiques continues orchestrées par un signal de commutation. Cette importante classe des systèmes dynamiques hybrides a été témoin des développements remarquables dans la théorie, aussi bien du point de vue théorique que les aspects pratiques, révelant ainsi des avantages potentiels pour de nombreuses applications du monde réel. Ce type de système peut représenter un très large éventail de systèmes tels que: l'électronique de puissance, les systèmes multi-contrôleurs, la robotique, les systèmes électromécaniques et les systèmes présentant des discontinuités. En conséquence, le développement d'outils permettant d'analyser les propriétés dynamiques telles que l'observabilité est essentielle avant toute étape de conception de la commande ou de la synthèse d'observateur .

La contribution de cette thèse est divisée en deux parties. La première partie propose, d'une part, l'analyse de l'observabilité des systèmes à commutations avec des modes non observable aux sens classique. Une conditions nécessaire et suffisant a été développé. d'autre part, une stratégies pour estimée les états discrètes et continus des systèmes à commutations a été proposée. Ce problème a été traité par deux observateurs algébriques. Le premier estimé le mode actif. Le second, estimer l'état continus. Cette stratégie est basée sur la technique robuste d'estimation de dérivés algébriques. Tenir compte de l'observabilité partielle de l'état continu, l'observateur algébrique reconstruit les états en utilisant la mesure de sortie et l'estimation de leurs dérivés. L'observateur hybride proposé fournit une estimation meilleure et rapide en temps réel.

La seconde partie de cette thèse est consacrée à la validation des approches théoriques pour l'observation des tensions flottantes du convertisseur multicellulaire. Les résultats de simulation du convertisseur deux et trois cellules confirment les résultats théoriques et montrent l'efficacité et la robustesse de l'observateur hybride malgré les variations de charge et d'entrée.

Mots-clés:

Systèmes à commutation, Automate hybride, Observabilité, Observateur algébrique, Mode active, Différenciation numérique, Approche algébrique, Convertisseur multicellulaire, Convertisseur de puissance.

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Notations

- $\mathbb{R}^n$: denotes the n-dimensional Euclidean space. We will drop the superscript for real numbers and write just $\mathbb{R}$.
- $\mathbb{R}^+$: is the set of real positive numbers.
- $\mathbb{N}$: denotes the set of natural numbers.
- $|x|$: represents the absolute value of the real number x or the cardinality of the set x.
- A : denotes a square matrix.
- T_N: denotes the hybrid time;
- x^T: denotes the transpose of the vector x.
- $ker(X)$: represents the nullspace of X.
- $Im(X)$: represents the image space of X.
- $X^\perp$: denotes the orthogonal complement of X.
- $\cup$: denotes the union operation.
- $\cap$: denotes the intersection operation denotes the Euclidean norm in $\mathbb{R}^n$.

General Introduction

In dynamical systems, most of the research work in systems and control has been concerned usually with dynamical systems that are described purely as either time-driven continuous variable dynamics or event-driven discrete logic dynamics. Yet, there are numerous dynamical systems that are of heterogeneous nature. By heterogeneity, we mean that both continuous variable and discrete-event dynamics are present and interacting with each other to generate complex dynamical behaviors. These are termed *hybrid systems* and have been covers a large class of systems, for example power electronics, multi-controller systems, robotics, electromechanical systems and systems with discontinuities and among many other fields. From the analysis viewpoint, detailed investigation of the discrete behavior has a lower priority for researchers in the control-theoretic area. For feasibility of analysis, we ignore the details of discrete dynamics and consider a more general class of systems, called *switched systems*.

Switched systems can be considered as a special class of *hybrid systems* that consists of collections of subsystems (or modes) with continuous dynamics, and a rule to regulate the switching behavior between them called *switching law* or *signal*. The history of switched system research can be treated back at least to the 1966s with the study of engineering systems that contain relays and/or hysteresis. However, switched systems began to attract people's attentions in the early 1990s, mainly because of the vast development and implementation of digital micro controllers and embedded devices. The last decade has seen considerable research activities in modeling, analysis and synthesis of switched systems involving researchers from a number of traditionally distinct fields, such as computer science and control system engineering.

To date, productive research has been conducted on the fundamental problems of *observability* and *observer design* of *switched linear systems*. The *observability* in switched systems has also been investigated by several researchers. In classical linear time-invariant systems, there is a single *uniform* notion of *observability* that deals with recovering the state from the knowledge of the output. However, in *switched systems* the *observability* problem can be approached in several ways. If we allow for the usage of the differential operator in the observer, then it may be desirable to determine the state of the system instantaneously from the measured output. This in turn requires each subsystem to be observable ; however, the problem becomes nontrivial when the *switching signal* is treated as a discrete state and simultaneous recovery of the discrete and continuous state is required for observability. In light of this, the first motivation of this thesis stems from the following question:

Question 1: How to detecte the active mode and estimated the switching signal in real time ?

On the other hand, with the knowledge of switching signal, even though the individual modes are not observable, it is possible to recover the initial state $x(t_0)$ when the output is observed over an interval that involves multiple switching instants. This phenomenon is inherent only to switched systems as the notion of *observability* over an interval coincide for linear time invariant systems. This variant of the observability in switched systems has been studied most notably by serval recherch. As well, the second motivation of this thesis comes from the following question:

Question 2: How the partial information about the state, that is available from each mode, can be combined to recover the complete information about the state even though the individual subsystems are unobservable ?

Therefore, we target the fundamental question as follows:

Question 3: if the system is observable, how can one go about designing the state estimators that are feasible for implementation on digital computers ? as an answer to previous questions, we go to the next subsection.

Research goals and developed results

The research goals of this thesis focus on developing a hybrid algebraic observer for estimated both the contiunous and discret sates of the switched linear systems. This observer takes into account the unobservable operating modes of the system. Proposed approach has been also validated on multicellular converter. We can summarized the developed results as follows :

- Development a necessary conditions for the *observability* of switched linear systems;

- Development a *hybrid observer* based on the *algebraic approach* to estimated the *hybrid state* ;
- Development of simple necessary and sufficient conditions for the observability analysis for the general case of *multicellular converter*, than applied the proposed observer.

Thesis organization

This thesis has two main parts organized as follows:

Part I. Focuses on the observer design of switched linear systems using algebraic approach. This part contains two chapters:

- **Chapter 1.**

  Introduces the switched linear systems class and gives some basics about this class e.g. the classification of switching signal. Also, it presents some unexpected results of stability of switched systems. In addition,we present a finding results on the observability of the switched linear system. Finally, this chapter provides different approches proposed for the estimation of both the discrete and continuous state of switched linear systems.

- **Chapter 2.**

  Includes the main results about the observer desgin of switched linear systems. For the identification of discrete state, we give a simple condition where the system matrices satisfy some algebraic relations. For the observability, we find that the observability is strongly depended to switching times. We focus our study on the switching sequences and we provide a simple necessary and sufficient condition that ensures the uniform observability respect to switching times. These latter based on the elimination of the switching times impact. The obtaining results are used in designing a new state observer where its idea is based on the algebraic derivative estimation technique. Taking into account the partial observability of the continuous state, the algebraic observer reconstructs the states using the output measurement and their derivatives estimation. Illustrative example are provided together with the developed results to highlight different contributions.

Part II: Algebraic observer of multicellular converters. like Part I, this part regroups the practical contributions of this thesis, mainly for the hybrid observer design for multicellular converters. we distinguish two chapters as below:

- **Chapter 3.**

  gives a thorough introduction to series multicellular converters. We presented the normal operation of the converter and different mathematical models proposed such as: average model, instantaneous model and hybrid model. Then, different obsevability condition presented in the literature and existing observer strategies for multicellular converter are discussed to comprehend and formulate the observation problem.

- **Chapter 4.**

  studies the observability of the converter. We give a necessary condition for the bservability analysis . The most important contribution in this chapter is to provide a real time estimation of the floating voltage using only the output measurement and their derivatives. The state observer where its idea came from the non-losing information property for the converter. Finally, we apply our results on the two and three-cells converter where the simulation results show the importance of our results whether for the analysis or for the design.

Scientific Contributions

Journal papers

- Kaouthar HADDADI, Noureddine GAZZAM ,Atallah BENALIA, "Algebraic observer desgin for switched linear systems applied to multicellular converters for estimating capacitor voltages",
  in Electrotechnica,Electronica,Automatica(EEA),2019.vol.67.no.1pp.28-34,ISSN 1582-5175.

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- Amina. HOCINE, Kaouthar. HADDADI ,Atallah BENALIA," Observability of multicellular converters series: an application to three- cells converter ", 2nd International Conference on Power Electronics and their Applications (ICPEA 2015),Djelfa 2015, Algeria.

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- Kaouthar. HADDADI, Noureddine GAZZAM, Atallah BENALIA,1st International Conference on Advanced Electrical Engineering 2019 (ICAEE 2019),19-21 November,Algeria, "Observability of Switched Linear Systems based Geometrical Conditions : An Application to DC/DC Multilevel converters".

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  (Submitted to Fourth International Conference on Electrical Engineering And Control Applications ICEECA'19. Constantine, Algeria).

- Noureddine GAZZAM, Kaouthar HADDADI, Atallah BENALIA, "Observability and Observer Design of a Class of Switched Systems".
  (Submitted to Fourth International Conference on Electrical Engineering And Control Applications ICEECA'19.Constantine, Algeria).

National Conference Papers

- Kaouthar. HADDADI, Atallah BENALIA, 1ère Journée D'Etude en Automatique et ses Applications, 1JEAA14 Laghouat, Algeria, 24 Avril 2014," observateur adaptative observer pour les convertisseurs multicellulaires".

- Kaouthar. HADDADI, Atallah BENALIA, 2ème Journée Doctorale de la Faculté de Technologie, 2JDF'15 Laghouat, Algeria," Observateur algébrique pour les systèmes à commutations".

- Noureddine GAZZAM, Kaouthar. HADDADI, Atallah BENALIA, 2ème Journée Doctorale de la Faculté de Technologie, 2JDF'15 Laghouat, Algeria," Analyse de l'observabilitè des systèmes linéaires à commutations".

Part I

Switched Dynamical Systems

Chapter 1

Switched Systems :An Overview

“ *In order to succed, we must first believe that we can* ”

N.Kazantzakis

The purpose of this chapter is to present an overview on the basic principles of hybrid dynamical systems, precisely switched systems, in order to clarify the structural notions presented in this work. First, it presents a concise history of the discovery and evolution of hybrid dynamical systems. A detailed of the hybrid modelling when, the main ingredients of hybrid models with special emphasize on hybrid automaton formalism is presented. Second, the presentation of the switched linear systems mainly contains the existing models, switching types and stabilizations problem. Finally, a literature review of the observer design challenges of switched systems with and without identification of discrete states, including the observability analysis is discussed to make the thesis contribution as clear as possible.

1.1 Hybrid Dynamical Systems History :1966 to Today

A Hybrid system arises when continuous and discrete dynamics interact. This is especially the case in many physical processes in which logic decision-making and embedded control actions are combined with continuous dynamics. The use of hybrid models is essential to adequately describe their behaviour. To capture the evolution of these systems, mathematical models are needed that combine the dynamics of the continuous parts of the system with the dynamics of the discrete events.

The actual term "hybrid systems" was used for the first time in 1966 by Hans Witsenhausen [Witsenhausen, 1966] to study engineering systems that contained relays and/or hysteresis. In 1987, Tavernini [Tavernini, 1987] used differential automata, he has given some basic defintions of hybrid systems such as: discrete modes, invariant and set of transitions, etc. However, the academic world started investigating the rich behaviour that is produced by these hybrid systems in the 1990s, mainly because of the widespread development and implementation of digital microcontrollers and embedded devices.

In 1993, Nerode and Kohn [Nerode and Kohn, 1992] presented an automata based theoretical approach to systems composed of interacting Ordinary Differential Equations (ODEs) and finite automata. Alur et al. [Alur and Dill, 1994] used hybrid automata, an extension of timed automata in 1994.

In 1995, [Guckenheimer, 1995] took a nonlinear control perspective, where switching is used to expand the domain of attraction of a nonlinear control system. The nonlinear control system admits a family of equilibria corresponding to constant control inputs. The idea is to switch at discrete time instants from a control input to another in a way that the system gradually progresses from one equilibrium to another towards the final equilibrium.

In 1996, stability and robustness issues of hybrid systems based on Lyapunov theory have been presented by [Pettersson and Lennartson, 1996]. The authors are concerned about the applicability of the results and they proposed strong conditions for stability in order to formulate the search for Lyapunov functions as an LMI problem. [Lygeros et al., 1996] presented a methodology for designing hybrid controllers for large scale, multiagent systems based on optimal control and game theory. The hybrid design is seen as a game between two players: the control, which is to be chosen by the designer, and the disturbances that encode the actions of other agents, that is the actions of high level controllers or unmodeled environmental disturbances.

In 1998, Benveniste [Benveniste, 1998] proposed a behavioral framework of hybrid systems modeling with emphasis on compositionality and the use of multiform time. [Dogruel, 1998] investigated the problem of partitioning the continuous state-space into specific regions in order to simplify the analysis of the hybrid system. In [Branicky et al., 1998] some analysis tools for switched and hybrid systems are presented. In particular, multiple Lyapunov functions are used for stability analysis of switched systems and iteracted function systems are used for Lagrange stability.

The last two decades have seen considerable research about hybrid systems, and these may be divided into four broad categories: modeling,analysis,control and design [Liberzon, 2003], [Lunze and Lamnabhi-Lagarrigue, 2009].

1.2 Hybrid Modelling (Hybrid Automaton)

A widely used model for hybrid systems is the hybrid automaton model that combines finite-state automata, as adopted in computer science, and differential equations, as used in mathematical control theory. This model was formalized for the first time by Alur et al. In 1995 and, since then, many variations and ramifications have been proposed in the literature (e.g. Lynch et al. 1996; Alur and Henzinger 1997; Branicky et al. 1998; Lygeros et al. 1998; Cassandras and Lafortune 1999) where the basic elements are the same. The following definition gives the main ingredients of a hybrid automaton [Birouche, 2006].

1.2.1 Analytical Presentation

Definition 1.1 (Hybrid automaton). A general hybrid automaton is a collection $\mathcal{H}$ given by:

$$\mathcal{H} = \{Q, \Sigma, \Psi, \Upsilon, X, U, Y, S_c, S, T, Inv, G, R, Int\} \tag{1.1}$$

Where

- $Q = \{q_1, \ldots, q_N\}$ is a finite set of discrete states.
- Σ, Ψ represent a finite set of discrete inputs and a finite set of discrete outputs respectively.
- X, U, Y are spaces with appropriate dimensions, which are associated to the continuous state $x(t)$, the continuous input $u(t)$ and the continuous output $y(t)$ respectively.
- S_c is a subclass of dynamic systems where $S_i \in S_c$ is defined by the following differential equation:

$$\dot{x}(t) = f_{q_i}(x(t), u(t)), q_i \in Q, t \in \mathbb{R} \tag{1.2}$$

- S is a map, which associates to each discrete mode $q_i \in Q$ a continuous dynamic $S_i \in S_c$;

- $T \in Q \times \Sigma \times Q$ represents a set of all possible transitions between discrete modes. We denote (q_i, ϱ, q_j), or simply $T_{i,j}$, the state transition from mode q_i to mode q_j activated by the discrete input ϱ.

- $\Upsilon : T \to \Psi$ is a function that assigns to each transition $T_{i,j} \in T$ a discrete output $\psi \in \Psi$.

- $Inv : Q \to 2^{Q \times U}$ denotes the invariants where each mode has an invariant region associated to it, which describes the conditions that the continuous state has to satisfy at this mode.

- $G : T \to 2^{X \times U}$ associates to each transition a continuous set where the transition is valid (called a guard condition).

- $R : \tau \times X \times U \mapsto 2^X$ represents a reset map, which specifies how new continuous states are related to previous continuous states for a particular transition.

- $Init \subseteq X \in Q$ gives the initial hybrid states.

The hybrid state variable of the system $\mathcal{H}$ is given by $(q, x) \in Q \times X$. The initial hybrid state (q_0, x_0) of 'trajectories' of a hybrid automaton must be contained in the initial set $Init$. Some elements of the hybrid automaton are usually omitted, which depends on the considered application. For example, the continuous inputs do not exist in power converters.

1.2.2 Graphical Presentation

The hybrid automaton ingredients can be represented graphically [Lunze and Lamnahbi-Lagarrigue, 2009] using the following defintion.

Definition 1.2 (Directed graph of $\mathcal{H}$). An oriented graph of a hybrid automaton defined by $\mathcal{H}$ is obtained by associating to each mode $q_i \in Q$ a node and to each transition an oriented arc from the node of mode q_i to the node of mode q_j. (Fig.1.1) illustrates the schematic representation of a hybrid automaton with three discrete modes represented by nodes and five transitions represented by arrows. Each node of the oriented graph includes a continuous differential equation and each oriented arc has a guard condition and a reset map. In this case, the initial discrete mode q_0 corresponds to mode q_1. Thus, for a given initial continuous state x_0, the continuous state of the hybrid system evolves according to the continuous dynamic of mode q_1. The trajectory of the system evolves as

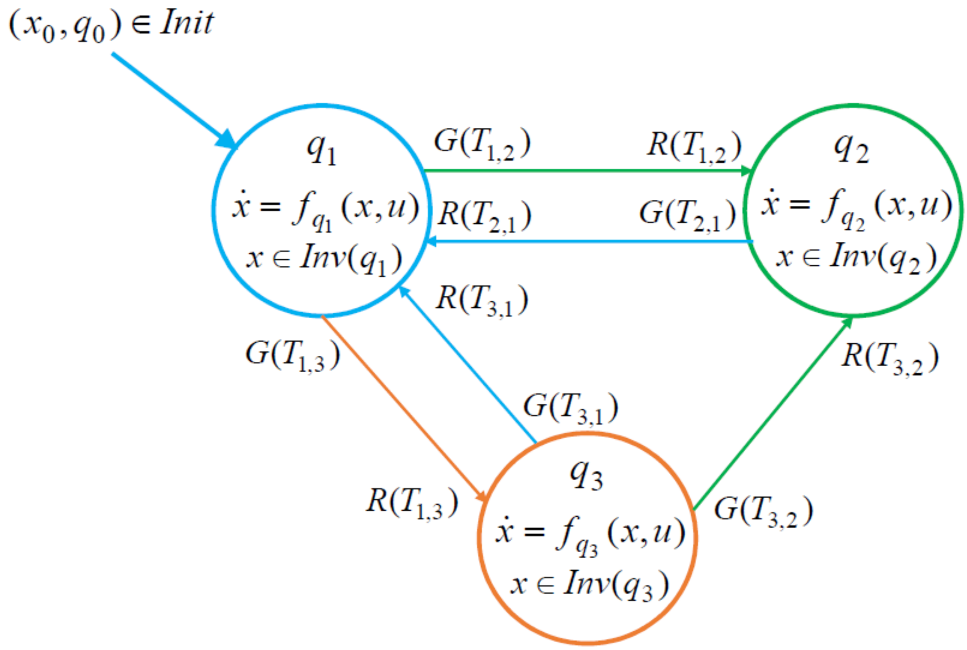

Figure 1.1: Graphical representation of a hybrid automaton with three discrete modes.

long as the invariant condition is verified ($x(t) \in Inv(q_1)$). If at some states this condition is no more verified, a transition to another mode has to be done. However, if at time t the continuous state reaches a guard condition, for example $x(t) \in G$, we say that the transition is enabled. In this case, the discrete mode may switch from mode q_1 to its successor mode q_3 and the initial continuous state in mode q_3 is given by the reset map. After this transition, the trajectory evolves according to the dynamics of mode q_3 and the whole process is repeated.

1.3 Switched Systems

As presented in previous section, switched systems are a class of hybrid systems. Switched systems are dynamical systems that have both continuous and discrete behaviors. Continuous behavior is represented by classical models named subsystem or mode, while the discrete behavior is simply a discrete event. The switching between modes is governed by logical rules that can be determined by the system's parameters. Switched systems are described by the following general form :

$$\begin{cases} \dot{x}(t) = f_{\sigma(t)}(x(t), u(t)) \\ y = h_{\sigma(t)}(x(t), u(t)) \\ x_0 \in \mathbb{R}^n, x(t^+) = J_{\sigma(t)}(x(t^-) \end{cases} \tag{1.3}$$

where $x(t)$ is the state vector, $u(t)$ is the control input, $y(t)$ is the measured output, $\sigma(t)$ is the switching signal which is a piecewise constant signal. $f_{\sigma(t)}$ and $h_{\sigma(t)}$ are vector fields, and $g_{\sigma(t)}$ are vector functions. $J_{\sigma(t)}(x(t^-)$ is a function that determines the jump at switching times. Throughout the thesis, we restrict our study for switched linear systems (SLS).

1.4 Switched Linear Systems

As a class of switched system, SLSs consist of several subsystems (modes) and a rule that orchestrates the switching among them. A SLS can be described by

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}(x(t)) + B_{\sigma(t)}(u(t)) \\ y = C_{\sigma(t)}(x(t)), x_0 \in \mathbb{R}^n, x(t^+) = J_{\sigma(t)}(x(t^-) \end{cases} \tag{1.4}$$

where A_σ , B_σ and C_σ are linear matrices in appropriate spaces. $J_\sigma \in \mathbb{R}^{n\times n}$, in this case is a matrix. For a SLS without jump the matrix J_σ becomes the identity matrix.

The *switching signal* $\sigma(t) : \mathbb{R}^+ \mapsto I_N = \{1, 2, \ldots, N\}$ might depend on time t, state $x(t)$, or both of them, or may be generated by a more sophisticated strategy involving memory, which is the aim of the next subsection.

1.4.1 Classification of Switching Signals

In switched systems, the switching mechanism is significant because of many reasons; one of these reasons is the stability, and also the observability criteria depends on the switching law $\sigma(t)$. In [Liberzon, 2003], the authors discussed three switching types which we will present here.

A- Time Dependent Switching

The switching signal in this case is only defined by the time, i.e. the continuous state and the other parameters in (1.4) do not affect the switching behavior. For a given initial time t_0, on the time interval $[t_0; t_3)$ with $t_0 < t_3 < \infty$, it can be given by

$$\sigma : [t_0; t_3) \mapsto I_N \tag{1.5}$$

This type of switching exists for the case of open-loop control. Figure(Fig.1.2) depicts this type of switching.

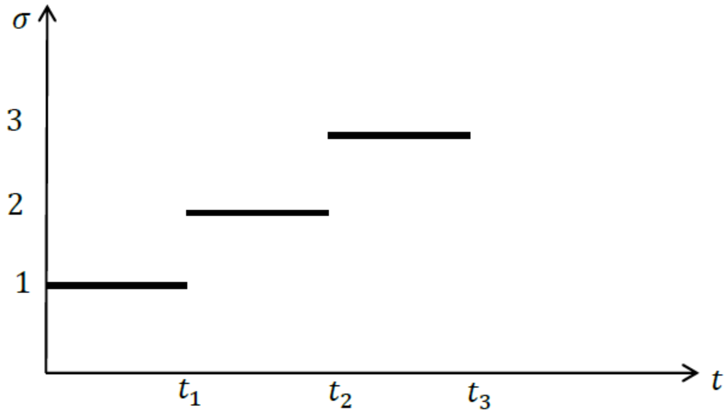

Figure 1.2: Time dependent switching signal.

B- State Dependent Switching

In this type of switching, the system (1.4)switches from one mode to another if the continuous state of the system satisfies the condition of transition. The idea of state dependent switching can be understood by considering a continuous state space which is partitioned into many operating regions by the switching surfaces. Each operating region defines its corresponding subsystem. When system trajectory crosses a switching surface it jumps to a new subsystem, as shows in figure(Fig.1.3). In this figure, the lines with arrow heads points out the continuous part of the trajectory and the dotted lines points out the switching surfaces. The switching signal can be presented as follows

$$\sigma(t) = \psi(x(t)) \quad t \geqslant t_0 \tag{1.6}$$

C- Controlled Switching

In this case, the switching can be controlled by the designer to get the desired behavior for the system. This kind of switching can be: time dependent switching and/or state

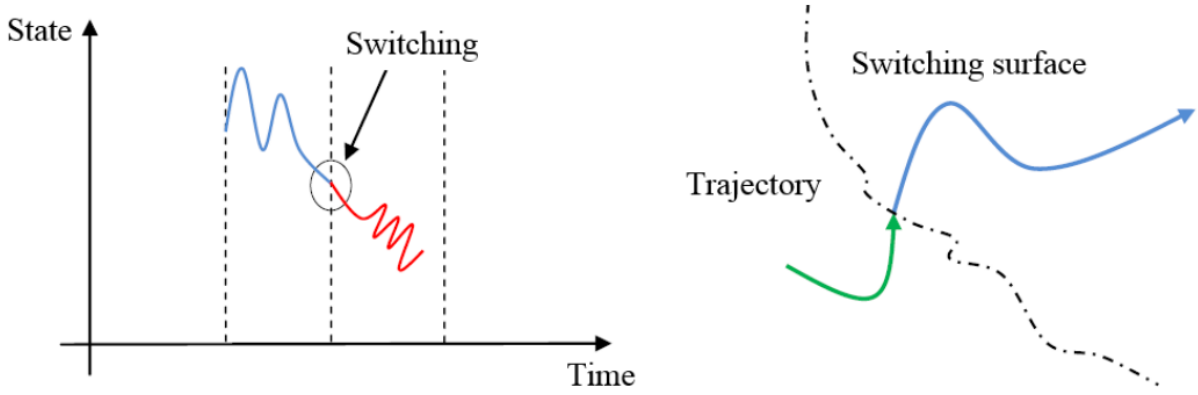

Figure 1.3: State dependent switching signal.

dependent switching [Benmiloud, 2011].

$$\sigma(t) = \psi(\sigma(t), x(t)) \quad t \geqslant t_0 \tag{1.7}$$

These switching signals can be completely characterized by the sequences defined in the next subsection.

1.4.2 Switching Sequences

We distinguish three main sequences to describe any given switching signal:

1- Switching Time Sequence(STS)

Represents the set of switching instants. Let $t_1, t_2, \ldots, t_N$ be the ordered switching times in $[t_0, t_f)$ with

$$ST = \{x_0, t_0, t_1, ..., t_N\} \tag{1.8}$$

It should be noted that N is strictly positive and is possibly infinite. The ordered sequence $\{t_0, t_1,, \ldots, t_N\} = \{t_i\}_{i=0}^{N}$ is named the *switching time sequence* of σ on $[t_0; t_f)$, denoted STS.

2- Switching Mode Sequence (SMS)

Represents the set of discrete modes corresponding to the switching instants in the set STS. We denote SMS, for the switching mode sequence of σ, which is given by

$$SMS = \{\sigma(t_0)), \sigma(t_1), ..., \sigma(t_N)\} \tag{1.9}$$

3- Switching Sequence (SS)

Switching sequence (SS): represents the sequence of ordered pairs $(t_i, \sigma(t_i))$ together with the initial condition x_0, denoted SS, which is given by

$$SS = \{x_0, (t_0, \sigma(t_0)), (t_1, \sigma(t_1)), ..., (t_N, \sigma(t_N))\} \tag{1.10}$$

At this level, the trajectory of the switched affine system can be obtained for a given switching sequence, which is the aim of the next subsection.

1.5 Solutions of Switched Linear System

Let us denote $\phi(t, t_0, x_0, \sigma)$ the state trajectory at time t of the continuous switched linear system given by (1.4) and initialized at $x(t_0) = x_0$ with a given switching signal .

Suppose that the switching signal is characterized by the switching sequence (1.9). For clarity, we suppose that the switching sequence is ordered as follows

$$SS(x_0) = \{x_0, (t_1, \sigma_1), (t_2, \sigma_2), ..., (t_{N-1}, \sigma_N)\} \tag{1.11}$$

As σ_1 is the initial activated mode during the time interval $[t_0, t_1)$, we have:

$$\dot{x}(t) = A_{\sigma_1}x(t) + B_{\sigma_1}u, \quad x(t_0) = x_0, \quad t \in [t_0, t_1) \tag{1.12}$$

The explicit solution of this diffrential equation is given by:

$$\phi(t, t_0, x_0, \sigma, u) = e^{A_{\sigma_1}(t-t_0)}x_0 + \int_{t_0}^{t_1} e^{A_{\sigma_1}(t_1-\tau)}B_{\sigma_1}u(\tau)d\tau \quad t \in [t_0, t_1)$$

We denote x_1 the state trajectory at the switching instant $x(t_1)$ hence:

$$x_1 = x(t_1) = \phi(t_1, t_0, x_0, \sigma) = e^{A_{\sigma_1}(t_1-t_0)} + \int_{t_0}^{t_1} e^{A_{\sigma_1}(t-\tau)}B_{\sigma_1}u(\tau)d\tau$$

Similarly, during the time interval $[t_1, t_2)$,mode σ_2 is activated and the explicit solution is given by:

$$\phi(t, t_0, x_0, \sigma, u) = e^{A_{\sigma_2}(t-t_1)}x_1 + \int_{t_1}^{t} e^{A_{\sigma_2}(t-\tau)}B_{\sigma_2}u(\tau)d\tau \quad t \in [t_1, t_2)$$

we can represente the solution of mode σ_1 by using the solution of mode σ_2 , and the explicit solution for $t \in [t_1, t_2)$ is given by:

$$\phi(t, t_0, x_0, \sigma) =$$
$$e^{A_{\sigma_2}(t-t_1)} e^{A_{\sigma_1}(t_1-t_0)} x_0 + e^{A_{\sigma_2}(t-t_1)} \int_{t_0}^{t_1} e^{A_{\sigma_1}(t-\tau)} B_{\sigma_1} u(\tau) d\tau + \int_{t_1}^{t} e^{A_{\sigma_2}(t-\tau)} B_{\sigma_2} u(\tau) d\tau$$

Following the same procedure as above, the solution of the switched linear system in the interval $t \in [t_0, t_N)$ can be computed as below:

$$\begin{aligned} \phi(t, t_0, x_0, \sigma, u) = e^{A_{\sigma_N}(t-t_{N-1})} e^{A_{\sigma_{N-1}}(t_{N-1}-t_{N-2})} ... e^{A_{\sigma_1} t_1} x_0 + \ldots + \\ e^{A_{\sigma_N}(t-t_{N-1})} \ldots e^{A_{\sigma_2}(t_2-t_1)} \int_{t_0}^{t_1} e^{A_{\sigma_1} t_1} B_{\sigma_1} u(\tau) d\tau + \ldots + \\ e^{A_{\sigma_N}(t-t_{N-1})} \int_{t_{N-1}}^{t_{N-2}} e^{A_{\sigma_{N-1}}(t-\tau)} B_{\sigma_{N-1}} u(\tau) d\tau + \int_{t_{N-1}}^{t} e^{A_{\sigma_N}(t-\tau)} B_{\sigma_{N-1}} u(\tau) d\tau \quad t \in [t_{N-1}, t_N) \end{aligned} \tag{1.13}$$

1.6 Stability and Stabilazation of Switched System

Stability of switched systems is critical for any application. For this reason, it has received the most attention as one of the important structural properties in the last decades. There have been several excellent survey papers addressing the basic problems involved in stability and desgin of switched systems appears in [Liberzon and Morse, 1999], [Liberzon and Morse, 1999]. Unlike traditional condition of stability, in switched systems the stability condition is totally depends on the *switching law*$\sigma(t)$. In fact, the stability of all subsystems does not imply the stability of the overall system. Furthermore, the instability of one or more subsystems does not imply the instability of the switched system [Liberzon, 2003] [Branicky, 1994]. In the case of switched systems, the stability analysis is carried out under one or the other hypothesis concerning the switching signal $\sigma(t)$, where the reader may refer to [Pettersson and Lennartson, 1996]for more details:

- Switched system is stable under arbitrary switching, when there is no restriction on the *switching signals*. This problem is usually called stability analysis under arbitrary switching,

- Switched system is stable under restricted switching and the stability condition is achieved for a limited class of *switching signals*.

Answering the first challenge, In order to obtain stability conditions for switching systems, various approaches existing in the literature. One of the first approaches has been the use of results on differential inclusions. These results show that the existence of a Quasi-Quadratic Lyapunov Function $V(x) = x^T R(x) x$ implies the stability of the differential inclusion, or of an equivalence its convexity. However, these results can not be applied directly because of the numerical complexity of the search for this function $R(x)$. As a result, numerous studies devoted to the search of a Common Lyapunov Function(CLF). Moreover, the converse Lyapunov theorem can also be derived in different settings Note that the uniform stability with respect to the switching signals requires the individual subsystems to be stable, under an additional condition that requires the vector fields of the subsystems to commute, which in turn leads to commutativity of the flows of corresponding subsystems, the overall system becomes stable. The results based on commutator operation or Lie-algebraic criteria appear in [Agrachev and Liberzon, 2001],

On the other hand, for stability under constrained switching, one uses the multiple Lyapunov function approach [Branicky, 1998]. The basic idea that follows in this approach is that if we look at the values of the Lyapunov Function associated to each subsystem at every time that subsystem is activated, then those values must form a non-decreasing sequence. One can extend this idea to develop the notion of dwell-time and average dwell-time stability in switched systems under which the stability among asymptotically stable systems is preserved when the switching is slow enough.

1.7 Illustrative Example: DC-DC Boost Converter

1.7.1 DC-DC Boost Converter Modeling

Power converters are good examples of hybrid systems where the continuous dynamics correspond to the inductor currents and capacitor voltages, and the discrete variables correspond to the ON/OFF power devices. Figure.1.4 depicts the topology of a DC-DC boost converter. It consists of four components: a controlled power device SW, an uncontrolled device (diode D)," a low-pass filter $L - C$", and a resistive load R. The output voltage v_c of the boost converter is always greater than the input voltage v_{in}. The boost converter can operate in a continuous conduction mode (CCM) or in a discontinuous conduction mode (DCM), depending on the waveform of the inductor current.

Let us consider $x = [i_L \quad v_c]^T$ as the state vector, which corresponds to the inductor current and the output voltage. Hybrid model captures the exact behavior of the circuit, without any approximation. The hybrid automaton of DC-DC boost converter is given by

$$\mathcal{H}_{Boost} = \{Q, \Sigma, \Psi, \Upsilon, X, U, Y, S_c, S, T, Inv, G, R, Int\} \tag{1.14}$$

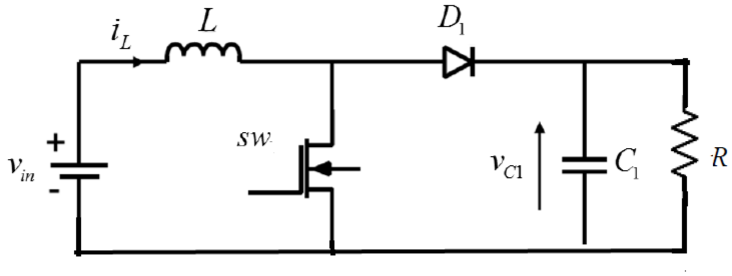

Figure 1.4: Topology of DC-DC boost converter supplying a resistive load.

Where

- $Q = \{q_1, q_2, q_3\}$ represents a set of two discrete modes corresponding to the states of the controlled switch (SW) and the uncontrolled device(D).
- $\Sigma = (\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5, \varrho_6)$,
- $X \subseteq R^{2+}$ is the continuous state space, $U = \emptyset$.
- $S : (Q \times X) \to \mathbb{R}^2$: denotes the application that assigns to every discrete mode,belongs to the set of discrete modes $Q = \{q_1, q_2, q_3\}$, a continuous dynamics belongs to $S_c = S_1, S_2, S_3$ and given by

$$\dot{x}(t) = f_{q_i}(x(t)) = A_{q_i}x(t) + B_{q_i} \quad q_i \in Q \tag{1.15}$$

With

$$A_{q1} = \begin{pmatrix} 0 & -\frac{1}{L} \\ \frac{1}{L} & -\frac{1}{RC} \end{pmatrix}, A_{q2} = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{pmatrix}, A_{q3} = \begin{pmatrix} 0 & 0 \\ 0 & \frac{-1}{RC} \end{pmatrix}$$
$$B_{q1} = \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix}, B_{q2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, B_{q3} = \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix}$$

- $T = \{T_{1,2}, T_{2,1}, T_{1,3}, T_{3,1}, T_{2,3}, T_{3,2}\}$: represents the transitions between three-discrete modes.
- $Inv(Q) = \mathbb{R}^2, \Upsilon : T \to \Psi$.
- $G : T \to 2^X$: associates to each transition a continuous set where the transition is validated.
- $Init \subseteq X \times Q$: gives the initial states.

1.7.2 Hybrid Control of DC-DC Boost Converter

The control scheme proposed takes into account the desired capacitor voltage v_d and the peak inductor current. The figure.1.5 show exactly the variations of v_c, i_L depending on time in the case of CCM or DCM. From figure.1.5, the change in inductor current during

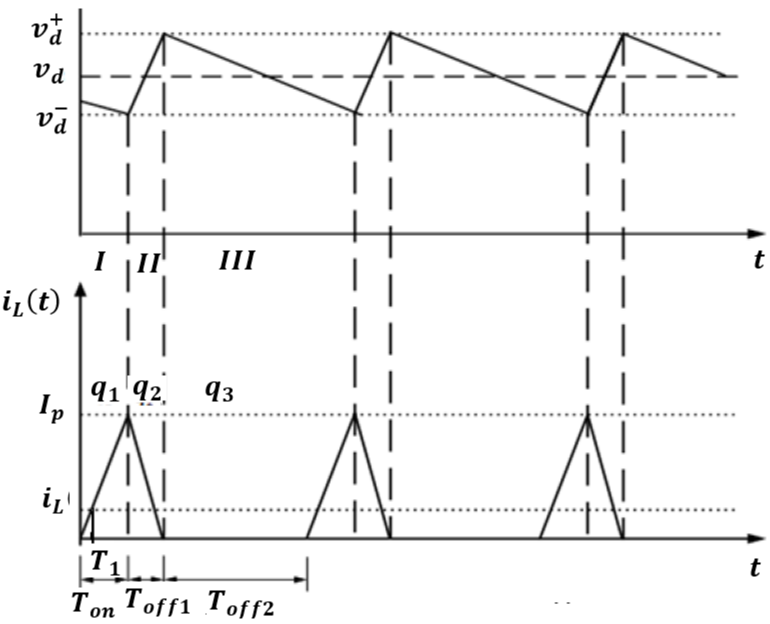

Figure 1.5: Typical capacitor voltage and inductor current waveforms in CCM and DCM operation of a DC-DC boost converter.

the period $(T_1 - T_{on})$, is

$$\Delta = \frac{1}{2}(i_p - i_L)(T_1 - T_{on}) \tag{1.16}$$

Where

T_1: Time required to wait in the load current; T_{on}: The ON time period, if Sw closes;

$$T_1 = \frac{Li_L}{v_{in}}, \quad T_{on} = \frac{Li_p}{v_{in}} \tag{1.17}$$

The change of the energy in inductor during the change of inductor current from i_L to i_p and the total energy lost in the capacitor, when the capacitor voltage falls from v_d^- to v_d^+ are given respectively as

$$\begin{cases} E_L = \Delta E = \dfrac{1}{2}L(i_p - i_L)^2 \\ E_C = \Delta E = \dfrac{1}{2}C(v_d^{+2} - v_d^{-2}) \end{cases} \tag{1.18}$$

Take in to account the energy balance principle, this principle is determined by equating the sum of energy gained by the inductor to the energy lost by the capacitor in the respective modes of operation. By equating both these energy terms, the peak current is given as

$$I_P = \sqrt{\frac{C}{L}(v_d^{+2} - v_d^{-2})} + i_L \tag{1.19}$$

This value can be calculated based on the maximum energy that can be supplied to the load this will happen marginally discontinuous mode of operation when there will be no mode q_3, thus $T_{off2} = 0$.

$$R_{min} = \frac{2v_d^2}{i_{p_{max}} v_{in}} \tag{1.20}$$

Where

$$i_{pmax} = \sqrt{\frac{C}{L}(v_d^{+2} - v_d^{-2})} + \frac{v_d}{R_{min}} \tag{1.21}$$

Substituting equation (1.21) in equation (1.20), the expression of becomes:

$$R_{min} = \frac{1}{\sqrt{\frac{C}{L}(v_d^{+2} - v_d^{-2})}} \frac{(2v_d - v_{in})v_d}{v_{in}} \tag{1.22}$$

For a DC-DC boost converter, the control problem is to regulate the output voltage. Furthermore, it is required to ensure that the switching transitions are stable. The condition for maintaining the circuit in each of the three modes is given as follows:

- **Case I**: The system is in mode q_1 if the inductor current is lower than inductor peak current $i_L \leq I_p$. If $i_L > I_p$ the system switch from mode q_1 to mode q_2.
- **Case II**: The system is always in the mode q_2, if $i_l > 0$. If $i_L = 0$ and $v_c < v_d$, the system switch from mode q_2 to mode q_1. If $i_L = 0$ and $v_c \geq v_d$, the system switch from mode q_2 to mode q_3.
- **Case III**: The system is always in the mode q_3 if $i_L = 0$ and $v_c \geq v_d$. If $v_c < v_d$, the system switch from mode q_3 to mode q_1.

Many hybrid systems exhibit periodic behavior. This limit cycle includes all operation modes(see [Haddadi and Atallah, 2012]). When the load current $i_L = I_{L_{max}} = \frac{v_d}{R_{min}}$, the third mode will not be present. When $i_L = I_{L_min} = \frac{v_d}{R_{max}}$, the trajectory of phase portrait

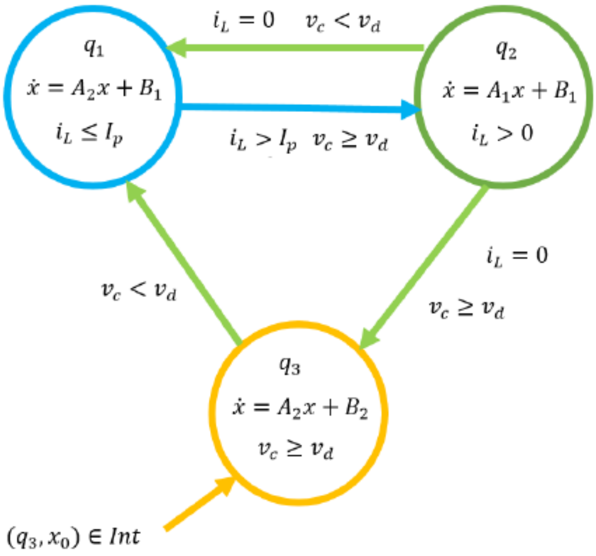

Figure 1.6: Switching scheme of DC-DC boost converter.

will go to mode q_3, the voltage fall slowly and the limit cycle will remain stable as show in figure.1.5.

For the stability condition, the idea here is that even if we have Lyapunov functions for each subsystem, we need to impose restrictions on switching to guarantee stability. As a particular class of MLF, multiple Lyapunov like function. The stability results developed in this context are based on the decay of the Lyapunov function at any successive instants for which a subsystem is switched to another mode (the proof of stability condition given in [Haddadi and Atallah, 2012]). The values of the parameters of DC-DC boost

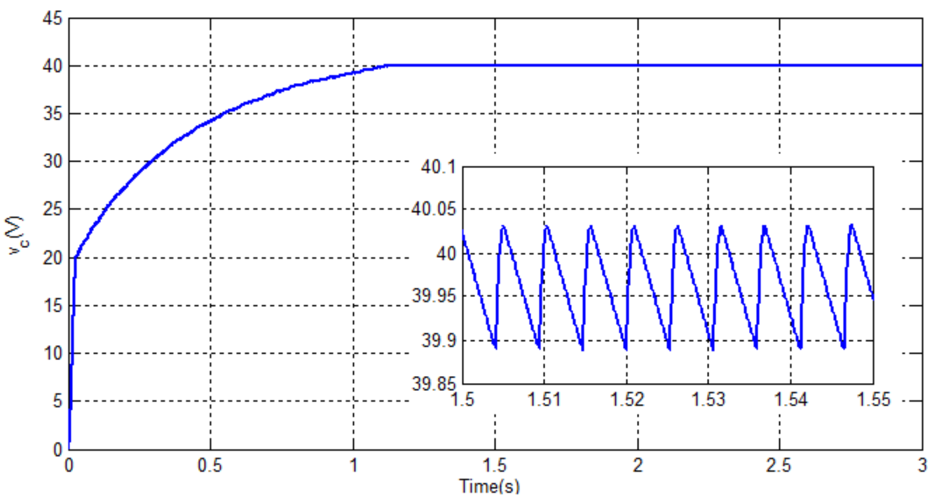

Figure 1.7: Output voltage evolution of DC-DC boost converter.

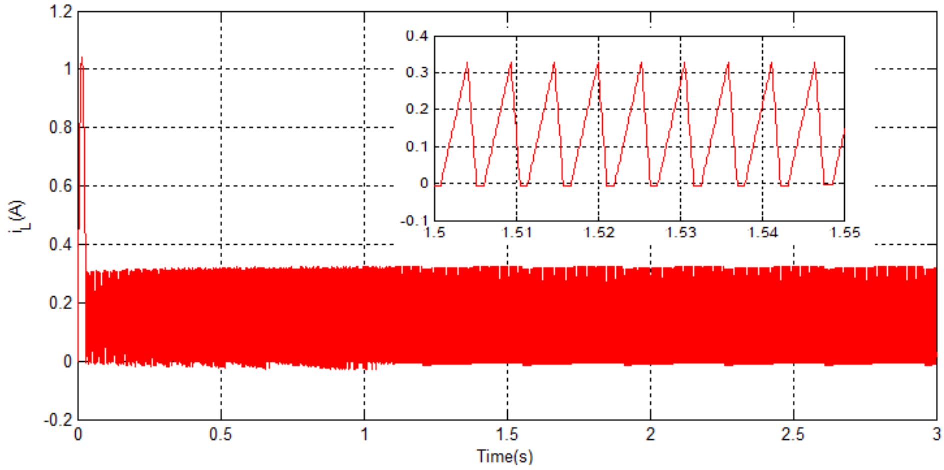

Figure 1.8: Current evolution of DC-DC boost converter.

converter are given in the following table. It is observed that the output voltage remains

Table 1.1: A typical DC-DC Boost converter parameters.

Parameter	L	C	R	v_{ref}
Value	$350mH$	$10\ \mu F$	$20\ \Omega$	40V

constant and stable without any overshoot (Fig.1.7). In the other side the waveforms of inductor current (Fig.1.8) present in the beginning the overshoot. In (Fig.1.9) the limit cycle presents only the first two modes. As we already mentioned in the previous section, the limite cycle found by simulation demonstrates the effectiveness of the proposed scheme(Fig.1.6).

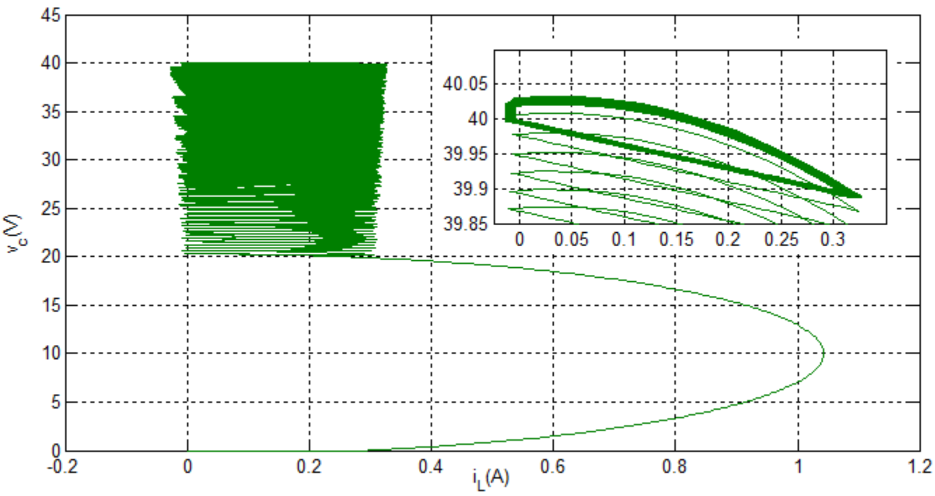

Figure 1.9: Limit cycle evolution of DC-DC boost converter.

1.8 Observation Problem of Switched Linear System

The observation problem consists of reconstructing the image of unknown variables involved in a system. To answer to this objective, a sensor can help to get in a direct way (physically) and in real time, measurement the variables. Nevertheless, for many systems, the installation of sensors, to measure $x(t)$ for example, is either too expensive or impossible. Therefore, the theory of observation interested in the synthesis of observers to estimate these variables.

An observer(Fig.1.10) is a mathematical tool based on the modeling of the system. It uses a known variables as the contol (input) $u(t)$ and the output $y(t)$ (measured by a sensor), to estimate the state (continuous and /or discrete)of the system in real time. Control and diagnosis of a system are the main issues in the theory of observation. Indeed, in the literature, control of a system is based on the assumption that continuous and discrete states can be measured or estimated. We can cite, for example, the works of [Zakaria et al., 2018], [Zhang et al., 2014]for feedback control, and observation theory can be applied to prgnostic of system (see [Pisano et al., 2014], [Belkhiat et al., 2011]).

The application of this theory assumes that the system studied verified the observability

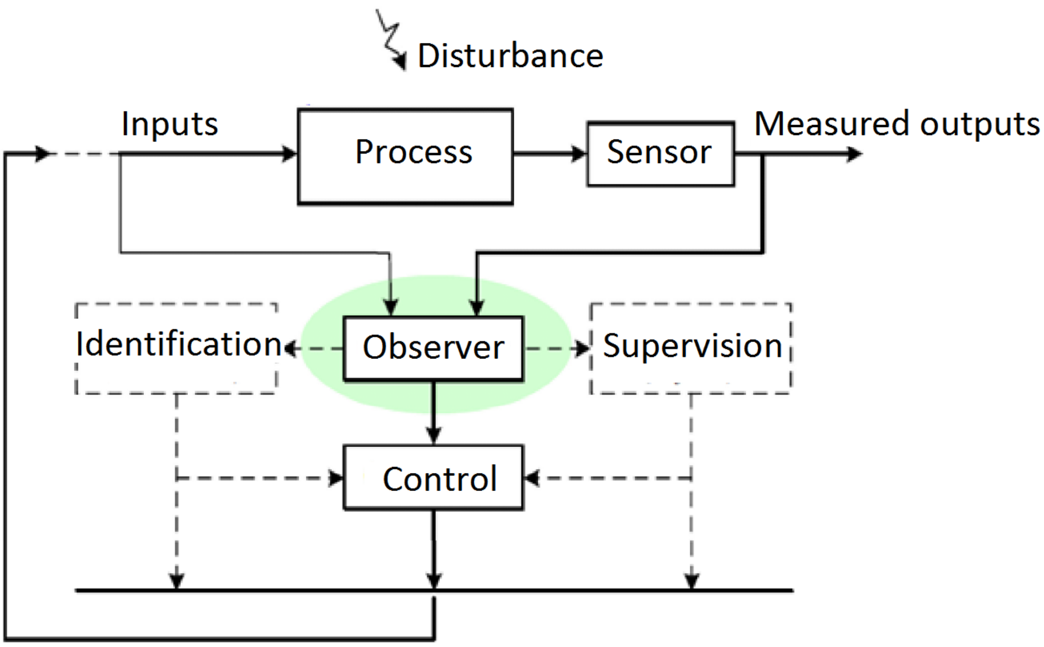

Figure 1.10: Observer: the essential part in the control scheme.

conditions. This last notion defines the capacity to estimate the variables of a system, from its modeling and different known variables. Observability issues for switched systems have been developed in the literature according to two approaches presentd in the next subsection.

1.8.1 Observability Analysis of Switched Linear System

The observability in switched systems has also been investigated by several researchers. In classical linear time-invariant systems [Basile and Marro, 1969], there is a single uniform notion of observability that deals with recovering the state from the knowledge of the output (see **Appendix A**). However, in switched systems the observability problem can be approached in several ways, we can cited two well know approches.

$A - Z(T_N)$**Observability**

This approach is based on the notion of $Z(T_N)$ -observability developed in [Kang et al., 2009]. First , let's remember this definition:

Definition 1.2. The function $z = Z(t, x, \sigma)$ is $Z(T_N)$ observable along the switching times sequence of T_N if for any pair of paths $(t, x^i(t), \sigma^i(t)), i = 1, 2$ defined in the time interval $[t_0, t_N)$, the equality

$$h^1(t, x^1(t), \sigma^1(t)) = h^2(t, x^2(t), \sigma^2(t)) \quad t \in [t_0, t_N) \tag{1.23}$$

This definition makes it possible to obtain the following result:

Theorem 1.1 . Let the system (1.2) and a time sequence T_N. Suppose only for any ,$z = Z(t, x, u)$ is always continuous. Suppose there is a series of linear projections $P_{i\{i=0,\ldots,N\}}$ such as :

- For all $i \in \{0, \ldots, N-1\}, P_i Z(t, x, \sigma)$ Z-observable $t \in I_i = [t_i, t_{i+1})$,
- Rank $([P_0^T, \ldots, P_{N-1}^T]) = dim(z)$,
- $\frac{d\bar{P}Z(t,x,\sigma)}{dt} = 0$, for $t \in I_i$ where$\bar{P}$ is the complement of P.
  Than z is Z-observable along the sequence of switching times T_N.

The first condition of this theorem implies that each of the projections$P_i Z(t, x, u)$ is Z-observable over a nonzero time interval. The second condition guarantees that we can project each of the components of Z, that is to say that all the components of Z is observable along the time sequence T_N. The last condition requires that when a component of Z is not observable at a given moment, it remains constant [Benmansour, 2009].

B- Observability by the Determination of the Unobservable Subspace

This approach is similar to that of Z-observability. The difference is that the concept of $[t_0, T^+)$ - observability allows to consider systems whose state presents jumps([Tanwani, 2012]), this approach represents a generalization of Z-observability. Should be defining formally the notion of $[t_0, T^+]$ - observability, specify the notation $[t_0, T^+)$. This is used to represent the interval $[t_0, T + \epsilon)$ where $\epsilon > 0$ is arbitrarily small. Indeed, the switching signal σ is continuous on the right, so the output $y(T)$ belongs to the following mode when T is the switching instant.The measurement of the output only at time T is insufficient to obtain information on the new mode, it is then necessary to consider an output signal on an interval $[t_0, T + \epsilon)$ with $\epsilon > 0$.

1.8.2 Observer Design

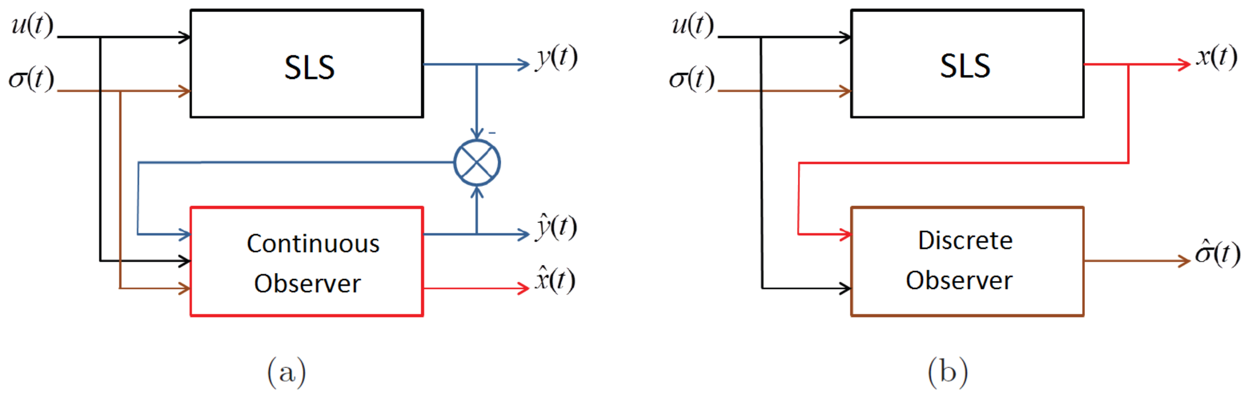

Figure 1.11: Principles of observations of continuous or discrete state. (a) State observer continuous when the active mode $\sigma(t)$ is known. (b) Discrete state observer when the continuous state x (t) is measured.

In the literature, several observer design have been proposed. The main difference between these approaches is related to knowledge of the discrete or continuous state. Indeed, several works considering only the estimation of continuous state respectively discrete state while the discrete state (Fig.1.11(a)) (respectively the continuous state (Fig.1.11(b)) is known and others are interested in the reconstruction of continuous and discrete states simultaneously(Fig.1.12). This latter led to propose three challenges problems of the observation of the switched systems. These problems are given as follows :

1.8.3 Problem A. Continuous state estimation

The active mode is assumed to be known following the structure of (Figure.1.11.(a)), is used in several studies. A Luenberger observer is designed on the basis of a Common Lyapunov Function for the dynamics of error in [Alessandri and Coletta, 2003]. A condition on Linear Matrix Inequality (LMI) is proposed in order to ensure an asymptotic convergence, from the estimation error, to zero. Where the authors have assumed that each mode in the system is in fact observable admitting a state observer.

1.8.4 Problem B. Discrete state identification

On the contrary of previous problem, consider that the continuous state can be measured in this case. A robust discrete state estimator, using sliding mode techniques, is defined in [Orani et al., 2011], for a class of uncertain switching systems. Others, ensure the detection of active mode by an observer based on the formalism of hybrid petri nets [Hamdi et al., 2009].

1.8.5 Problem C. Discrete and Continuous state identification

[Floquet et al., 2012] addresses the problem of simultaneous discrete and continuous state reconstruction in the framework of linear autonomous switched systems. A sliding mode observer is developed in [Bejarano and Fridman, 2011], [Djemai et al., 2005], [Saadaoui et al., 2006].

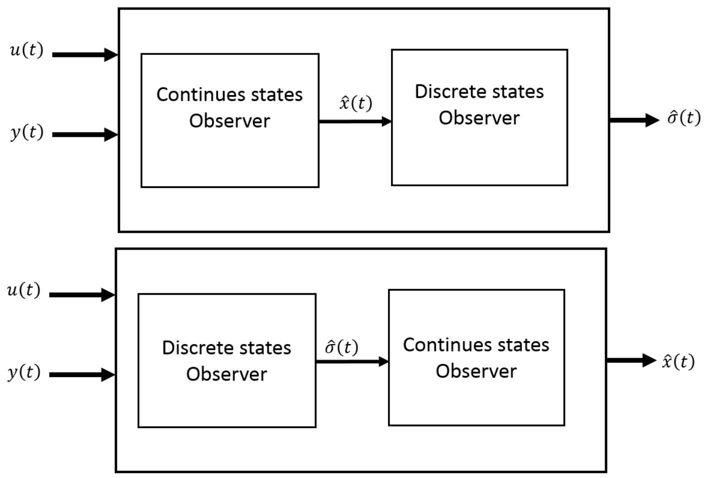

Figure 1.12: Principles of observations of continuous and discrete state.

For these studies, the conditions on detectability of the active mode are proposed. Indeed, these approaches assume that the available variables, such as the measurement of output $y(t)$ and /or the estimation of the continuous state $x\hat{(t)}$, make it possible to differentiate the modes and reconstructed of the discrete state. The problem can also be addressed in an inverted way, that is an observer based on a petri net, in [Hamdi et al., 2009], or an automaton in [Balluchi et al., 2002], can be defined to estimate the discrete state $\sigma(t)$. Then a second observer ensures the reconstruction of the continuous state $x(t)$. We can also mention [Fliess et al., 2008], where an algebraic approach is used to define the conditions of distinguishability and to estimate the discrete state of a switched system. Other approaches have therefore been developed using a battery of continuous

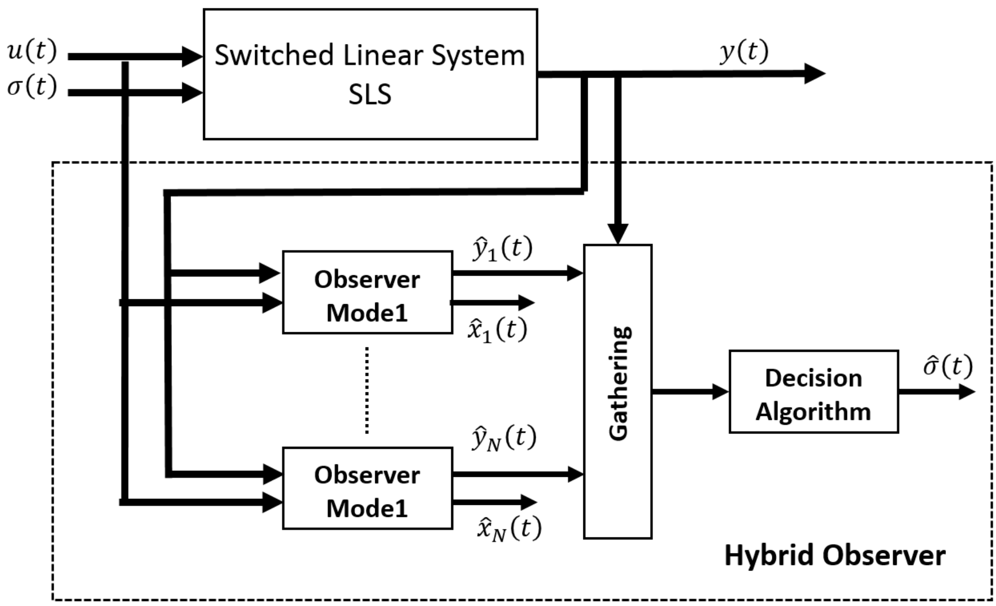

Figure 1.13: Hybrid observer defined by a battery of continuous observers and an identification algorithm of the active mode.

observers associated with operating modes (Figure 1.13). A high-gain observers is defined in [Veluvolu and Soh, 2009] to ensure the estimation of the continuous state. On the basis of a Lyapunov function, a function associated with a decision algorithm is proposed to reconstruc $\sigma(t)$. In [Ahrens and Khalil, 2009] A higher order sliding mode observation approach is also discussed in following the same structure. This observation technique makes it possible to consider a wider range of systems because the synthesis of each continuous observer, consider among a battery of observers, is associated with only one mode of operation. Nevertheless, the computing load generated is the main disadvantage of this approach for real-time applications.

In the majority of the approaches mentioned above, each mode is considered observable in the classic sens. However, in the case of switched systems, it may be possible to reconstruct the continuous state while the observability condition of each mode is not verified. These systems are not observable in the classical sense, that is, the goal of the present work.

1.9 Conclusion

In this chapter, we have provided some basics for the switched linear systems and the motivation of studying their structural properties. We have presented the different class of switching and the different model of such systems. In addition to this, a DC-DC boost converter has been considered as an illustrative example to highlight the existing theoretical results. A challenging observation issue and the observabiltiy condition has been raised where we will address it in Chapter 2 of this thesis. An algebraic approach will be proposed to observe the discrete and continuous state of a system subjected whose modes (or sub-systems) are not observable in the classical sense.

Chapter 2

Observer Desgin of SLCS: Main Results

“ *A journy of a thousand miles begin with a single step* ”

Lao-Tzu

This chapter addresses the estimation of hybrid state of switched linear systems. In particular, we discuss the distinguishability of discrete state, the observability condition of continuous state as well as the observer design. We firstly start with the study of the identification of discrete state using the algebraic tools. Secondly, for the observability of continuous state, general conditions will be provided. Then, we suggest a new algebraic observer based on the robust calculation of output derivatives and the gathering partial information from individual modes estimated the continuous state. After that, the hybrid algebraic observer algorithm is presented. Finally, illustrative example shows the effectiveness of proposed observer.

2.1 Problem Formulation

Consider the class of switched linear systems modeled as follows

$$\begin{cases} \dot{x} = A_{\sigma(t)}x + B_{\sigma(t)}u \\ y = C_{\sigma(t)}x \end{cases} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the vector of continuous states , $y(t) \in \mathbb{R}^p$ is the vector of outputs and $u(t) \in \mathbb{R}^m$ is the vector of outputs. The exogenous, yet observed, switching signal $\sigma(t) : \mathbb{R} \mapsto Q$ is assumed to be piecewise constant right-continuous function where $Q = \{1, \ldots, N\}$ is a finite discrete set. The constant matrices $A_{\sigma(t)} \in \mathbb{R}^{n \times n}, B_{\sigma(t)} \in \mathbb{R}^{n \times m}, C_{\sigma(t)} \in \mathbb{R}^{p \times n}$ are respectively the state, input and output matrices. Over a given time interval $[t_0, t_f[$, the switching signal $\sigma(t) : [t_0, t_f[\mapsto Q$ is defined by

$$\sigma(t) = q_k \quad t_{k-1} \leqslant t < t_k \quad q_k \in Q, k = \{1, \ldots, l\} \tag{2.2}$$

Thus, the mode (also called subsystem) $(A_{q_k}, B_{q_k}, C_{q_k})_{k \in \{1, \ldots, l\}}$ are active at the switching times $\{t_k\}_{k=0,\ldots,l}$ during the subintervals $I_k = [t_0, t_f[$. For brevity, the switching signal is noted by $\sigma(\tau)_{\tau \in [t_0, t_f[}$. Note that, the switching signals is completely characterized by $STS, SS, ..$ presented in the previous chapter.

2.1.1 Hypotheses and Assumptions

Through this thesis, it is assumed that:

- ***A.1.*** System (2.1) satisfies the minimal dwell time $t_{k+1} - t_k \geq T_\delta$,where $T_\delta > 0$.
- ***A.2.*** The dimension of the continuous state is invariant when the transition is enable. Note that assumption ***A.1*** excludes Zeno phenomena [Johansson et al., 1999].
- ***A.3.*** The switching signal considered to be unknown for designing the global observer.
- ***A.4.*** Assuming that the individual subsystems are unobservable in classical sens(the rank condition is not verified).

The majority of the approaches mentioned in Chapter 1, the problem of the observation is not considered for the class of switched system including unobservable modes. Each mode is considered observable in the classical sense. However, in the case of switched systems, it may be possible to reconstruct the continuous state while the condition (**A.4**) is not verified. These systems are not observable in the classical sense, that is, a classical observer can not be applied. In the following, the concept of observation of both the discrete and continuous state by an algebraic approach for switched systems is presented. We will presente in next section the principle of the proposed approach and how to adapt it to build a hybrid observer.

2.2 Identification of Discrete Mode : Active Mode Estimation

The observer design for a discrete mode in switched system, is the determination of switching time as well as the active mode(actual one). This process needs only the knowledge of inputs, outputs and their derivatives. In [Fliess et al., 2008], M. Fliess and H.S Ramirez propose a method for identifying the active mode in real time. This method comes from the techniques of differential algebra and operational calculation. It is clear that this theory is complicated but its application remains quite simple. For our case, we are interested in its utility in the Laplace space and we are restricted to linear systems. We will present in this section the principle of the method and how to adapt it for the estimation of the discrete state. Before begin, we give a necessary condition for the distinguishability between different modes.

2.2.1 Distiguishability Condition

Consider the switched linear systems(2.1). In this section we will give notions on the distinguishability between the modes, for that we will rewrite the modes that we will study their distinguishability. Let i and j are two modes, their *grouping* is given by

$$\begin{cases} \dot{x}(t) = Ax + Bu \\ Y = Cx \end{cases} \tag{2.3}$$

With

$$A = \begin{pmatrix} A_i & 0 \\ 0 & A_j \end{pmatrix}, B = \begin{pmatrix} B_i \\ B_j \end{pmatrix}, C = \begin{pmatrix} C_i \\ C_j \end{pmatrix}$$

Definition 2.1 Two modes i, j of (2.1) have the same input are indistinguishable if their outputs are identical

$$y_i(t) \equiv y_j(t) \tag{2.4}$$

Two no indistinguishable modes are distinguishable.
The latter is equivalent to

Definition 2.2 Two modes i, j are indistinguishable if

$$y_i^{(k)}(0) = y_j^{(k)}(0), Y^{(k)}(0) = 0, k = \{0, 1, \ldots, n\} \tag{2.5}$$

Mathematically, this condition can be written as

$$M_N[x_0 \;\; u^{(1)}(0) \;\; \dots u^{(n)}(0)]^T = 0$$

With

$$M_N = \begin{pmatrix} C & 0 & 0 & \dots & 0 \\ CA & CB & 0 & \dots & 0 \\ CA^2 & CAB & CB & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CA^n & CA^{n-1}B & CA^{n-2}B & \dots & 0 \end{pmatrix}$$

The system (2.1) can be written as

$$OBS_{ij}x_0 + COM_{ij}U = 0 \tag{2.6}$$

With

$$OBS_{ij} = \begin{pmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^n \end{pmatrix}, COM_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ CB & 0 & 0 & \dots & 0 \\ CAB & CB & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CA^{n-1}B & CA^{n-2}B & \dots & CB & 0 \end{pmatrix}$$

Proposition 2.1 Consider W a matrix verified $W \times OBS_{ij} = 0$,the modes i and j are distinguishable if

$$W \times COM_{ij} = 0 \tag{2.7}$$

This equivalent to

$$Ker(OBS_{ij}) \subseteq Ker(COM_{ij}) \tag{2.8}$$

With

$$Rank(|OBS_{ij}|) = Rank(|OBS_{ij} \;\; COM_{ij}|) \tag{2.9}$$

This last condition(2.9)is a necessary and sufficient condition.

2.2.2 Algebraic Discrete States Observer: Identification of Active Mode

Assume from now that any of pair modes are distinguishable. From now we want to obtain effective real-time algorithm to determine the current mode \i”.

Let consider the signale-input signale-output systems(SISO), with the time derivative operator $\dfrac{d}{dt}$ of order $n \in \mathbb{N}^{*+}$

$$(\frac{d^n}{dt^n} + a_n \frac{d^{n-1}}{dt^{n-1}} + ... + a_1)y(t) = (b_m \frac{d^m}{dt^m} + ... + b_1)u(t) \tag{2.10}$$

Without loss of generality, we are interested in strictly proper second order systems

$$\ddot{y}(t) + a_2\dot{y}(t) + a_1 y(t) = b_2\dot{u}(t) + b_1 u(t) \tag{2.11}$$

Taking the Laplace transformation

$$(s^2Y(s) - sy(t_0) - \dot{y}(t_0)) + a_2(sY(s) - y(t_0)) + a_1Y(s) = b_2(sU(s) - u(t_0)) + b_1U(s) \tag{2.12}$$

To eliminate the terms $y(t_0), \dot{y}(t_0)$, eq(2.12) is derived twice with respct to the Laplace variable

$$\begin{aligned} a_2[s\frac{d^2Y(s)}{ds^2} + 2\frac{dY(s)}{ds}] + a_1[\frac{d^2Y(s)}{ds^2}] - b_2[s\frac{d^2U(s)}{ds^2} + 2\frac{dU(s)}{ds}] - b_1[\frac{d^2U(s)}{ds^2}] \\ +s^2\frac{d^2Y(s)}{ds^2} + 4s(dYds + 2Ys) = 0 \end{aligned} \tag{2.13}$$

Multiplying eq(2.13) by s^{-2} inverse Laplace transformation to the time domain results in

$$a_2D_2(t) + a_1D_1(t) + b_2N_2(t) + b_1N_1(t) + R(t) = 0 \tag{2.14}$$

With

- $D_1(t) = \int_0^t \int_0^{\jmath_1} \jmath_2^2 y(\jmath_2) d\jmath_2 d\jmath_1$
- $D_2(t) = \int_0^t \jmath_1^2 y(\jmath_1) d\jmath_1 - 2\int_0^t \int_0^{\jmath_1} \jmath_2 y(\jmath_2) d\jmath_2 d\jmath_1$
- $N_1(t) = -\int_0^t \int_0^{\jmath_1} \jmath_2^2 u(\jmath_2) d\jmath_2 d\jmath_1$
- $N_2(t) = -\int_0^t \jmath_1^2 u(\jmath_1) d\jmath_1 + 2\int_0^t \int_0^{\jmath_1} \jmath_2 u(\jmath_2) d\jmath_2 d\jmath_1$
- $R(t) = t^2 y(t) - 4\int_0^t \jmath_1 y(\jmath_1) d\jmath_1 + 2\int_0^t \int_0^{\jmath_1} y(\jmath_2) d\jmath_2 d\jmath_1$

The eq(2.14) has the advantage to be totally independent of the initial conditions, it is sufficient to measure the input and output to evaluate the integrals, this constitutes the major advantage about exploiting this results in our observer desgin.

Now, apply the differential algebra to the selection of active mode. We associate with each discrete state q_i the linear continous dynamics as

$$\ddot{y}(t) + a_{2i}\dot{y}(t) + a_{1i}y(t) = b_{2i}\dot{u}(t) + b_{1i}\dot{u}(t) \quad i \in 1, ..., N \tag{2.15}$$

With $y(t), u(t) \in \mathbb{R}$ and $a_{2i}, a_{1i}, b_{2i}, b_{1i}$ parametres are constant in $\mathbb{R}$.

We take eq(2.14) for the system in eq(2.15)

$$a_{2i}D_2(t) + a_{1i}D_1(t) + b_{2i}N_2(t) + b_{1i}N_1(t) + R(t) = 0 \tag{2.16}$$

The functions $D_2(t), D_1(t), N_2(t), N_1(t), R(t)$ are evaluated in real time, and either $V(t)$ a column vector such as

$$V(t) = [D_2(t) \;\; D_1(t) \;\; N_2(t) \;\; N_1(t) \;\; R(t)]^T$$

The parameters a_{li}, b_{li}, l with $l = \{1, 2\}$ are known for each dynamic associated with q_i and θ_i be a vector line of parameters such as

$$\theta_i = [a_{2i} \;\; a_{1i} \;\; b_{2i} \;\; b_{1i} \;\; 1]$$

So let's take eq(2.16) that we will allows us to propose the following criterion

$$\begin{cases} t_k < \hat{t} < t_k + 1 \\ if \;\; \theta_i V(t) = 0 \;\; \hat{q} = q_i \\ if \;\; \theta_i V(t) \neq 0 \;\; \hat{q} \neq q_i \end{cases} \tag{2.17}$$

The criterion (2.17), is applicable if the parameters are identifiable.

Consider two second order linear systems with parameter vectors θ_i, θ_j the parameters are identifiable if and only if:

$$\theta_i V(t) = \theta_j V(t) \implies \theta_i = \theta_j$$

We have

$$\theta_i V(t) = \theta_j V(t) \implies (\theta_i - \theta_j)V(t) = 0 \tag{2.18}$$

Now, we have five successive samples of $V(t)$ with a switching time T

$$V(kt) = [D_1(kT) \;\; D_2(kT) \;\; N_2(kT) \;\; N_1(kT) \;\; R(k)]^T$$

With

- $D_1(kT) = \int_{(k-1)T}^{KT}\int_{(k-1)T}^{KT} \jmath_2^2 y_k(\jmath_2)d\jmath_2 d\jmath_1 = \frac{T^4}{12}(6k^2-8k+3)y_k$
- $D_2(kT) = \int_{(k-1)T}^{KT} \jmath_1^2 y_k(\jmath_1)d\jmath_1 - 2\int_{(k-1)T}^{KT}\int_{(k-1)T}^{KT} \jmath_2^2 y_k(\jmath_2)d\jmath_2 d\jmath_1 = T^3(k-1)^2 y_k$
- $N_1(kT) = \int_{(k-1)T}^{KT}\int_{(k-1)T}^{KT} \jmath_2^2 u_k d\jmath_2 d\jmath_1 = \frac{T^4}{12}(6k^2-8k+3)u_k$
- $N_2(kT) = \int_{(k-1)T}^{KT} \jmath_1^2 u_k d\jmath_1 + 2\int_{(k-1)T}^{KT}\int_{(k-1)T}^{KT} \jmath_2^2 u_k d\jmath_2 d\jmath_1 = -T^3(k-1)u_k$
- $R(kT) = T^2 y(t) - 4\int_{(k-1)T}^{KT} \jmath_1 y_k d\jmath_1 + 2\int_{(k-1)T}^{KT}\int_{(k-1)T}^{KT} y_k d\jmath_2 d\jmath_1 = T^2(k^2-4k+3)y_k$

For $k = 1, 2, 3, 4, 5$ the matrix $\gamma = [V(t)\ \ V(2T)\ \ V(3T)\ \ V(4T)\ \ V(5T)]$

$$\gamma = \begin{pmatrix} 0 & T^3 y_2 & T^3 4y_3 & T^3 9y_4 & T^3 16y_5 \\ \frac{T^4}{12}y_1 & \frac{T^4}{12}y_2 & \frac{T^4}{12}y_3 & \frac{T^4}{12}y_4 & \frac{T^4}{12}y_5 \\ 0 & -T^3 u_2 & -T^3 4u_3 & -T^3 9u_4 & -T^3 16u_5 \\ -\frac{T^4}{12}u_1 & -\frac{T^4}{12}11u_2 & -\frac{T^4}{12}33u_3 & -\frac{T^4}{12}67u_4 & -\frac{T^4}{12}133u_5 \\ 0 & -T^2 y_2 & 0 & T^3 3y_4 & T^2 8y_5 \end{pmatrix} \tag{2.19}$$

The eq(2.18) admits a unique and null solution $(\theta_i - \theta_j) = 0$ if and only if γ is invertible. Note that the matrix is not invertible if $u = 0\ \ or\ \ y = 0\ \ or\ \ y = \alpha u$ with $\alpha \in \mathbb{R}$. We quote in particular the following cases:

Case 1: Systems without control $B_i = B_j = [0\ \ 0]^T$, so we take that the samples concerning the output $y(t)$ and γ becomes

$$\gamma = \begin{pmatrix} 0 & T^3 y_2 & T^3 4y_3 \\ \frac{T^4}{12}y_1 & \frac{T^4}{11}y_2 & \frac{T^4}{33}y_3 \\ 0 & -T^2 y_2 & 0 \end{pmatrix} \tag{2.20}$$

this matrix is always invertible if $y(t) \neq 0$ then criterion (2.18)is discriminating if $\theta_i \neq \theta_j$

Case 2: if the u is constant : these derivatives are null, and therefore whatever the parameter applied to $\dot{u}(t)$ it does not appear in the function criterion. We take $V(kT)$ without $N_2(kT)$ so the case where we have two subsystems with the same eigenvalues (i.e. the same denominators in their transfer functions) the parameters applied should be different.

$$\gamma = \begin{pmatrix} 0 & T^3 y_2 & T^3 4y_3 & T^3 9y_4 \\ \frac{T^4}{12}y_1 & \frac{T^4}{11}y_2 & \frac{T^4}{11}y_3 & \frac{T^4}{67}y_4 \\ -\frac{T^4}{12}u & -\frac{T^4}{12}11u & -\frac{T^4}{12}33u & -\frac{T^4}{12}67u \\ 0 & -T^2 y_2 & 0 & -T^2 3y_4 \end{pmatrix} \tag{2.21}$$

We have seen how the criterion inspired by the differential algebra makes it possible to know from which discrete state (mode)is actif. Then we required $u(t), y(t)$ that always be continuous and coming from the same dynamic associated with q_i what is not always the case for the switched system where, present discontinuities during switching or are continuous but resulting from different dynamics in switching. During a transition the observer of the discrete state (diverges) $\theta_j V(t) \neq 0$ and this situation of (divergence) is interesting because it allows us to detect the switching time but does not provide any information on the discrete state.

A solution to this problem is to periodically (reset) the observer of the discrete state, just choose a very small period compared to the minimum dwell time, then the observer can be reset several times between two successive transitions which introduces a maximum delay of one period to provide information on the active discrete state.

2.3 Observability of Continuous State

The problem of observability deals with extracting information about the state of the system from the measured output while the input and the switching signal are assumed to be known. This property plays a fundamental role in state estimation as the construction of state estimators, or observers, requires the actual system to be observable. Even though the basic problem of observability has been well studied for non-switched systems, the presence of the switching signal and the hybrid nature of switched systems make the problem more significant. In general, the individual subsystems of a switched system may not be observable, so the information about the state cannot be estimated in arbitrarily fast time using the classical methods, such as Luenberger observers. Thus, an interesting question is "*how the partial information about the state, that is available from each mode, can be combined to recover the complete information about the state even though the individual subsystems are unobservable ?*". Moreover, if the system is observable, then can be designing the state estimators that are feasible for implementation on digital computers. Fundamental questions of this sort, i.e., seeking observability of the entire system especially with unobservable modes and constructing asymptotic observers for estimating state are not only theoretically interesting but also possess significant applicability.

As montioned in previous chapter, there are many works that discuss the observability of switched linear systems. The considered problem of observability of switched linear systems has been studied but there are still some shortcomings about the observability of the system(2.1) which will be rectified in this chapter using some geometrical tools and matrices properties. First, let us present some existing results of this problem and then we will give our contribution for the considered problem. Before that, let us give the different definitions about the observability of continuous state of system(2.1).

2.3.1 Definitions

***Definition 2.1*.**
The switched system (2.1) is called state *observable* on $[t_0, T], t_0 < T = t_f$, if any *initial condition state* $x(t_0) = x_0 \in \mathbb{R}^n$ is *uniquely* determined by the control *input* u and the measurement *output* y for $t \in [t_0, T]$.
This definition is the most general existing ones and it means that there exists at least a *switching signal* (switching sequence) $\sigma : [t_0, T] \to Q$ that guarantees the *unique* determination of the continuous state.
***Definition 2.2*.**
The switched system 2.1 is called state *observable* on $[t_0, T], t_0 < T = t_f$ or *simply observable* for a given *switching sequence* $SS = ((t_0, \sigma(t_0)), ..., (t_{N-1}, \sigma(t_{N-1})))$ if any *initial condition state* $x(t_0) = x_0 \in \mathbb{R}^n$ is *uniquely* determined by the control *input* u and the measurement *output* y for *switching sequence* SS.
***Definition 2.3*.**
The switched system 2.1 is called state observable on $[t_0, T], t_0 < T = t_f$ for a given *switching sequence* $SS = ((t_0, q_N), ..., (t_{N-1}, q_N))$ and only if:
$y(t) = y(t, t_0, x_0, u, SS) = 0, t \in [t_0, T]$
implies that $x_0 = 0$.
Remarks 2.1

- It is obvious that the system (2.1)is observable if there is at least one discrete mode that satisfies the classical observability condition;
- It is clear that the system (2.1) is observable for a given switching sequence if there is at least one discrete mode that satisfies the classical observability condition;
- The control input does not affect the global observability of (2.1) i.e. the observability can be studied only with the linear part of (2.1).

2.3.2 Sufficient Conditions for Observability

In this section, we present a characterization of the unobservable subspace for system (2.1) with a given switching signal σ. Towards this end, let $N_{q_1}^{q_N}$ denote the set of states for system (2.1) that generate identically zero output over $[t_0, t_f)$.

For fixed switching times, it is easily seen that $N_{q_1}^{q_N}$ is actually a subspace due to linearity of (2.1), and we call $N_{q_1}^{q_N}$ the *unobservable subspace* for $[t_0, t_f)$.

Then, the *unobservable subspace* on the interval $[t_0, t_f)$ is simply given by the largest A_q-invariant subspace contained in $kerC_{q_1}$, i.e., $ker < C_{q_1}|A_{q_1} >= kerQ_{q_1}$, where

$$Q_{q_1} = col(C_{q_1}, C_{q_1}A_{q_1}, \ldots, C_{q_1}A_{q_1}^{n-1})$$

So it is clear that $N_{q_1}^{q_1} = kerQ_{q_1}$. Now, when the measured output is available over the interval $[t_0, t_N)$ that includes switchings at $\{t_0, \ldots, t_{N-1}\}$, more information about the state is obtained in general so that $N_{q_1}^{q_N}$ gets smaller as the difference $q_1 - q_N$ gets larger, and we claim that the subspace $N_{q_1}^{q_N}$ can be computed recursively as follows:

$$\begin{cases} N_{q_N}^{q_N} = KerQ_{q_N} \\ N_{q_1}^{q_N} = KerQ_{q_j} \cap (e^{-A_{q_j}\tau_j} N_{q_j}^{q_N}) \end{cases} \tag{2.22}$$

Where $\tau_j = t_j - t_{j-1}$.

In [Goshen-Meskin and Bar-Itzhack, 1992], [Goshen-Meskin and Bar-Itzhack, 1992] the authors showed that the observability of system(2.1) for the sequence SS can be checked using the following matrix

$$Q_{SS} = \begin{pmatrix} Q_{q_1} \\ Q_{q_1}e^{A_{q_1}\tau_1} \\ \vdots \\ Q_{q_N}e^{A_{q_N}\tau_N}e^{A_{q_{N-1}}\tau_{N-1}} \ldots e^{A_{q_1}\tau_1} \end{pmatrix}, \tag{2.23}$$

Where

Q_{q_j}, is the observability matrix of the mode q_j and it is given as follows

$$Q_{q_j} = \begin{pmatrix} C_{q_j} \\ C_{q_j}A_{q_j} \\ \vdots \\ C_{q_j}A_{q_j}^{n-1} \end{pmatrix}, \tag{2.24}$$

2.3.3 Observability of switching sequence SS

The authors in [Goshen-Meskin and Bar-Itzhack, 1990], [Goshen-Meskin and Bar-Itzhack, 1992] provided the general geometrical condition for the observability and it is given in the following theorem.

Theorem 2.1.

The *continuous state* of system 2.1 is *completely observable* for the *sequence SS* if and only if:

$$ker(Q_{SS}) = \{0\} \tag{2.25}$$

The condition given in this theorem is based on the *unobservable subspace* of (2.14)for the sequence SS. When the system (2.1) is *partially observable* for the sequence SS, the *unobservable subspaces* N_{SS} is given by:

$$N_{SS} = ker(Q_{SS}) \tag{2.26}$$

In [51], the authors proved that the *observability* condition given above can be significantly simplified for a particular case for the system(2.1) These simplifications are given by the following lemma and corollary.

Lemma 2.1.

If $ker(Q_j) \subset ker(Aq_j), j = 1, \ldots, N$ then

$$ker(Q_{SS}) = ker(SS) \tag{2.27}$$

Where

$$ker(SS) = \begin{pmatrix} Q_{q_1} \\ \vdots \\ Q_{q_N} \end{pmatrix} \in R^{nN \times n} \tag{2.28}$$

Based on this lemma,[44] could state the following corollary, which enables the avoidance of the exponential matrix computation in the observability analysis.

Corollary 2.1.

If $ker(Q_{q_j}) \subset ker(A_{q_j}), j = \{1, \ldots, N\}$then the system 2.1 is observable, for a given switching sequence SS, if and only if,

$$ker(Q_{SS}) = \{0\} \tag{2.29}$$

The matrix S_{SS} for is called the stripped observability matrix (SOM).

The observability of the switching sequence SS is developed by Tanwani and his co-workers in [Tanwani et al., 2012] using geomitrical tools. Their basic idea is to compute the unobservable set of (2.1) for the sequence SS. [Tanwani et al., 2012] provided an algorithm that gives the unobservable subspace of the sequence. For the sequence, the unobservable subspace can be determined by using the following algorithm

$$N_{q_1}^{q_N} = N_{q_1}^{q_1} \cap (\cap_{i=2}^{N} \prod_{j=1}^{i-1} e^{-A_{q_j}\tau_j} N_{q_i}^{q_i}) \tag{2.30}$$

Where $N_{q_i}^{q_i}$ is the unobservable subspace of the mode q_i and it is given by

$$N_{q_i}^{q_i} = ker \begin{pmatrix} C_{q_i} \\ C_{q_i}A_{q_i} \\ \vdots \\ C_{q_i}A_{q_i}^{n-1} \end{pmatrix} \tag{2.31}$$

From this algorithm, [Tanwani et al., 2012] gave the following theorem.

Theorem 2.2 the continuous state of the system 2.1 is completely observable for the sequence SS if and only if:

$$N_{q_1}^{q_N} = \{0\} \tag{2.32}$$

In [Tanwani, 2012], the authors proved that the choice of switching times can play an important role not only for the partial observability, but also for the total observability for a given switching sequence. This leads to provide the following corollarys that characterize the influence of switching times.

corollary 2.2 a switching mode sequence $SS = (q_1, \ldots, q_N)$ is satisfying the observability uniform respect to time property if and only if: its observable subspace is invariant for all switching times

$$ker(Q_{SS}) = ker \begin{pmatrix} Q_{q_1} \\ Q_{q_1}e^{A_{q_1}\tau_1} \\ \vdots \\ Q_{q_N}e^{A_{q_{N-1}}\tau_{N-1}}e^{A_{q_{N-1}}\tau_{N-2}} \ldots e^{A_{q_1}\tau_1} \end{pmatrix} = ker \begin{pmatrix} Q_{q_1} \\ Q_{q_2} \\ \vdots \\ Q_{q_N} \end{pmatrix} \tag{2.33}$$

with $\tau_i > 0$ are arbitrary.

corollary 2.3 the system (2.1) is satisfying the observability uniform respect to time property if and only if: the observable subspace of any switching modes is invariant for all switching times.

$$ker(Q_{SS}) = ker \begin{pmatrix} Q_{q_1} \\ Q_{q_1} e^{A_{q_1}\tau_1} \\ \vdots \\ Q_{q_N} e^{A_{q_{N-1}}\tau_{N-1}} e^{A_{q_{N-1}}\tau_{N-2}} \dots e^{A_{q_1}\tau_1} \end{pmatrix} = ker \begin{pmatrix} Q_{q_1} \\ Q_{q_2} \\ \vdots \\ Q_{q_N} \end{pmatrix} \tag{2.34}$$

with $\tau_i > 0, q_i$ are arbitrary.

The last two theorems are significant regarding the uniform time observability property of (2.1) (for switching sequences). However, they are quite trivial since their demonstration idea is easy. The following theorems characterize the uniform time observability of (2.1) for switching sequences and provide some nontrivial conditions.

corollary 2.4 a switching mode sequence $SS = (q_1, \dots, q_N)$ is satisfying the observability uniform respect to time property if and only if

$$ker(A_{q_i}^{k_i} Q_{q_{i+1}}) \subseteq ker(Q_{q_{i+1}}) \tag{2.35}$$

with $k_i \geq 0$

Proof ***corollary 2.4***

We have from

$$x \in ker(Q_{SS}) \Rightarrow Q_{q_i} e^{A_{q_i}\tau_i} e^{A_{q_{i-1}}\tau_{i-1}} \dots e^{A_{q_1}\tau_1} = 0$$

Taking $\tau_j = 0, ij < i$, one obtain

$$e^{A_{q_i}\tau_i} x = 0$$

Differentiate it k^{th} respect to τ_i and taking $\tau_i = 0$, one get:

$$Q_{q_i} A_{q_i}^{k_i} x = 0$$

which means exactly the given condition in corollary 2.4.

The uniform observability respect to time of (2.1) for a switching sequence can be extended to all possible sequences for the system 2.1. This extension is given by the next corollary

corollary 2.5 the system (2.1) is satisfying the observability uniform respect to time property if and only if:

$$ker(A_{q_i}^{k_i} Q_{q_i}) \subseteq ker(Q_{q_j}) \tag{2.36}$$

where $k_i \geq 0$, the two modes are arbitrary. The given condition in ***corollary 2.4***. can be easily extended for the case of many matrices, as

$$ker(Q_i A_{i_1}^{k_{i_1}} A_{i_2}^{k_{i_2}} \dots A_{i_j}^{k_{i_j}}) \subseteq ker(Q_{q_j}) \tag{2.37}$$

Example 2.1 To clarify the ideas, we consider the following switched linear system

Where $\sigma : \mathbb{R}^+ \longrightarrow I_N = \{1, 2, 3\}$ is the switching control law. The state and output matrices are given by,

$$A_{q_1} = \begin{bmatrix} 1 & 1 & 5 \\ 2 & -1 & 4 \\ -2 & -1 & -6 \end{bmatrix}, C_{q_1} = \begin{bmatrix} 1 & 1 & 3 \end{bmatrix}$$

$$A_{q_2} = \begin{bmatrix} 9 & -3 & -11 \\ -4 & -7 & -10 \\ 4 & 3 & 6 \end{bmatrix}, C_{q_2} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

$$A_{q_3} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -3 & -1 \\ -1 & 1 & -1 \end{bmatrix}, C_{q_3} = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$$

The sequence that we want to analyze is $Q = \{q_1 q_2 q_3\}$ the switching instants are $t_0 = 0, t_1 = 1, t_2 = 2$. Than, $SS = \{(t_0, q_1), (t_1, q_2), (t_2, q_3), \}$

Before calculating the unobservable subspace of the sequence SS , we must calculate the unobservable subspace of each mode of the sequence, we have:

$N_{q_1}^{q_1} = ker(\begin{bmatrix} C_{q_1} \\ C_{q_1} A_{q_1} \\ C_{q_1} A_{q_1}^2 \end{bmatrix}) = ker(C_{q_1}) = \{x \in \mathbb{R}^3 : x_1 + x_2 + 3x_3 = 0\}$

$N_{q_1}^{q_1} = span\left\{ \begin{bmatrix} 0 & 3 & -1 \end{bmatrix}^T, (\begin{bmatrix} 0 & 3 & -1 \end{bmatrix})^T \right\} = ker(C_{q_1}) = \{x \in \mathbb{R}^3 : x_1 + x_2 + 3x_3 = 0\}$

$N_{q_2}^{q_2} = ker(\begin{bmatrix} C_{q_2} \\ C_{q_2} A_{q_2} \\ C_{q_2} A_{q_2}^2 \end{bmatrix}) = ker(C_{q_2}) = x \in \mathbb{R}^3 : x_1 + x_3 = 0$

$N_{q_2}^{q_2} = span\left\{ \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}^T, (\begin{bmatrix} 1 & 1 & -1 \end{bmatrix})^T \right\} = ker(C_{q_2}) = \{x \in \mathbb{R}^3 : x_1 + x_2 = 0\}$

$N_{q_3}^{q_3} = ker(\begin{bmatrix} C_{q_3} \\ C_{q_3} A_{q_3} \\ C_{q_3} A_{q_3}^2 \end{bmatrix}) = ker(C_{q_3}) = \{x \in \mathbb{R}^3 : x_1 + x_2 = 0\}$

$N_{q_3}^{q_3} = span(\begin{bmatrix}0 & 1 & -1\end{bmatrix})^T, (\begin{bmatrix}1 & 1 & -1\end{bmatrix})^T = ker(C_{q_3}) = \{x \in \mathbb{R}^3 : x_2 + x_3 = 0\}$

From(2.34), the unobservable subspace of the sequence SS is

$N_{q_1}^{q_3} = N_{q_1}^{q_1} \cap N_{q_2}^{q_2} \cap N_{q_3}^{q_3} = \{0\}$

Now, we can given another methodology for obtain the observable states using the properties of the switching sequence.

2.3.4 Observability of Periodic Sequence of Discrete Modes(PSDM)

Consider the system(2.1)presented by periodic sequence of discrete modes ($PSDM$), we define a periodic sequence of discrete modes as

$$PSDM = S(S_1, T, N) = S_1 S_1 \dots S_1$$

With $S1$ switching sequence repesents during the period T,and N represent the number of the periodic sequence in S. The condition given by

$$rank(Q_{ss}) = n$$

is a suffisant condition for the observability of the system(2.1)during the switching sequences SS, but it requires a lot of calculation that is why we are looking for conditions that make this analysis easy to check than this condition. in the following, we will give simpler conditions using the geometric approach.

The matrix of the observability of SS can be simplified as

$$Q_S = \left(\begin{bmatrix} Q_{S_1} \\ Q_{S_1} M \\ Q_{S_1} M^2 \\ \vdots \\ Q_{S_1} M^{N-1} \end{bmatrix}\right) \tag{2.38}$$

Where

$M = e^{A_{q_N}\tau_N} e^{A_{q_{N-1}}\tau_{N-1}} e^{A_{q_{N-2}}\tau_{N-2}} \dots e^{A_{q_1}\tau_1}$.

Q_{S_1}, is the observability matrix under the sequence S_1

$$Q_{S_1} = \left(\begin{bmatrix} Q_{q_1} \\ Q_{q_2} e^{A_{q_1}\tau_1} \\ \vdots \\ Q_{N-1} e^{A_{q_{N-1}}\tau_{N-1}} e^{A_{q_{N-2}}\tau_{N-2}} \dots e^{A_{q_1}\tau_1} \end{bmatrix}\right)$$

The observability analysis using the formulation (2.39) is similaire to the observability analysis using in linear case, where M represent the matrix of state and Q_{S_1} represent the matrix of output. Then we can use the geomitric algorithm to obtain the unobservable subspace as follows

$$\begin{cases} J_{K+1} = Ker(Q_{S_1}) \cap M^{-1} J_k \\ J_0 = Ker(Q_{S_1}) \end{cases} \tag{2.39}$$

The unobservable subspace is $N_{q_1}^{q_N} = J_k$,where $k < n + 1$ is the little natural whole that check.

Remark.2.2

the unobservable subspace of the sequence S is the maximum unobservable subspace contained in $ker(Q_{S_1})$, with $ker(Q_{S_1})$ is M-invariant.

The algorithm given in(2.39) can provide us with interesting conclusions about the observability condition of $PSDM$, these conclusions will be given by proposals followed by demonstrations.

Proposition 2.2

The observability analysis of S is restricted to S_1 iff unobservable subspace $ker(Q_{S_1})$ is M-invariant.

Proof if.if $ker(Q_{S_1})$ is M-invariant we will have

$$J_1 = Ker(Q_{S_1}) \cap M^{-1} J_0 = Ker(Q_{S_1}) \cap M^{-1} Ker(Q_{S_1}) \tag{2.40}$$

According to (2.40), The observability analysis of S is restricted to S_1.

iff.if $ker(Q_{S_1})$ is M-invariant ,than

$$J_1 = Ker(Q_{S_1}) \cap M^{-1} Ker(Q_{S_1}) \neq Ker(Q_{S_1}) \neq J_0 \tag{2.41}$$

According to (2.40)The observability analysis of S is not restricted to S_1.

Proposition 2.3

The observability of S is depends of the n first period i.e if $N > n$ The observability is restricted to first period.

Proof. the proof and an immediate consequence of the algorithm given in condition $rank(Q_{ss}) = n$.(more detail see [Haddadi and Atallah, 2019]).

In the following we will show the importance of the results found by the analysis of PSDM in academic exemple.

Example 2.2

To clarify the ideas, we consider the following switched linear system(2.1) ,we analysis the observability of two $PSDM$, with $\sigma(t) \in Q = \{q_1, q_2, q_3\}$ and the matrices of state and output given by:

$$A_{q_1} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & -3 \end{bmatrix}, C_{q_1} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$A_{q_2} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & -3 \end{bmatrix}, C_{q_2} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$A_{q_3} = \begin{bmatrix} 0 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 2 & -4 \end{bmatrix}, C_{q_3} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

The $PSDM$ is given by :

$$S^1 = S^1(S_1^1, 1, 4) \text{ where } S_1^1 = (0.1)(0.6, 3)$$

$$S^2 = S^2(S_1^2, 1, 4) \text{ where } S_2^2 = (0.2)(0.3, 3)(0.7, 1)$$

we can easily demonstrate that these modes are not observable in the classical sens, which makes the analysis of the observability of these sequences no trivial.

Observability analysis of S_1^1 **.**

as $N = 4 > 3$ then the analysis of the observability of this sequence is restricted to the three periods.

The unobservable subspaces of the sequence S_1^1 and the matrix are:

$$ker(Q_{S_1^1}) = ker\left\{\begin{bmatrix} Q_1 \\ Q_3 e^{0.6A_1} \end{bmatrix}\right\} = span\left\{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right\}$$

With

$$M^1 = e^{0.4\times A_3} e^{0.6\times A_1} = \begin{bmatrix} 0.956 & 0.043 & -0.375 \\ 0.101 & 0.898 & 0.247 \\ -0.27 & 0.27 & 0.236 \end{bmatrix}$$

Than,

$$J_1 = ker(Q_{S_1^1})(M^1)^{-1}J_0 = span\left\{\begin{pmatrix}1\\1\\0\end{pmatrix}\right\} \cap span\left\{\begin{pmatrix}1\\1\\0\end{pmatrix}\right\} = span\left\{\begin{pmatrix}1\\1\\0\end{pmatrix}\right\} = J_0$$

Observability analysis of S_1^2 **.**

The unobservable subspaces of the sequence S_1^2 and the matrix are :

$$ker(Q_{S_2^1}) = ker\left\{\begin{bmatrix}Q_2\\Q_3e^{0.8A_2}\\Q_1e^{0.5A_3}e^{0.8A_2}\end{bmatrix}\right\} = span\left\{\begin{pmatrix}0.517\\0.855\\0\end{pmatrix}\right\}$$

$$M^2 = e^{0.4\times A_3}e^{0.6\times A_1} = \begin{bmatrix}1.747 & 0.672 & 0.018\\0.37 & 1.501 & 0.615\\0.079 & -0.047 & -0.02\end{bmatrix}$$

$$J_2 = ker(Q_{S_1^2})(M^2)^{-1}J_0 = span\left\{\begin{pmatrix}1\\1\\0\end{pmatrix}\right\}\cap span\left\{\begin{pmatrix}0.517\\0.855\\0\end{pmatrix}\right\} = span\left\{\begin{pmatrix}0.31\\-0.073\\1.382\end{pmatrix}\right\} =$$

$\{0\}$

as $J_1 = \{0\}$, than the sequence S^2 is observable.

2.4 Algebraic Continuous State Observer

Algebraic approaches have been recently used for systems control and observation with emphasis on their characteristics of non-asymptotic convergence, robustness to uncertainties and capacity to deal with measured noise without any assumption on its statistical properties [Li et al., 2006]. Some examples for controller design are given in [Diehl et al., 2002], [Conte et al., 1999]. For the case of observers based on algebraic methods, the estimation is obtained by means of algebraic calculation of the output derivatives and, for observable systems, can be locally mapped to the state space [Martínez-Guerra and De León-Morales, 1996]. Those approaches have been related to the theory of linear observers in [Corless and Tu, 1998].

2.4.1 Algebraic Observer Scheme

the algebraic observer's idea is based on the robust differentition of the output and the gathring partial information from individul modes.

We suppose that the system is conditionally observable for the sequence SS .

From system (2.1), it is possible to define the successive differentiation of output for the mde q_i gives:

$$\begin{cases} y(t) = C_{q_i}x(t) \\ \dot{y}(t) = C_{q_i}A_{q_i}x(t) + C_{q_i}B_{q_i}u(t) \\ \ddot{y}(t) = C_{q_i}A_{q_i}^2 x(t) + C_{q_i}A_{q_i}B_{q_i}u(t) + C_{q_i}B_{q_i}\dot{u}(t) \\ \quad \vdots \\ y^{(r_i-1)}(t) = C_{q_i}A_{q_i}^{(r_i-1)}u^{(0)}x(t) + \ldots + C_{q_i}B_{q_i}u^{(r_i-2)}(t) \end{cases} \tag{2.42}$$

Which can but in the compact form, one obtains

$$Y_{q_i}(t) = \mathbf{C}_{q_i}x(t) + M_{q_i}U(t) \tag{2.43}$$

With

$$Y(t) = \begin{pmatrix} y(t) \\ \dot{y}(t) \\ \ddot{y}(t) \\ \vdots \\ y^{(r_i-1)} \end{pmatrix}, \quad U(t) = \begin{pmatrix} u(t) \\ \dot{u}(t) \\ \ddot{u}(t) \\ \vdots \\ u^{(r_i-2)} \end{pmatrix}$$

Where

$\mathbf{C}_{q_i} \in R^{(r_{i_k} \times n)}$ is the reduced

Proposition 2.4

if the following conditions :

- $Rank[\mathbf{C}_{q_i}^T \;\; \mathbf{C}_{q_i}^T] = n$;
- $Rank[\mathbf{C}_{q_i}^T \;\; \overline{\mathbf{C}}_{q_i}^T] = n, k \in 1, ..., l$; with $\overline{\mathbf{C}}_{q_i}$ is the complement of $[\mathbf{C}_{\mathbf{q_i}}$;
- $\frac{d\overline{\mathbf{C}}_{q_i}^T}{dt} = 0$ for $t \in I_i = [t_{i-1}, t_i]k \in \{1, ..., l\}$.

Then the system is conditionally observable for the sequence SS. The algorithm of the proposed observer is diveded into three steps

1- *First step*: Compute robustly the needed derivatives of the output y and input u

2- *Second step*: Gathering the partial information from individual modes over the hybrid trajectory

3- *Third step*: Reconstructing the states using the following expression

$$\hat{x}(t) = \mathbf{C}^{-1}[\hat{Y}(t) - M\hat{U}(t)]$$

Where:

$$\hat{x} = \begin{pmatrix} \hat{x}_{q_1} \\ \vdots \\ \hat{x}_{q_N} \end{pmatrix}, \quad \hat{Y} = \begin{pmatrix} \hat{Y}_{q_1} \\ \vdots \\ \hat{Y}_{q_N} \end{pmatrix}, \quad \hat{U} = \begin{pmatrix} \hat{U}_{q_1} \\ \vdots \\ \hat{U}_{q_N} \end{pmatrix},$$

$$M = \begin{pmatrix} M_{q_i} & ... & M_{q_i} \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} \mathbf{C}_{q_i} & ... & \mathbf{C}_{q_i} \end{pmatrix}$$

2.4.2 Estimation of Algebraic Derivatives : Numerical differentiation

To execute the above scheme, we need an efficient and fast method for getting the nedded derivatives of the output and the input.

In what follows, we recall to the algebraic method which can initially developed in [Zehetner et al., 2007] that gives a no-asymptotic and robust estimation of the derivatives. This method introduced a newway to get the derivatives up to a desired order of a given signal $y(t)$. It is based on the algebraic combinations of time-integrations of $y(t)$ to compute its derivatives.

Consider a real-valued function $y(t)$ that has the following convergent Taylor series:

$$y(t) = \sum_{i=0}^{\infty} \frac{y^{(0)}(0)}{i!} t^i, t \geq 0 \tag{2.44}$$

around $t = 0$. The approximation of $y(t)$ around 0 by the truncated Taylor series of order N is given by

$$y_N(t) = \sum_{i=0}^{N} \frac{y_N^{(i)}(0)}{i!} t^i \tag{2.45}$$

The goal is to get the estimation of the derivatives of y(t) up to order N. The operational calculus yields for the expression (2.45)is

$$y_N(s) = \sum_{i=0}^{N} \frac{y^{(i)}(0)}{s^{i+1}} \tag{2.46}$$

Multiply both sides of this expession s^{N+1} and the algebraic derivative $\frac{d^k}{ds^k}, k = 1, \ldots, N-1$ respectively, one obtains

$$\frac{d}{ds^{k+1}}[s^{N+1} y_N(s)] 0 =< k <= N-1 \tag{2.47}$$

From that the derivatives $\frac{d^k y(0)}{ds^k}, k = 1, \ldots, N-1$ are linearly identifiable.
Multiplying(2.45) by $s^{-\tilde{N}}, \tilde{N} > N$,allows to get rid of time derivatives of $y(t)$. The following theorem gives a complete characterization of this estimation.

Theorem 2.3 For all $t \geq T$, the j ordre time derivative estimate $\hat{y}^{(j)}(t)$ of the polynominal $y(t)$ as defined in(2.44) satisfies the convolution

$$\hat{y}^{(j)}(t) = \int_0^T H_j(T,\tau) y(t-\tau) d\tau, j = 0, \ldots, N \tag{2.48}$$

Where

$$H_j(T,\tau) = \frac{(N+j+1)!(N+1)!}{T^{(N+J+1)}} \textstyle\sum_{k_1=0}^{N-j} \sum_{k_2=0}^{j} \psi(k_1,k_2,j)$$

And

$$\psi = \frac{1/k_1!k_2!(k_1+k_2)!)(T-\tau)^{k_1+k_2}(-\tau)^{N-k_1-k_2}}{(N-j-k_1)!(j-k_2)!(N-k_1-k_2)!(N-k_1+1)}$$

Depends on the order j of derivative to be estimated,and on an arbitrary constant time window length $T > 0$. The proof of the last theorem is given in [Zehetner et al., 2007].

2.4.3 General Algorithm of Hybrid State Estimation

The algorithm in Fig.2.1 decribes the proposed hybrid algebraic observer scheme.

2.5 Illustrative Example

In order to highlight these results and show the efficiency of proposed observer , we consider the following example of a switched system with three-discrete modes as follows

$$A_{q_1} = \begin{bmatrix} -8 & -2 \\ 2 & -1 \end{bmatrix}, C_{q_1} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$A_{q_2} = \begin{bmatrix} -1 & 4 \\ -2 & -7 \end{bmatrix}, C_{q_2} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$A_{q_3} = \begin{bmatrix} -18 & 3 \\ 9 & -12 \end{bmatrix}, C_{q_3} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

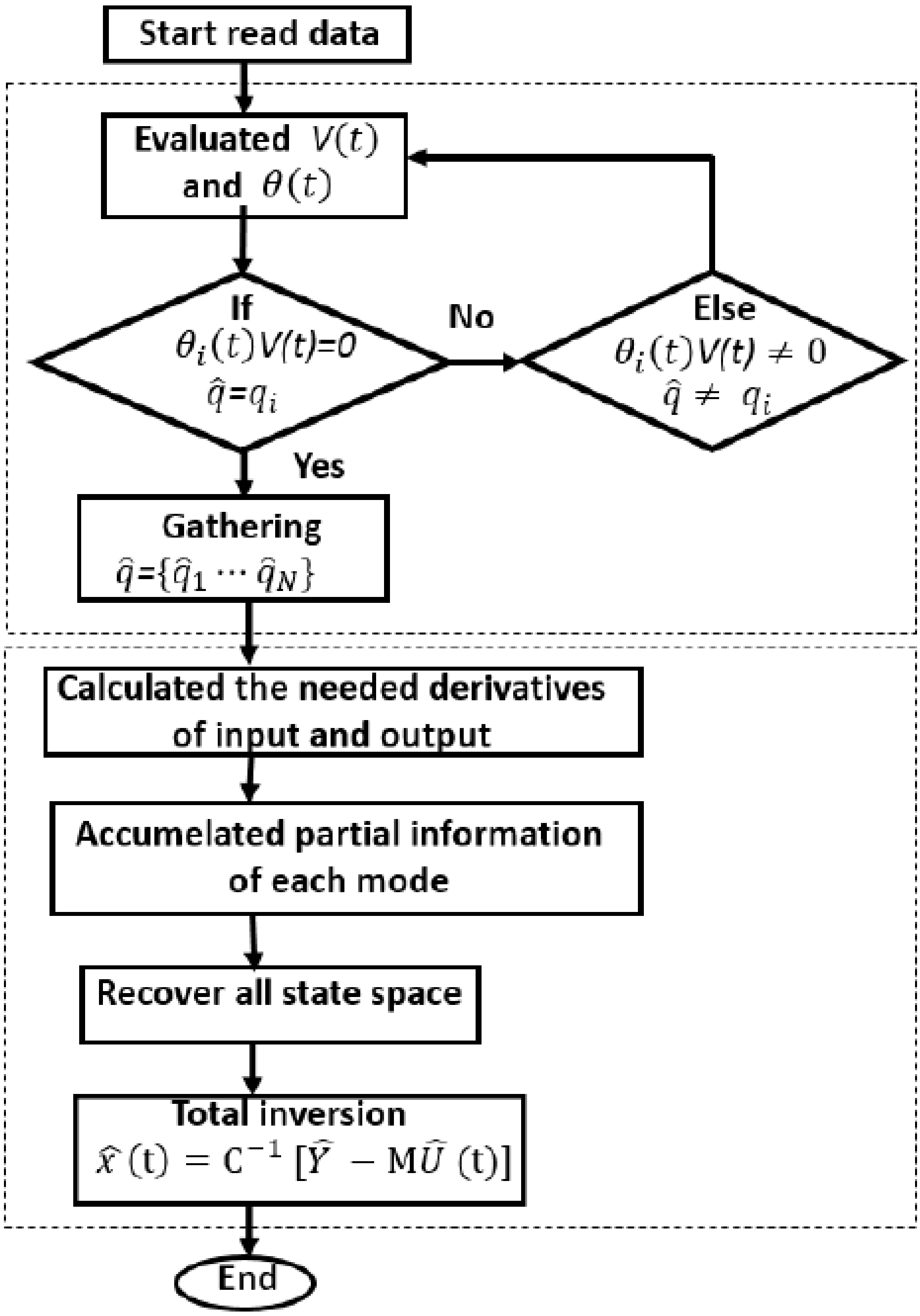

Figure 2.1: Flow-chart of hybrid state estimation.

$$B_{q_1} = B_{q_2} = B_{q_3} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Note that the modes $1, 2, 3$ are not observable in the classical sense. The switching sequences are, $SS_1 = (0, \sigma = 3), (t_1 = 1, \sigma = 1), (t_2 = 3, \sigma = 2)$, is observable using the condition (2.34) for this sequence. The simulation results of the proposed hybrid observer are presented. (Fig.2.2) gives the estimation of the discrete state. (Fig.2.3),(Fig.2.4) show

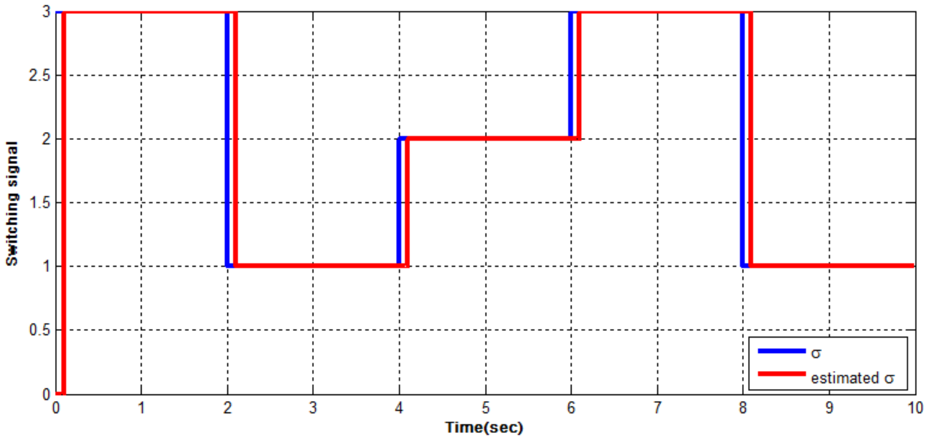

Figure 2.2: Switching signal and its estimated .

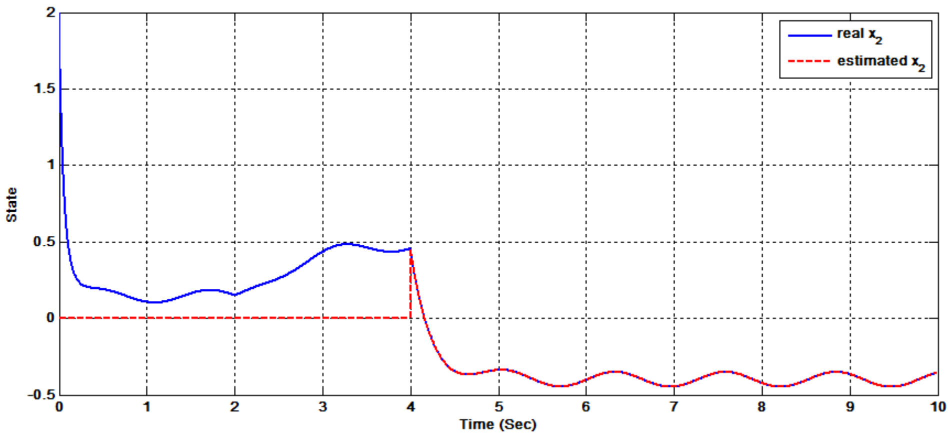

Figure 2.3: The first state and its estimated for the sequence SS_1

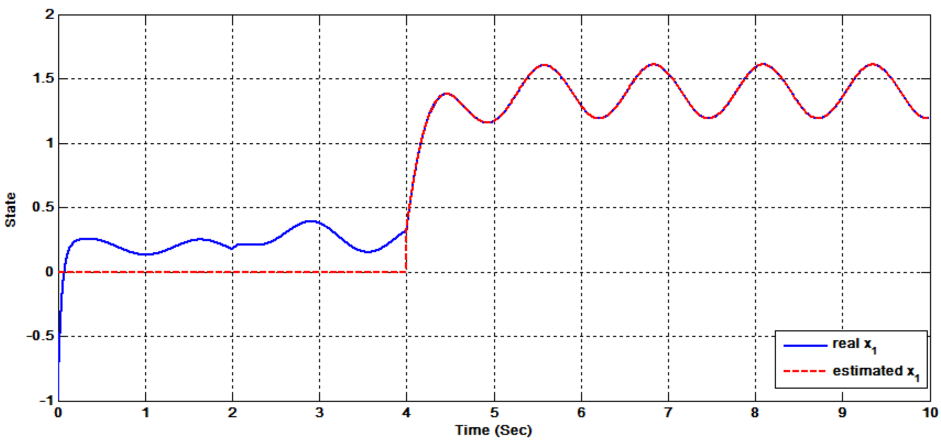

Figure 2.4: The second state and its estimated for the sequence SS_1

the states and their estimated for the sequences SS_1. These figures show the finite-time convergence of the estimated states to real states. The error estimation becomes zero for the states and this show the efficiency of our observer.

2.6 Conclusion

In this chapter, we have proposed a hybrid algebraic observer for estimated the discrete and continuous sate of the switched linear system while the existence of unobservable modes. Taking into account the hybrid observabiltiy of the continuous state, the algebraic observer can reconstruct the state using the output measurement and their robust derivatives estimation. The proposed hybrid state observer provides better and fast estimation as presented in the illustrative exemple.

Part II

Application on Multicellular Converter

Chapter 3

Background : Multicellular Converter

“ *it does not matter how slowly you go as long as you do not stop* ”

Confucius *Confucius*

This chapter is devoted to the description and the analysis of DC-DC series multicellular converter. Such converter belongs to the category of multilevel converters where the output voltage has more than two levels as the case for conventional converters (buck converter, boost converter, etc). This chapter introduces different mathematical models of the converter and gives some of their dynamical properties. In addition, it includes control requirements together with observability analysis. At the end, we motivate the need for more effcient observer by a comparative study between different existing observer schemes.

3.1 Introdction

Power electronics has witnessed increasing challenges in various technological applications via switched converters as the case for renewable energy systems electrical vehicles, microprocessors, etc. In these applications, the voltage and current stress can go beyond the range that one power device can handle and unsatisfactory performances are usually faced due to the limited switching frequency. To meet the constraints in voltage or in current, several solutions have been proposed based on the association of several power converters or the connection of multiple power devices to obtain a macro-component having satisfactory characteristics in voltage and/or current. Although these solutions seem to be attractive, non-straightforward synchronous control of the multiple elements must be ensured. To this end, an innovative solution has been proposed at the beginning of the 1990s by H. Foch and T. Meynard [Meynard and Foch, 1992] using the concept of a commutation. This suggested topology known as series multicellular converter for high voltage applications. This structure offers a *(p+1)* output voltage level, which makes it possible to obtain a remarkable improvement of the output waveforms (Low ripples and high apparent frequency), allowing a significant reduction of the filtering elements. It should be noted that these structures can operate in several configurations:chopper, half-bridge inverter or fullbridge inverter that can be used for observer design.

3.2 Modeling of a Series Multicellular Converter

The multi-cell converter consists of a series of p elementary switching cells (Fig.3.2)(one cell is composed of two complementary switches). Between which a floating voltage source is inserted. These floating voltage sources are realized by capacitors. The multicellular series structure can be adapted to all configurations: chopper or inverter (with a capacitive midpoint).

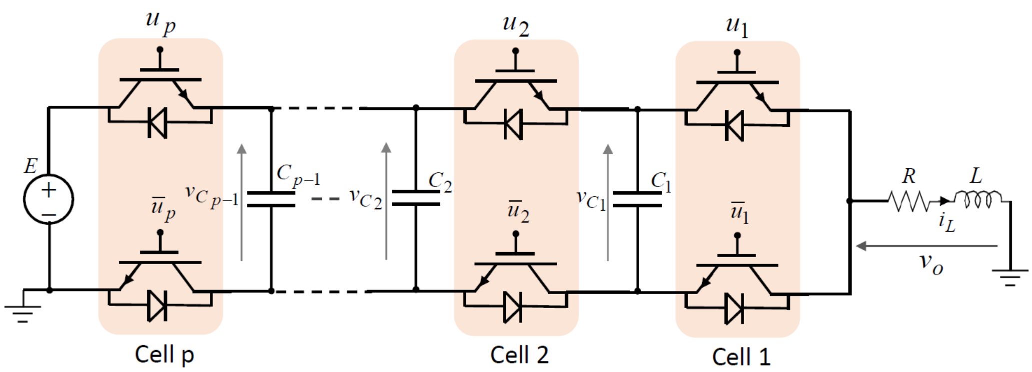

Figure 3.1: The multi-cell converter with p elementary switching cells.

The first advantage of these converters is the reduction of voltage constraints on the switches. Floating voltage sources impose on each cell a voltage constraint equal to E/p. On the other hand, the current rating of the switches is identical to that a classic structure: it's the current of charge. Serial multicellular converters also improve the waveform of the output voltage and allow more flexibility to achieve different voltage levels (compared to the NPC structure [Barbosa et al., 2005]).

On the other hand, the constraint of these converters is the need for a large number of capacitors, in particular for a three-phase configuration. The multicellular structure also called Meynard and foch structure results from the connection of p floating voltage sources placed in series so as to obtain $p + 1$ discrete levels of output voltage, indexed from 0 to p. the sources of tension are the supply bus voltage E (constant value)and $p - 1$ capacitors used as floating sources.

3.2.1 Elementary Switching Cell

The chopper is p cells (Fig.3.1). The function of each cell i is represented by u_i. $\overline{u}_i$ will also be called cell state i. The capacitor voltages are denoted by $v_{c_i}, i = \{1...p - 1\}$ the output voltage by V_o.

The principle of an elementary switching cell as the basis of the following source interconnection rules:

- Voltage source should never be short-circuit;
- Current source must never operate in an open-circuit;
- Sources of the same nature cannot be connected between them, but sources of different natures connect between them (voltage-current).

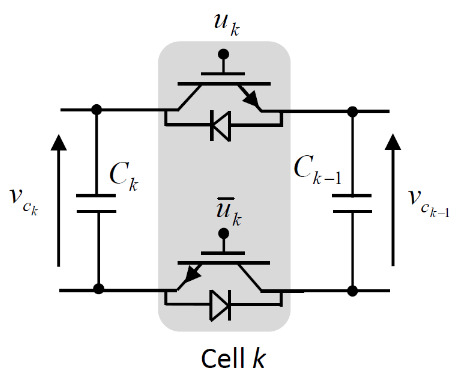

Figure 3.2: Elementary switching cells.

In order to respect the rules of the interconnection of sources, the control signals of the switches u_i and $\overline{u_i}$ must be complementary in nature, when one is passing the other is blocked. Thus, a cell of commutation can present only by two states. Conventionally the switching cell is set to state 1 when the switch u_i is in the passant state. In the same way it is said in the state 0 when the same switch is blocked. The switching cell can be considered as a binary system. Table (Tab.3.1) characterize of a switching cell.

Table 3.1: Elementary switching cells principle.

State	u_i	$\overline{u}_i$	V_o
1	Passing	Bloked	E
0	Bloked	Passing	0

Basic structure of a multicellular series chopper begins with the association of two switching elementary cells , in our case we used two cells and three cells as an illustrative examples. Since the cell can be considered as a binary system, the association of two -cells gives 2^2 states, and three- cells gives 2^3 states whose main characteristics are summarized in the next chapter.

3.2.2 Instantaneous Model

The assumptions used for the implementation of the Instantaneous model at the multicellular converter are

- The switches are ideal (saturation voltage, leakage current and zero switching time).
- Dead time is zero (the switches are considered perfect).
- The switches of the same switching cell operate in a complementary way.
- The values of the floating capacitors are such that the voltages at their terminals are constant over a decoupling period.
- The load current i_L is constant over a decoupling period and corresponds to the average value of this one over this same period.
- The supply voltage E is constant.

From the (Fig.3.1) one can write the evolution of voltage capacitors equation in kth cell as follows

$$\frac{dv_{c_k}}{dt} = \frac{u_{k+1} - u_k}{c_k} i_L \quad , k = 1, ..., n-1 \tag{3.1}$$

The equation that describes the load current is as follows

$$\frac{di_L}{dt} = \frac{-R}{L} i_L + \frac{E}{L} u_n + \frac{u_k - u_{k+1}}{L} v_{c_k} \tag{3.2}$$

The output voltage for this topology is depending on the cell state and it is given by

$$V_0 = \frac{u_k - u_{k+1}}{L} v_{c_k} \tag{3.3}$$

Hence, the instantaneous model of the converter is described by the following system of differential equations:

$$\begin{cases} \frac{dv_{c_k}}{dt} & = \frac{u_{k+1} - u_k}{c_k} I_L, i = 1, ..., n-1 \\ \frac{dI_L}{dt} & = \frac{-R}{L} I_L + \frac{E}{L} u_n + \frac{u_k - u_{k+1}}{L} v_{c_k} \end{cases} \tag{3.4}$$

This model can also be written in the following compact form:

$$\dot{x} = A(u)x + B(u) \tag{3.5}$$

with

$$A(u) = \begin{pmatrix} 0 & ... & 0 & \frac{u_2 - u_1}{C_1} \\ \vdots & \ddots & \vdots & \vdots \\ 0 & ... & 0 & \frac{u_p - u_{p-1}}{C_1} \\ \frac{u_2 - u_1}{L} & ... & \frac{u_p - u_{p-1}}{L} & \frac{-R}{L} \end{pmatrix}, \quad B(u) = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \frac{Eu_p}{L} \end{pmatrix}$$

Where the continuous state vector $x = [v_{c_1} \quad ... \quad v_{c_p - 1} \quad i_L]^T$ corresponds to the capacitor voltages and the load current. The vector $u = [u_1 \quad ... \quad u_p]^T$ represents the applied control input and it corresponds to the state of the cells. Moreover,it can be rewritten as a nonlinear affine model :

$$\dot{x} = f(u)x + g(x)u \tag{3.6}$$

with

$$g(x) = \begin{pmatrix} \frac{-i_L}{C_1} & \frac{-i_L}{C_1} & 0 & 0 & ... & 0 & 0 \\ 0 & \frac{-i_L}{C_2} & \frac{-i_L}{C_2} & 0 & ... & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & ... & ... & ... & ... & \frac{-i_L}{C_{p-1}} & \frac{i_L}{C_{p-1}} \\ \frac{v_{c_1}}{L} & \frac{v_{c_2} - v_{c_1}}{L} & \frac{v_{c_3} - v_{c_2}}{L} & ... & ... & \frac{v_{c_{p-1}} - v_{c_{p-2}}}{L} & \frac{E - v_{c_{p-1}}}{L} \end{pmatrix}, \quad f(x) = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ -\frac{R}{L} i_L \end{pmatrix}$$

3.2.3 Average Model

The main idea is to replace the system variables by their average values over a switching period, which must be too small compared to the time constants of the system. Hence, the obtained model is continuous and allows the possibility of several linear and nonlinear controllers design.

Let us denote $X(t) =< x(t) >= [v_{c_1} \quad ... \quad v_{c_p-1} \quad i_L]^T, U(t) =< u(t) >= [U_1 \quad ... \quad U_p]^T$ where $< x(t) >$ and $< u(t) >$ are the average values of the state and the input over a switching period T, defined as below:

$$\begin{aligned} < x(t) > &= \frac{1}{T}\int x(\tau)d\tau \\ < u(t) > &= \frac{1}{T}\int u(\tau)d\tau \end{aligned} \tag{3.7}$$

Therefore, the average model of series multicellular converter is given by:

$$\dot{X} = A(u)X + B(U) \tag{3.8}$$

with

$$A(u) = \begin{pmatrix} 0 & \dots & 0 & \frac{U_2-U_1}{C_1} \\ \vdots & \ddots & \vdots & \vdots \\ 0 & ... & 0 & \frac{U_p-U_{p-1}}{C_1} \\ \frac{U_1-U_2}{L} & ... & \frac{U_{p-1}-U_p}{L} & \frac{-R}{L} \end{pmatrix}, \quad B(U) = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \frac{EU_p}{L} \end{pmatrix}$$

3.2.4 Switched affine Model

From (Fig.3.1) we observer that there are both contionus varaibles (load current and capacitors voltages) and discrete variables (boolean input). Thus ,the converter has a hybrid behaviour and therfore we can derive a hybrid model. The switched affine model for the converter is given as follows

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)} \\ y(t) = C_{\sigma(t)}x(t) \end{cases} \tag{3.9}$$

Where $x = [v_{c_1} \dots v_{c_{n-1}} i_L]^T$ is the vector state, y is the output of the system. $\sigma(t)$ is the switching signal that takes its values in a finite set of discrete modes i.e. $\sigma(t) \in Q = \{q_1, q_2, \dots, q_N\}$ where $N = 2^n$. The system matrices are completely depending on the switching signal. For $\sigma(t) = q_k$, they are as follows:

$$A_{q_k} = \begin{pmatrix} 0 & \dots & 0 & \frac{u_2-u_1}{c_1} \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & \frac{u_n-u_{n-1}}{c_{n-1}} \\ \frac{u_1-u_2}{L} & \dots & \frac{u_{n-1}-u_n}{L} & -\frac{R}{L} \end{pmatrix}, B = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ -\frac{E}{L}u_n \end{pmatrix}, C = \begin{pmatrix} 0 & \dots & 0 & 1 \end{pmatrix} \quad (3.10)$$

The choice of the measured output of the converter is justified by some practical reasons. The index of the discrete state is given by [Gazzam and Benalia, 2018]

$$j = 1 + \sum_{k=0}^{n-1} 2^k u_k^j$$

The developed switched affine model given is widely used for different purposes such that: the study of the structural properties as observability, control, observer design and fault diagnostic. In our work, we will use this model to study almost the mentioned purposes above using some interesting dynamical and matrix properties of the converter.

3.3 Properties and Control Strategies

For converter safety, which corresponds to an equilibrated distribution of the input voltage among the p cells, the capacitor voltages must be regulated around the following average references

$$v_{ref_k} = k\frac{E}{p}; \quad k \in Ip - 1 \quad (3.11)$$

In this case, the voltage across each cell in average is equal to $\frac{E}{p}$ and the output voltage has $(p+1)$ level $(0, \frac{1}{pE}, \dots, \frac{(p-1)}{pE}, E)$. In order to maintain the capacitor voltages around their equilibrium values, the average value of the currents through them should be zero, that is:

$$< I_{Cki} >=< u_{k+1} - u_k > I_L = 0$$

Assume that the load current constant over a switching period, one can get

$$< I_{C_{k_i}} >=< u_{k+1} - u_k >= 0$$

Therefore, we must require identical average values of cells signals in steady state. The average value of the latter is known as the duty cycle, denoted α :

$$\alpha =< u_1 >=< u_2 >= \dots =< u_p > \quad (3.12)$$

In fact, the duty cycle value depends on the desired load current. In steady state (identical voltages across switching cells), the average value of the output voltage has the following reference value

$$v_0 = \alpha E$$

From the load current dynamic, one can get

$$\alpha = R\frac{i_{Lref}}{E} \tag{3.13}$$

The remaining degree of freedom corresponds to the phase shift between the cells signals. Using a harmonic model of the converter, the authors in [Amet et al., 2011]. have proved that a regular phase displacement given below allows the cancellation of the $(p-1)$first harmonics, which present the highest values and are the most difficult to filter.

$$\phi_k = (k-1)\frac{2\pi}{p} \tag{3.14}$$

In this case, we restate some interesting properties [Meradi et al., 2013] of a multicellular converter topology:

- ***Property 1***. The output voltage ripple is divided by p and the output voltage has $p+1$ level $(0, \frac{1}{pE}, \dots, \frac{(p-1)}{pE}, E)$
- ***Property 2***. if the duty cycle $\alpha \in [\frac{k-1}{p}, \frac{k}{p}[$, then the output voltage will be between $(k-1)\frac{E}{p}$ and $k\dfrac{E}{p}$ over a switching period.
- ***Property3***. The apparent frequency of the output voltage is multiplied by p.

In summary, the control objectives of a series multicellular converter are: stabilization of the load current i_L around its desired state I_{ref}. For converter safety, the capacitor voltages must be fast regulated around their references v_{ref_k} . For optimal operation of the converter, the controller should guarantee the above properties during steady state where the phase shift between successive cells signals has to be equal to $\dfrac{2\pi}{p}$.

3.3.1 Control of Multicellular Converter :Open-Loop

The multicellular converter is controlled in open loop when Feedback loop is not used. Several control of this kind have been made. We can cite the example where the signal control of the semiconductor components are given by the PWM (Pulse Width Modulation) strategy, the current and output voltage are delivered to themselves [Gateau, 1999].

1. **Pulse-Width-Modulation Strategy**

The objective of this strategy is to jointly control the load current and the capacitor voltages across each commutation cell. The idea is to generate p binary signals with phase shift $\frac{2\pi}{p}$ duty cycles are computed and allow to stabilize the load current and the voltages of the floating capacitors around the desired values. The control signals of each k cell, in the case of the natural PWM, are generated by the intersection of a triangular carrier and the modulating signal (constant in the case of a chopper). In addition, the carriers are all regularly phase shifted together by an angle α.

The phenomenon of natural balancing of the floating voltages allows the multicellular converter to operate in open loop, without any asservisement of these.

3.3.2 Control of Multicellular Converter :Close-Loop

As we have seen in the previous section, it is possible to control the multicellular converter in open-loop, benefiting from natural balancing of the floating capacitors. However, for applications requiring a greater rebalancing dynamic, different closed-loop control strategies have been developed, we can summary by:

- In [Gateau et al., 2002] a nonlinear control has been investigated based on input/output linearization. In order to generate the cells signals, a pulse width modulator(PWM) is used where the basic idea is to compare the average control signals U issue from the feedback linearization control with p carriers regularly out of phase by $2\pi/p$. The function that can generate these carriers is given by

$$p_k = \frac{1}{2}[\frac{2}{\pi}arcsin(sin(\frac{2\pi}{p}f_s t - \phi_k - \frac{\pi}{2}) + 1)] \quad k \in I_p \tag{3.15}$$

  The robustness of this control scheme has been validated through simulations and experimental tests. However, it has some limitations due to the average model used for controller design. The latter may generate average input signals that does not belong to the interval [0 1]. Thus, a saturation function must be added, which limits the closed loop performances.

- A hybrid predictive control law has been investigated in [Trabelsi et al., 2008] for a three-cells converter, which determines the next discrete mode minimizing the distance between the predicted state ($\tilde{x}$) and the desired state. For real-time constraint, a simplified model of the converter has been used for prediction

$$\tilde{x}(k+1) = x(k) + T_s \dot{x}(k) \tag{3.16}$$

with T_s is the sampling period. During each sampling period, the control strategy proceeds as follows:

- Measure capacitor voltages ($v_{c_j}(k), j \in I_{p-1}$) and load current ($i_L$).
- Predict the state vector $\tilde{x}(k+1)$ using (3.11) for each discrete mode.
- For a desired state $x_{ref} = [v_{ref_1} \dots v_{ref_{p-1}} \;\; I_{ref}]^T$, the control selects the discrete mode which gives the smallest distance between $x(k+1)$ and x_{ref}.

This strategy is efficient and allows a good tracking compared to a classic PWM control scheme. However, no stability proofs are provided and the steady state behaviour is not considered[Benmiloud.M,2016].

- In [Patino et al., 2010], a predictive controller has been developed to guarantee a fast convergence to steady state and ensure a good tracking of an optimized limit cycle. The latter represents the best cycle in the sense given by a criterion, which can be defined for reducing the quadratic norm of the tracking error or to minimize the harmonic content in output waveforms. Different optimized solutions are learned through a neural network that yields a state feedback control. This approach combines the performance of PWM control techniques and direct control strategies. However, extensive computations make this strategy not straightforward. Besides, no proof for the stability of the desired limit cycle has been done.

- Another approach based on state space partition has been developed in [Kamri et al., 2012] by solving a bilinear matrix inequality to guarantee a practical stability of a desired state of a switched system. The objective is to assign for each discrete mode a quadratic region. The investigated results have been applied to a multicell converter where simulation results show the effectiveness of the approach. This control scheme has the advantage to be a general systematic method for the practical stabilization of switched systems. However, the steady state behaviour is not controlled, which corresponds to the main operation of the converter and judges its optimal operation depending on the harmonics in the output waveforms (output voltage and load current)[Benmiloud.M,2016].

The authors in [Van Gorp et al., 2011] have proposed a direct control method using a sliding mode approach. Under this controller, the multicellular converter supplies the load current i_L as close as possible to the desired load current Iref and ensures an equilibrated distribution of the input voltage across each cell. In fact, these results are obtained by requiring the time derivative of the candidate Lyapunov function to be negative. Hence, the stability of the system is fulfilled. This approach is quiet simple and does not require extensive computations. It guarantees a fast convergence to the desired state with a direct decision of the cells states (no PWM is needed). However,no limitation on the switching frequency is considered (theoretically it is infinite).

3.4 Observability Analysis

In general,the observation of state variables in power converters is not interested in the literature. Because these variables are generally accessible to the measure and do not justify the presence of a heavy observer in calculation. With the advent of multicellular converters, it was seen that the number of variables to be known was greater and increased proportionally to the number of switching cells. In this case, an observer of the floating voltages is totally justifiable. Since it eliminates sensors, often expensive and fragile, and thus reduce the cost and size of the installation. However, before attempting to build an observer, it is necessary to ensure its feasibility, by testing the observability of the converter. The first paper addressing the observability of floating voltages of the multicellular converters in 2001 by Bensaid [Bensaid, 2001]. it was developed two discrete model based on instantaneous and averge model, to illustrate this method, we take three-cells converter as an exemple.

3.4.1 Observability Analysis based Classical Approch

For the three-cells converter connected to a load $R-L$, the instantaneous model is defined by

$$\begin{cases} \dot{v}_{C_1} &= \frac{1}{C_1}(u_2-u_1)i_L \\ \dot{v}_{C_2} &= \frac{1}{C_1}(u_3-u_2)i_L \\ \dot{i}_L &= -\frac{R}{L}i_L - \frac{1}{L}(u_3-u_2)v_{c_2} - \frac{1}{L}(u_2-u_1)v_{c_1} + \frac{1}{L}Eu_3 \end{cases} \tag{3.17}$$

The observability matrix of the system is given by

$$O = \begin{pmatrix} 0 & 0 & 1 \\ \frac{1}{L}(u_1 - u_2) & \frac{1}{L}(u_2 - u_3) & -\frac{R}{L} \\ -\frac{R}{L^2}(u_1 - u_2) & -\frac{R}{L^2}(u_2 - u_3) & \frac{R^2}{L^2} - \frac{1}{LC_1}(u_1 - u_2)^2 - \frac{1}{LC_2}(u_2 - u_3)^2 \end{pmatrix} \tag{3.18}$$

The obteined observability matrix of the converter shows that the last row is a linear combination of the preceding lines. Than, the Kalman test is not checked $rank[O] = 2 < 3$ as well as the converter is unobservable. For this purpose, a good model representation of the system must be adopted.

1- Sampled Exact Model (SEM)

Discrete-time model based on a state average technique is obtained for a three-cells converter. The main idea of this model is to solving the system of equation (3.17) for each of these sequence $(sej =, j = 1, ...7)$(Fig.3.3). We thus obtain seven relations describing the evolution of the state x at time t_{j+1} as a function of the state at time t_j. Noting by S_j the value taken by the input during the sequence j, and $t_j = t_{j+1} - t_j$ by the duration of this sequence, we obtain

$$x(t_{j+1}) = F_j x(t_j) + G_j E, \quad j = 1, \ldots, 7 \tag{3.19}$$

With

$$\begin{cases} F_j & = e^{A(S^j)\Delta t_j} \\ G_j & = [\int_{t_j}^{t_{j+1}} (e^{(A(S^j)(t_{j+1} - \tau))} d\tau)].B(S^j) \end{cases} \tag{3.20}$$

Then the exponential matrix in (eq.3.19) can be approximated to the third order (Taylor series development)

$$\begin{cases} F_j & \approx I + A(S^j)\Delta t_j + \frac{1}{2}A^2(S^j)\Delta t_j^2 + \frac{1}{6}A^3(S^j)\Delta t_j^3 \\ G_j & \approx (I + \frac{1}{2}A(S^j)\Delta t_j + \frac{1}{6}A^2(S^j)\Delta t_j^2)B(S^j) \end{cases} \tag{3.21}$$

The sampled exact model over T_d is then obtained by

$$x(k+1) = F(\alpha)x(k) + G(\alpha)E(k) \tag{3.22}$$

With

$$\begin{cases} F(\alpha) = \prod_{j=1}^{7} F_j \\ G(\alpha) = \prod_{l=1}^{7} (= \prod_{j=l+1}^{7} F_j).G_l \end{cases} \tag{3.23}$$

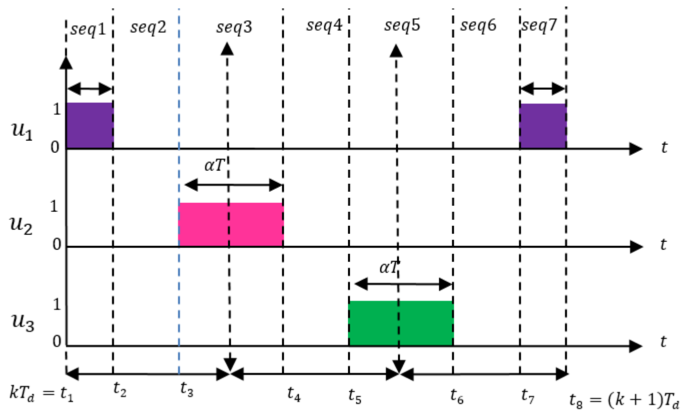

Figure 3.3: Control signals respresentation and their exact model over one switching period.

The sampled exact model cannot be implemented in real time from its complexity of implementation and the important size of computation that it requires. Indeed, once the matrices F and G have been determined, it is necessary to calculate in real time seven matrix products to determine only the matrices F. One can plan to use simple model based on average values over one-third of switching period is presented in the next subsection.

2- Sampled Average Model (SAM)

Discrete-time model based on a state average technique is obtained for a three-cell converter. In the following, the main idea is to replace the instantaneous variables in(3.1)by their average values over a switching period. In this model, we obteined the averge of the

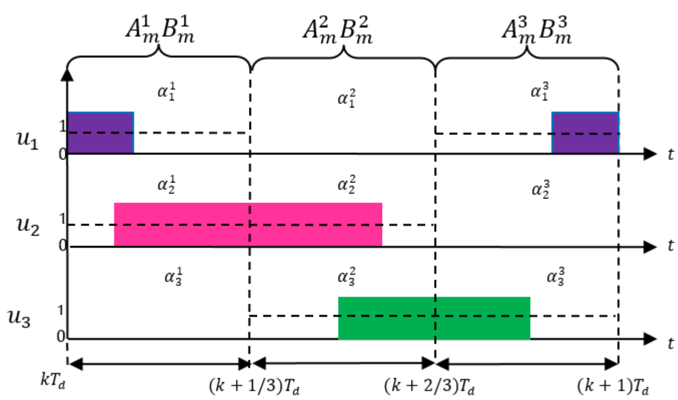

Figure 3.4: Control signals respresentation and their average model over one switching period.

input u_i in each interval $t \in [(k+(j+1)/3)T, (k+j/3)T]$ by [Bensaid, 2001]

$$u_i^j = \frac{3}{T}\int_{\frac{j-1}{3}}^{\frac{j}{3}T} u_i dt, i, j = \{1, 2, 3\} \tag{3.24}$$

Than, we obteined p continuous equations over a switching period given by

$$< \dot{x} >^j = A_m^j < x >^j + B_m^j E, j = \{1, 2, 3\} \tag{3.25}$$

With

$$A_m^j = \begin{pmatrix} 0 & 0 & \cdots & 0 & u_1\delta_1^j \\ 0 & 0 & \cdots & 0 & u_2\delta_2^j \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & u_{p-1}\delta_{p-1}^j \\ -\frac{1}{L}\delta_1^j & -\frac{1}{L}\delta_2^j & \cdots & -\frac{1}{L}\delta_{p-1}^j & -\frac{R}{L} \end{pmatrix}, B_m^j = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \frac{U_p^j}{L}E \end{pmatrix}$$

These equations are then integrated on their interval respectively. We obtiend than $p = 3$ recurrent equations represent the sampled average model at $T/3$

$$x(k + \frac{j}{p}) = F_m^j x(k + \frac{j-1}{p}) + G_m^j E(k), j = \{1, 2, 3\} \tag{3.26}$$

with

$$F_m^j = e^{A_m^j \frac{T}{p}} = I + A_m^j \tfrac{T}{p} + \tfrac{1}{2}(A_m^j)^2(\frac{T}{p})^2$$
$$G_m^j = (I \tfrac{T}{p} \tfrac{1}{2}(A_m^j)(\frac{T}{p})^2) B_m^j$$

But this solution leads to a heavy computational load and consequently remains complex for a real-time implementation. the Sampled average model at T given by (Fig.3.5)

$$x(k+1) = F_m(u)x(k) + G_m(u)E(k), \quad k \in N \tag{3.27}$$

With

$$F_m(u) = \prod_{j=1}^{p} F_m^j = F_m^p F_m^{p-1} ... F_m^1$$
$$G_m(u) = \sum_{i=1}^{p} (\prod_{j=i+1}^{p} F_m^j) G_m^i$$

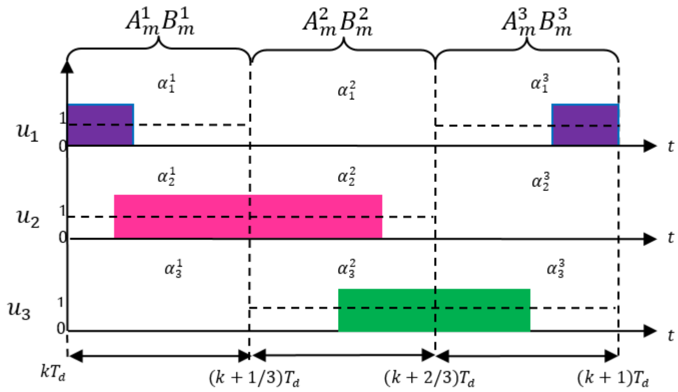

Figure 3.5: Control signals respresentation and their exact model over one switching period.

3-Static Method

This approach is not directly an approach to treat the observability of the multicellular converter as a non-linear system, but it has allowed the formulation of a method allowing the reconstruction of the floating capacitor. This approach being static, we consider a fixed control u. Therefore, whatever the chosen control, the multicellular converter becomes an affine system in the state.

For affine systems it suffices to look at the rank of the observability matrix O defined by

$$O = [C \ \ CA(u) \ \ \dots \ \ CA^{p-1}(u)]^T$$

If the rank of O is equal to the size of the system, then the system is observable for the control u considered. We can easily notice that, whatever the order of O, rank is at most two. The system is therefore not observable. Using the rank condition, we get again a rank equal to two. From a static point of view, the load current and the voltage across the capacitors are therefore simultaneously observable. Rewrite the system (3.17) in the form

$$\begin{cases} \dot{i_L} & = -\frac{R}{L} i_L + \frac{E}{L} u^p + V_s \\ \dot{V_s} & = -\sum_{j=1}^{p-1} \frac{i_L}{Lc_j} (u^{j+1} - u^j)^2 \end{cases} \tag{3.28}$$

[Gateau, 1999] proposed to find the values of floating capacitors voltages v_{c_j} by measuring the voltage Vs over a sufficient time interval

$$V_s = -\frac{1}{L}\left[u_i^2 - u_i^1 \quad \dots \quad u_i^p - u_i^{p-1}\right]\begin{bmatrix} v_{c_1} \\ \vdots \\ v_{c_{p-1}} \end{bmatrix} \tag{3.29}$$

Remark 3.1 As a hybrid approaches applied to the observability of multicellular converter, The authers in [Benmansour, 2009, Van Gorp et al., 2012] applied the theory of the $Z(T_N)$-observability and observability by the determination of the unobsrvable subspace to verified the observability condition. In the next chapter we demontrate a simple condition to verified the observability based on the unobsrvable subspace method.

3.5 Observer desgin

Several observers were designed to estimate capacitor voltages for the multicellular converter (Three-cells), We can recall the most important.

3.5.1 Sampled Observer from Luenberger

In [Bensaid et al., 1999], Using the exact sampled model, the observer equation of Three-cells converter is written as

$$\begin{cases} \hat{x}(k+1) = (F(\alpha) - L(\alpha)C)\hat{x}(k) + G(\alpha)E(k) + L(\alpha)i_L(k) \\ \hat{y}(k) = C\hat{x}(k) \end{cases} \tag{3.30}$$

Where
$x(k) = [v_{c_1}(k) \;\; v_{c_2}(k) \;\; i_L(k)]^T, C = [0 \;\; 0 \;\; 1], L(\alpha) = [L_1(\alpha) \;\; L_2(\alpha) \;\; L_3(\alpha)]^T$
Figure(Fig.3.6) shows the circuit diagram of the observer. In open loop, the duty cycles applied to the converter are constant and the equations of the observer are stationary. The Ackermann formula can be used to calculate the gain vector $L(\alpha)$. In the Figures(Fig.3.7),(Fig.3.8) we show the simulation results obtained with adynamic triple pole $(z_1 = z_2 = z_3 = 0.92)$ it has been noting that the estimated voltage converges towards the real voltage in the steady state not in the stationary state, because of calculation drawing their from the discritization of model and the use of SEM over switching period T_d.

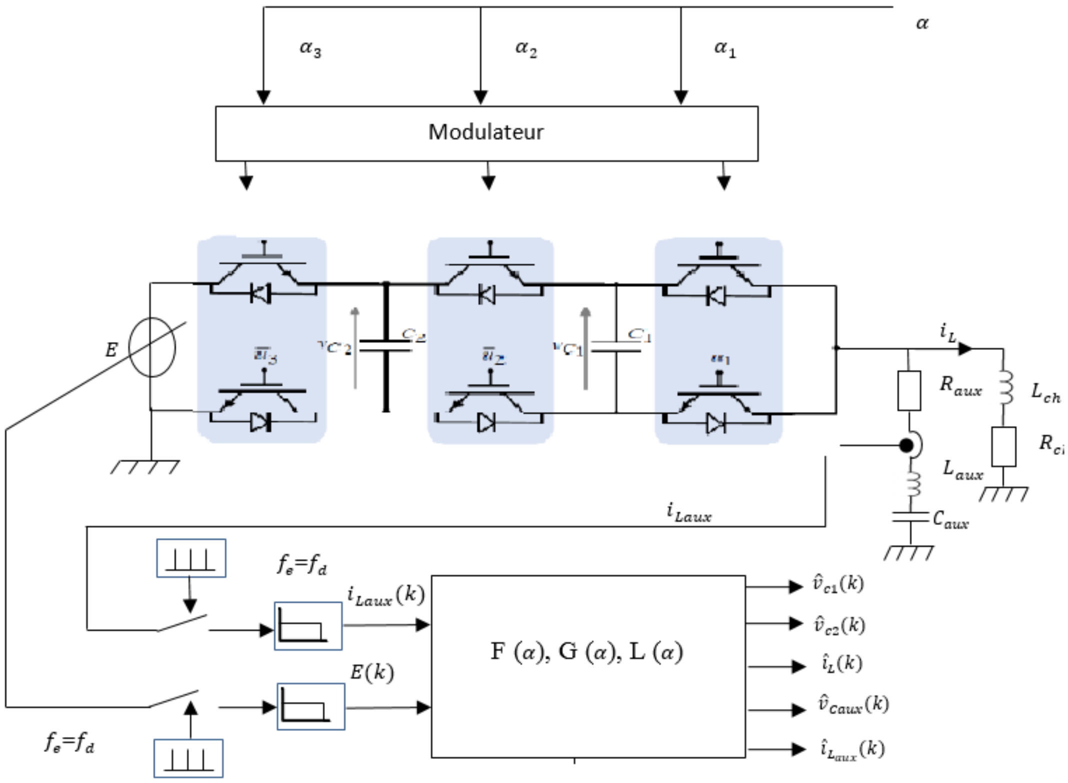

Figure 3.6: Sampled Luenberger observer scheme of Three-cells converter.

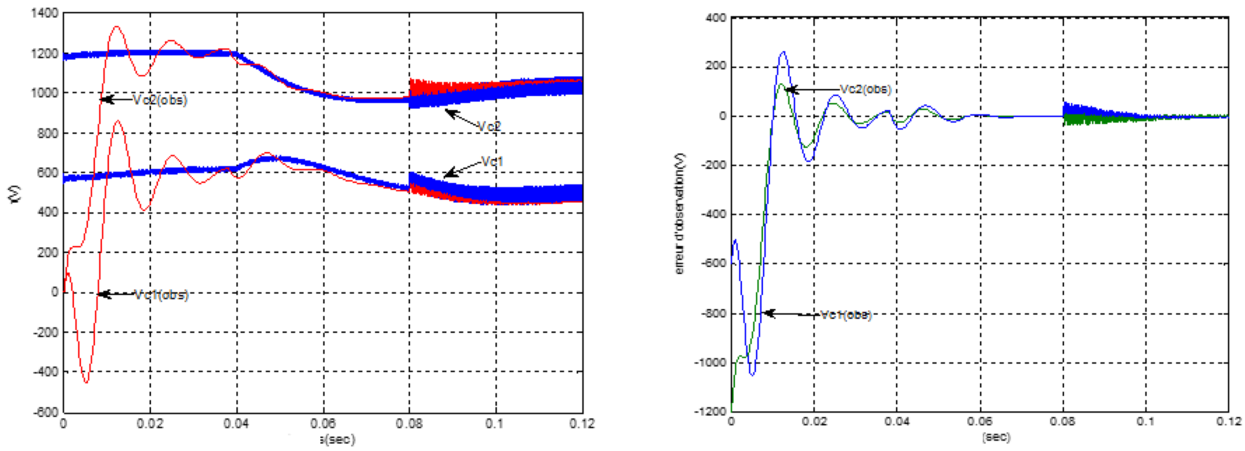

Figure 3.7: Floating voltages of Three-cells converter.

3.5.2 Discrete-Kalman Filter

One of the first observers applied to the multicellular converter is a recursive Kalman filter in discrete time presenred in [Bensaid et al., 1999]. This one is based on the use of a samlped average model over a $T/3$ interval (presented in section 3.4.1). Using a PWM

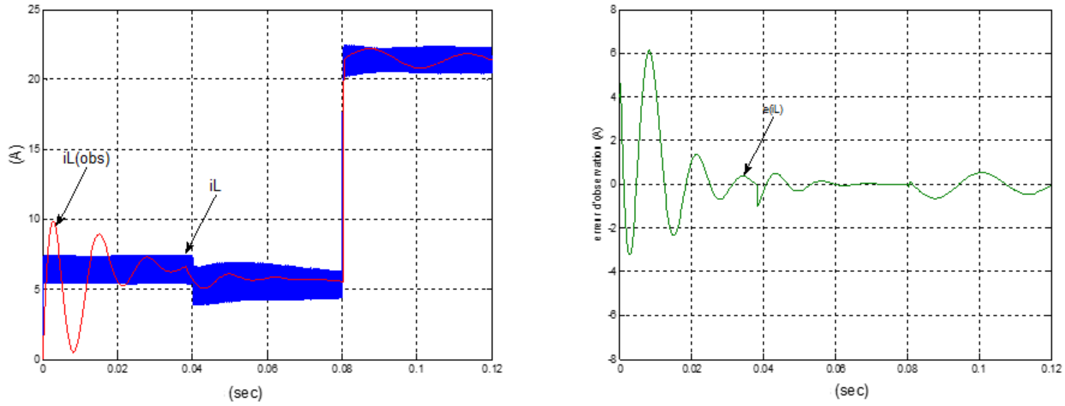

Figure 3.8: Load current of Three-cells converter.

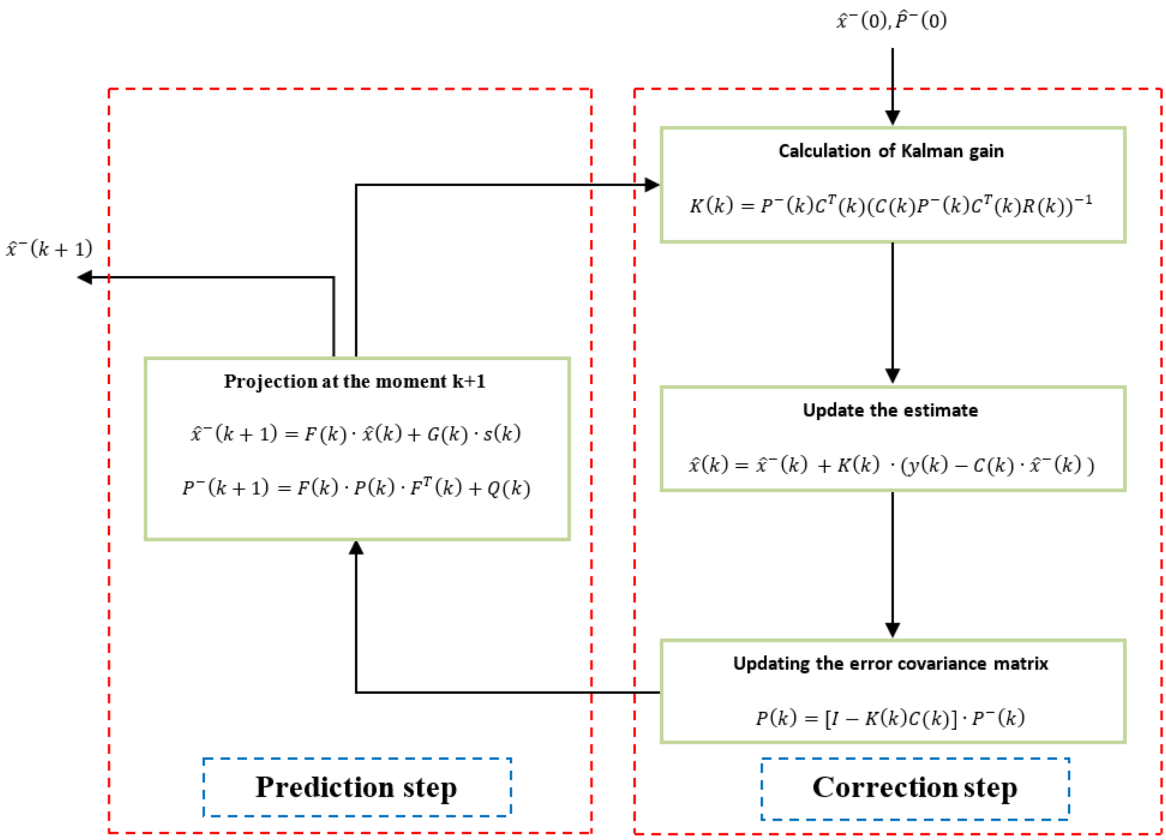

Figure 3.9: Discrete-Kalman filter algorithm.

control, the observer has good performance with respect to noise as well as parametric uncertainties (fig(3.10),fig(3.11)). No guarantee is formulated as to the stability of the closed loop. A second application of the Kalman filter on the multicell converter is done in [Aguilera and Quevedo, 2009], This is based on a model in discrete time of the converter. A predictive control is used to lead the system to the given reference. However, the observability condition is not discussed.

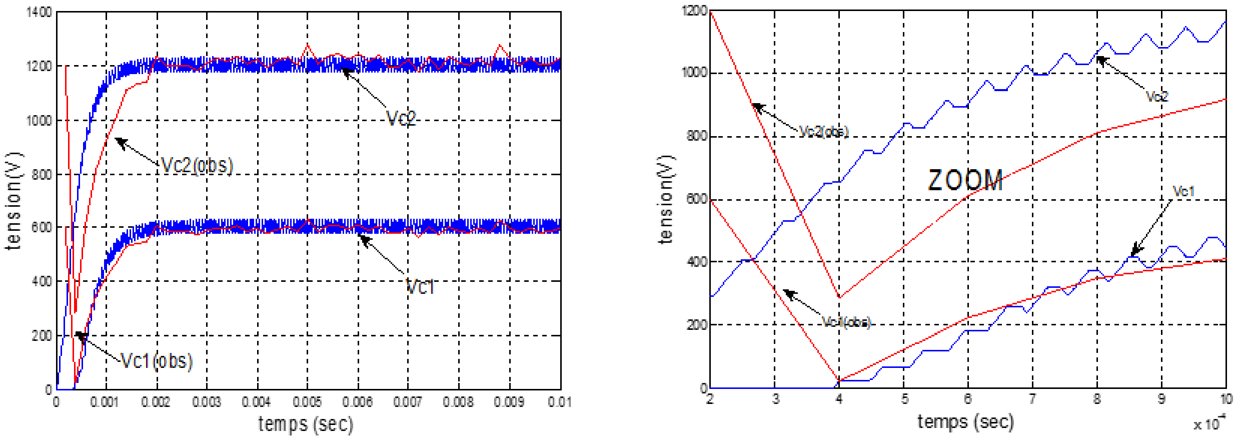

Figure 3.10: Floating capacitor voltage and its estimeted.

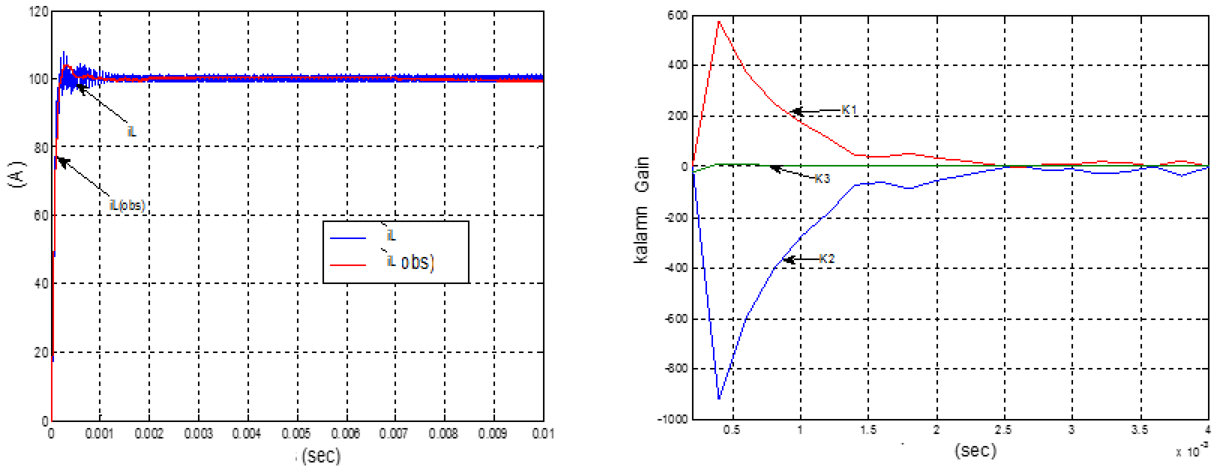

Figure 3.11: Load current variation and Kalman Filter gains.

3.5.3 Switched- Luenberger Observers

The basic idea of this observer is to use the partial information obtained from different modes of the system in order to estimate the floating voltages. This hybrid observer is based on the geometric approach developed by [Tanwani et al., 2011]. He has the interest important not to require the explicit knowledge of a minimum stay of modes, unlike other observers seen in this section. The application of this observer in the case of the multicellular converter is developed by [Van Gorp et al., 2012]. The hybrid observer's equations are as follows:

$$\begin{cases} \dot{\hat{x}}(t) = A(u_{k-1})\hat{x}(t) + A(u_{k-1}) \\ \hat{x}(t) = \hat{x}(t_k^-) - \epsilon(u_{k-1}, t_k^-) \end{cases} \tag{3.31}$$

This observer consists of two equations. The first gives the dynamics of the system that one seeks to observe while the second ensures the convergence of the error estimation. This second equation worked as a reset to the estimated vector: every switching, a little more information is known about the state and is obtained by a Luenberger type observer to ensure the convergence of the error.

3.5.4 Adaptive-gain Second-Order Sliding Mode Observer(SOSM)

In 2014,A novel adaptive-gain SOSM observer was presented in [Liu et al., 2014] for the multi-cell power converter system, which belongs to a class of switched systems. With the use of $Z(_N)$-observability, the capacitor voltages were estimated under a certain condition of the input sequences. The proposed adaptive-gain SOSM algorithm combines the nonlinear term of the super-twisting algorithm and a linear term, the so-called SOSML algorithm. The proposed observer for the three-cells converter is formulated as

$$\begin{cases} \hat{i}_L = -\frac{R}{L} i_L - \frac{u_1}{L}\hat{v}_{c_1} - \frac{u_2}{L}\hat{v}_{c_2} + \frac{u_3}{L}E + \mu(e_1) \\ \hat{v}_{c_1} = \frac{u_1}{C_1} i_L + k_1\mu(e_1) \\ \hat{v}_{c_2} = \frac{u_2}{C_2} i_L + k_2\mu(e_1) \end{cases} \tag{3.32}$$

Where $\mu(e_1) = \lambda(t)|e_1|^{\frac{1}{2}} sign(e_1) + \alpha(t)\int_0^t sign(e_1)d\tau + k_\lambda(t)e_1 + k_\alpha(t)\int_0^t e_1 d\tau$

The adaptive gains $\lambda(t), \alpha(t), k_\lambda, k_\alpha(t)$ and the design parameters k_1, k_2 are to be defined. The robustness of the proposed observer and the Luenberger switched observer were compared in the presence of load resistance variations and output measurement noise. It was found that the adaptive-gain SOSML observer is more robust. Two main advantages of the proposed method are: Only one parameter k has to be tuned; A-priori knowledge of the perturbation bounds is not required. but, From implementation point of view, the calculations required for the adaptive-gain SOSML are more intensive than those of Luenberger observer. However, the correction term and two design parameters $k1; k2$ entails low real-time computational , as the computational capabilities of digital computers have greatly increased and the additional processing requirements can be easily accomplished. Regardless of the number of cells, the complexity of the calculation increases linearly with the number of cells. This means that, for an n-cell system with $2n$, the additional computational burden comes only from the calculation of new parameters $k_3, \ldots, k_{n-1}$.

3.5.5 Finite Time Observers

A fourth class of observers is widely studied in the literature for multicellular converters, observers in finite time. Indeed, they own the advantage of a convergence in finite time, and thus to be able to guarantee a maximum time for the convergence of the observer. As previously, the reconstruction of floating voltages by the knowledge of the load current under the hypothesis that the system is observable, from that, we can find in the different literatures observers in finite time

- Sliding modes observer of order one [Bensaid et al., 1999, Benmansour et al., 2008, Haddadi and Atallah, 2013];
- Super-twisting observer [Bensaid, 2001, Ghanes et al., 2009, Djemaï et al., 2011];
- Finite-time observer [Defoort et al., 2010, Hauroigne et al., 2012] .

1- Sliding Mode Observer

In this strategy, we will use the sliding mode theory to observe the floating voltage of the multicellular converter. The sliding mode observer of three-cells converter given by

$$\begin{cases} \dot{\hat{v}}_{C_1} = \frac{u_2-u_1}{c_1}\hat{i}_L - \Lambda_1 sign(i_L - \hat{i}_L) \\ \dot{\hat{v}}_{C_1} = \frac{u_3-u_2}{c_2}\hat{i}_L - \Lambda_2 sign(i_L - \hat{i}_L) \\ \dot{\hat{i}}_L = -\frac{R}{L}\hat{i}_L - \frac{u_2-u_1}{L}\hat{v}_{C_1} - \frac{u_3-u_2}{L}\hat{v}_{C_2} + \frac{u_3}{L}E - \Lambda_3 sign(i_L - \hat{i}_L) \end{cases} \tag{3.33}$$

The Figure(Fig.3.12) shows the diagram of sliding mode observer of three-cells converter. Indeed, it is well known that first order sliding mode exhibits chattering phenomena which is undesirable in practice. When using second order sliding mode observer, based on super twisting observer, we will remove the chattering phenomena since the sign function acts on the derivative and we maintain the robustness properties of first order sliding mode sliding mode observer of three-cells converter .

2-Super-Twisting Observer

A Super-twisting observer has been designed for a multicellular converter. The problem of the capacitor voltages estimation is solved.In the case of three-cells the observer given

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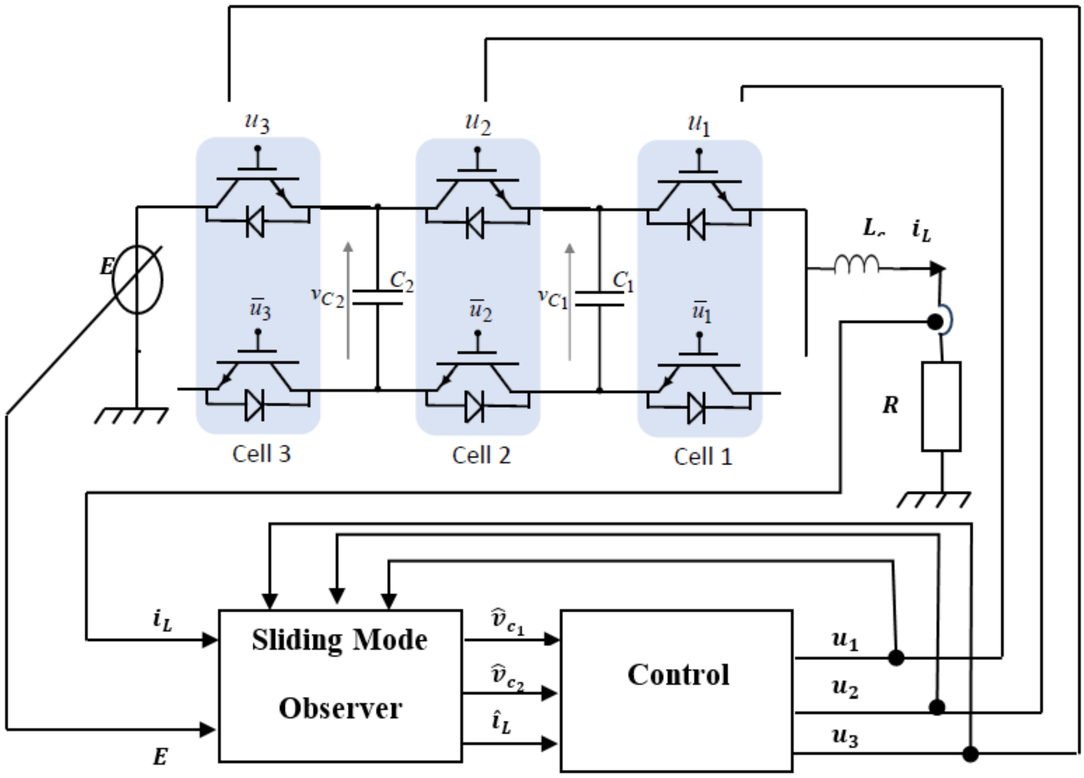

Figure 3.12: Blok diagram of sliding mode observer of three-cells converter.

by

$$\begin{cases} \dot{\hat{v}}_{C_1} = \frac{u_2-u_1}{c_1}\hat{i}_L - \alpha(u2-u1)sign(i_L-\hat{i}_L) \\ \dot{\hat{v}}_{C_1} = \frac{u_2-u_3}{c_2}\hat{i}_L - -\alpha(u3-u2)sign(i_L-\hat{i}_L) \\ \dot{\hat{i}}_L = -\frac{R}{L}\hat{i}_L - \frac{u_2-u_1}{L}\hat{v}_{C_1} - \frac{u_3-u_2}{L}\hat{v}_{C_2} + \frac{u_3}{L}E - \lambda\sqrt{|i_L-\hat{i}_L|}|u3-u2|sign(i_L-\hat{i}_L) \end{cases} \tag{3.34}$$

Where $\lambda = \dfrac{1+\theta}{1-\theta}\sqrt{\frac{2\alpha}{L}}0 < \theta < 1$

A Lyapunov function has been introduced in order to study the stability and properties of the proposed homogeneous observer.the simulations resuts of this observer is presented in (Fig.3.14).

Remark 3.1

- All the simulation results of the observer desgin for three-cells converter simulated under the same parameters of the converter(Tab.3.2).
- In the next chapter, for the comparative studies with the algebraic observer we

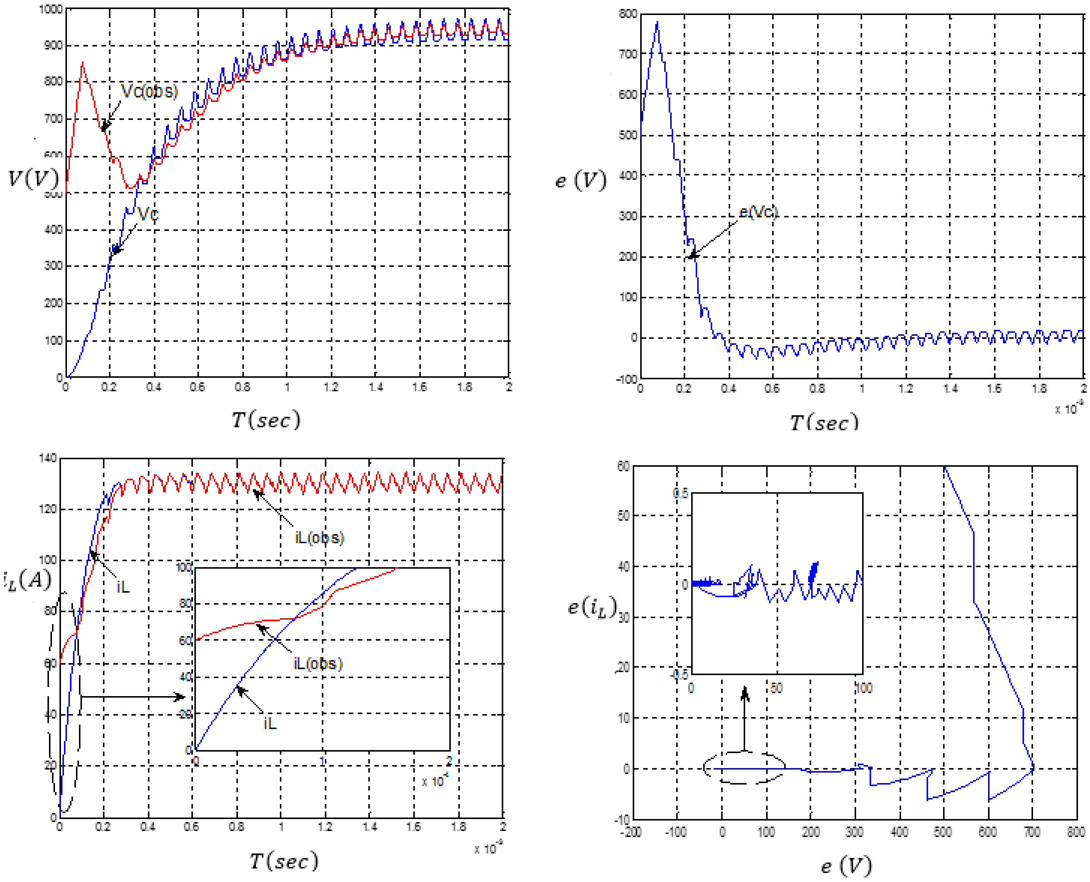

Figure 3.13: Floating voltages evolution of three-cells converter.

choose the super-twisting observer by contribution to the other observer presented above(Tab.3.3)

Table 3.2: Three-cells converter parameters.

Parameter	E	L	C	R
Value	$1500V$	$1mH$	$40\ \mu F$	10Ω

3.6 Comparative Study

Table (Tab.3.3) illustrates a comparative study between different proposed observers for series multicellular converter based on some criteria : the used model for observer design, observability cretaria, control method used if in Open-Loop or Closed loop, complexity of the method. The aim of the next chapters is to develop hybrid observer that achieve a

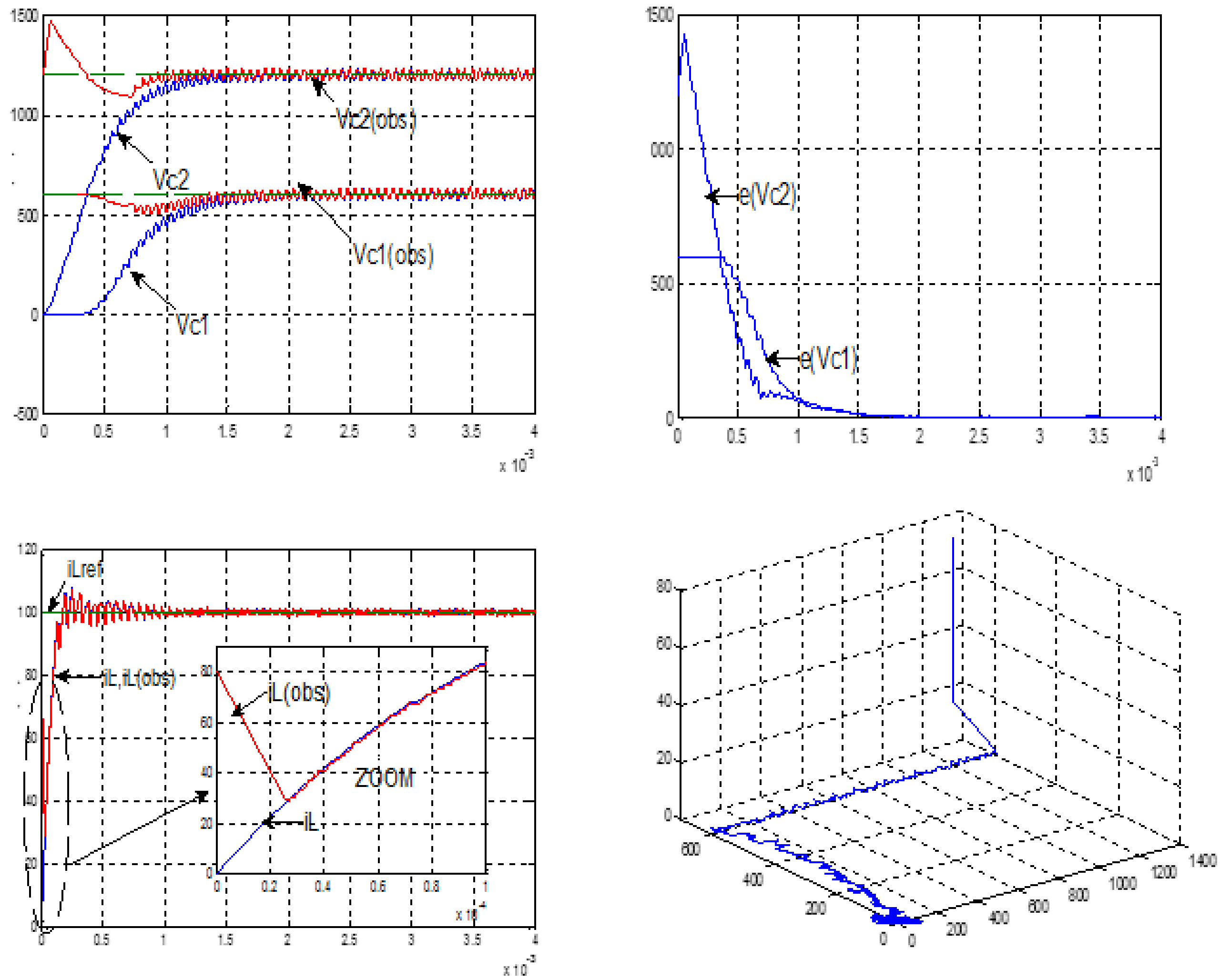

Figure 3.14: Floating voltage evolution of three-cells converter.

fast and robust convergence to the desired estimated state and guarantee an simple study in general case.

3.7 Conclusion

In this chapter, we have presented the multicellular converters topologies and their properties. One of their many advantages compared to classical converter is the reduction of the voltage across switches. This allows the use of faster switches which, in turn, guarantee high performance. The operating principle of the multicellular structure was described. We will exposed different observers existing in order to propose an algebraic observer to estimate the floating capacitors voltages.

Observers	**Sampled-O Luenberger**	**D-Kalman Filter**	**Switched Luenberger**	**Adaptive-G SOSM**	**Sliding Mode**	**Super Twisting**
Model	Instantaneous Model	Average Model	Switched Model	Switched Model	Instantaneous Model	Instantaneous Model
Observability Condition	SEM Approach	SAM Approach	Hybrid Approach	$Z(T_N)$	$Z(T_N)$	$Z(T_N)$
Control method	Nonlinear Decoupler Control	Nonlinear Decoupler Control	PWM	PWM	PWM	PWM
Complexity	High off-line computation	High off-line computation	Medium	Medium	Quite Simple	Simple

Table 3.3: Qualitative comparaison between different observer scheme for the multicellular converter.

Chapter 4

Observability and Observer Design of MC

> “*What you get by achieving your goals is not as important as what you become by achieving your goals.*”
>
> Henry David Thoreau

This chapter is devoted to the analysis of observability of the multicellular converter as well as the design an observer. The study in this chapter is strongly based on the dynamical properties (algebraic and geometric) presented in the last chapter. For the observability, we will use the techniques of the switched affine systems in order to give some interesting simplifications that will be used later. Then, we will design an algebraic observer that is based on the work of the second chapter. At the end of this chapter, an application with simulation to the two-cells and three-cells converter will demonstrate the importance in term of efficiency and robustness of the developed results of the general case of the converter.

4.1 Observability of Multicellular Converter

Let $SS = ((t_0, q_1), \dots, (t_{N-1}, q_N))$ be a switching sequence that guarantees the control objectives given in eq(3.11). The observability of the system (3.10) for the sequence SS

can be checked through the condition

$$Ker(O) = Ker(O_{SS}) = Ker \begin{pmatrix} O_{q_1} \\ O_{q_2} e^{A_{q_1}\tau_1} \\ \vdots \\ O_{q_N} e^{A_{q_{N-1}}\tau_{N-1}} \dots e^{A_{q_1}\tau_1} \end{pmatrix} = \{0\} \tag{4.1}$$

Where

$\tau_j = t_{j+1} - t_j$, $O_j = [C^T \ \ (CA_{q_j})^T \dots \ \ (CA_{q_j}^{n-1})^T]^T$ is the observability matrix of the mode q_j.

Note that the dimension of the matrix O_{SS} is $(nN \times n)$. From the dimension of O_{SS}, one can notice that the condition (4.1) becomes difficult to check for a high number of cells n or/and when the sequence SS has a high number of modes M. Moreover, the exponential matrices appear in can also make the observability condition hard to check. In this study, we will reduce the dimensional of the observability matrix as well as omit the exponential matrices appear in using some dynamical properties of the converter . [Liu et al., 2014] showed that the dimension of the observable subspace of any mode of the converter is at most 2, this means that all matrices O_j can be replaced by the following simplified observability matrix of the mode q_j

$$\bar{O}_j = \begin{pmatrix} C \\ CA_{q_j} \end{pmatrix} \tag{4.2}$$

From (4.1) and (4.2), the observability analysis of the sequence can be checked by the following condition,

$$Ker \begin{pmatrix} \bar{O}_{q_1} \\ \bar{O}_{q_2} e^{A_{q_1}\tau_1} \\ \vdots \\ \bar{O}_{q_N} e^{A_{q_{N-1}}\tau_{N-1}} \dots e^{A_{q_1}\tau_1} \end{pmatrix} = \{0\} \tag{4.3}$$

Although the dimension of the matrix observability of SS reduces from to $(nN \times n)$ to $(2N \times n)$ the observability condition(4.3) is not easy to verify due to the exponential matrices. In order to overcome this difficulty, we use the Sylvester's formula of the exponential matrix. We have for all square matrix $A \in R^{n\times n}$

$$e^{At} = \alpha_0(t)I + \alpha_1(t)I + \dots + \alpha_{n-1}(t)A^{n-1} \tag{4.4}$$

where $\alpha_j(t), j = \{1, \dots, n-1\}$ are real functions. For the multicellular converter, we can use the following lemma to simplify the Sylvester's formula 4.4.

Lemma 4.1

The exponential matrix of the mode q_j of the multicellular converter is given by:

$$e^{A_{q_j}t} = \alpha_0^j(t)I + \alpha_1^j(t)A_{q_j} + \alpha_2^j A_{q_j}^2 \tag{4.5}$$

Where $\alpha_0^j(t), \alpha_1^j(t)$ and $\alpha_2^j(t)$ are real functions. The simple formula of $e^{A_{q_j}t}$ comes from the fact that the degree of the minimal polynomial of A_{q_j} is 3, the matrix can be simplified as follows:

$$Ker\begin{pmatrix} \begin{pmatrix} C \\ CA_{q_1} \end{pmatrix} \\ \begin{pmatrix} C \\ CA_{q_2} \end{pmatrix}\pi_1 \\ \vdots \\ \begin{pmatrix} C \\ CA_{q_N} \end{pmatrix}\pi_{N-1} \end{pmatrix} = \{0\} \tag{4.6}$$

where

$\pi_j = \prod_{k=1}^{j-1}\alpha_0^k(t)I + \alpha_1^k(t)A_{q_k} + \alpha_2^k A_{q_k}^2$. From the condition(4.6), one can observe (after the development of the product term) that the component of this matrix composed of the terms $CA_{q_1}^{p_0}\dots CA_{q_N}^{p_{N-1}}$ where $p_0, p_1, \dots, p_{N-1} \in \{0,1,2\}$.To characterize the observability of the sequence SS, it is necessary to use some properties of the state and output matrices presented in the *chapter.2*. We have from

$$Ker(CA_{q_i}A_{q_j}) \in Ker(C) \cap Ker(CA_{q_j})$$

where q_i and q_j are two modes. This property can be extended when the number of modes is greater than 2. Therefore, we have

$$Ker\left(\begin{pmatrix} C \\ CA_{q_j} \end{pmatrix}\pi_j\right) = Ker\left(\begin{pmatrix} C\prod_{k=1}^{j-1}\alpha_0^k(t)I + \alpha_1^k(t)A_{q_k} + \alpha_2^k A_{q_k}^2 \\ CA_{q_j}\prod_{k=1}^{j-1}\alpha_0^k(t)I + \alpha_1^k(t)A_{q_k} + \alpha_2^k A_{q_k}^2 \end{pmatrix}\right)$$

$$= Ker\begin{pmatrix} \begin{pmatrix} C \\ CA_{q_{j-1}} \end{pmatrix} \\ \begin{pmatrix} C \\ CA_{q_{j-1}} \end{pmatrix} \end{pmatrix}$$

Which can be simplified as follows

$$Ker\left(\begin{pmatrix} C \\ CA_{q_j} \end{pmatrix}\pi_j\right) = Ker\begin{pmatrix} C \\ CA_{q_j} \end{pmatrix} \tag{4.7}$$

Replacing(4.7)into(4.6), we obtain

$$Ker\begin{pmatrix}\begin{pmatrix}C\\CA_{q_1}\end{pmatrix}\\\begin{pmatrix}C\\CA_{q_2}\end{pmatrix}\\\vdots\\\begin{pmatrix}C\\CA_{q_N}\end{pmatrix}\end{pmatrix}=\{0\}$$

Since the output vector C is common, one maintains only the first C . The last condition can be reduced as follows

$$Ker\begin{pmatrix}C\\CA_{q_1}\\CA_{q_2}\\\vdots\\CA_{q_N}\end{pmatrix}=\{0\}$$

The following theorem gives the simplest condition for the observability of the converter when the system evolves under the switching sequence.

Theorem 4.1 The continuous state of the multicellular converter is observable under the switching sequence if and only if

$$rank\left(\begin{pmatrix}1&\dots&1\\u_1^0&\dots&u_n^0\\\vdots&\ddots&\vdots\\u_1^N&\dots&u_n^N\end{pmatrix}\right)=\{0\} \tag{4.8}$$

Proof of this theorem given in [Gazzam and Benalia, 2016].

These results lead to the following major conclusions that characterize completely the observability of the multicellular converter for a given switching sequence.

- Observability of the multicellular converter is uniform (independent of the time);
- Repetition of the discrete modes in a given switching sequence can be omitted. If some discrete modes have the same observability matrix then we can take in consideration only one of them;

- The order of discrete modes in a sequence is not important and does not affect the observability of the converter which leads to more degrees of freedom in the control scheme;
- A necessary condition for the observability of the multicellular converter is restricted by the existence of different discrete modes;
- A switching sequence SS is observable if and only if it contains $n-1$ modes, which form an observable sequence.

4.2 Hybrid Algebraic Observer Design of Multicellular Converter

In the case of multicellular converter, the hybrid algebraic observer's idea is based on the robust differentition of the output and the gathring partial information from individul modes using the general algorithm of hybrid state estimation presented in (**section 2.4.3**). No, we can detailed also. For, the estimation of discrete states, because $(u = 0)$ the vectors given by four sampled as in the (***case2***). In the case of continuous state, We suppose that the converter is conditionally observable for the sequence SS . From system (3.17), it is possible to define the successive differentiation of output for the mode q_1 ,using the same process in section 2.4.

Since the synthesis of the algebraic observer is dependent on the dimension of the observable subspace, it is necessary to determine the dimension of the observable subspace of each mode. It is noticed in eq(4.2), that the dimension of the observable subspace of the multicellular converter series of a given mode is less than or equal to 2, which means that one needs to estimate only the first derivative of the measurement(see [Gazzam et al., 2019]). This derivative can be estimated using ***theorem 2.3***, by taking N =1, j =1,

$$\hat{y}^(1) = \int_0^T \frac{-12\tau + 6T}{T^3} y(t-\tau)d\tau \tag{4.9}$$

4.3 Application to Two-cells Converter

Figure(Fig.4.1) depicts the topology of two- cells converter. The converter that can be represented by:

$$\begin{cases} \dot{x}(t) = A_{q_i}x + B_{q_i} \\ \quad y = Cx \end{cases} \tag{4.10}$$

where $x(t) = [v_c \ i_L]^T$ is the continuous state vector corresponding to the floating capacitor voltage and the load current. The pair of matrices (A_{q_i}, B_{q_i}) for each discrete mode can be obtained by replacing the corresponding cells states in the following matrices

$$A_{q_i} = \begin{pmatrix} 0 & \frac{u_2-u_1}{C} \\ \frac{u_1-u_2}{L} & -\frac{R}{L} \end{pmatrix}, B_{q_i} = \begin{pmatrix} 0 \\ \frac{u_2}{L}E \end{pmatrix}$$

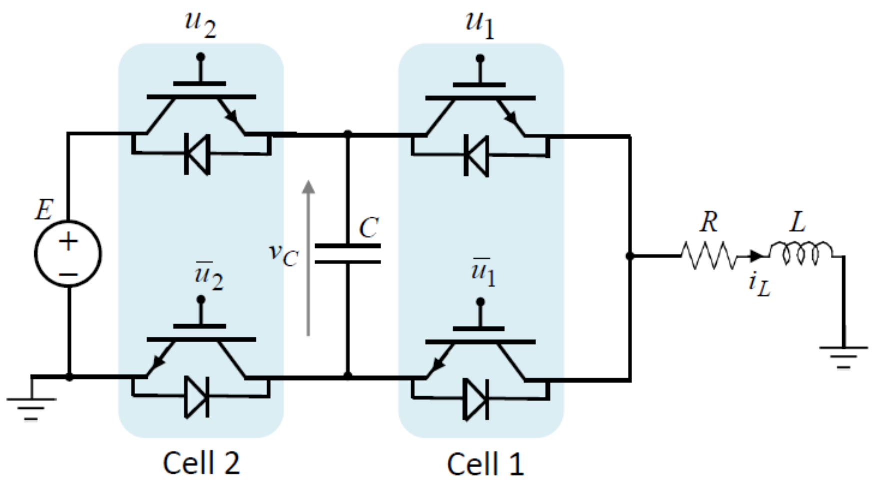

Figure 4.1: Two cells converter associated with an inductive load.

4.3.1 Hybrid Algebraic Observer of Two-Cells

In this section, we use the previous results of the observability analysis and observer design for the two-cells multicellullar converter. Firstly, we give a switching control law (PWM) and we apply the observer given in previous section(4.2). Note that the converter has 4 different discrete modes(Tab.4.1).

4.3.2 Observability Analysis

For texting observability of the multicellular converter we can used one of the two developed approaches, in this case we can used the observability analysis with PSDM.

Table 4.1: Two-cells Discrete modes.

Discrete modes	Cell states	Dynamics
Mode q_1	$u_1 = 0, u_2 = 0$	$A_{q_1} = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{R}{L} \end{pmatrix}$
Mode q_2	$u_1 = 1, u_2 = 0$	$A_{q_2} = \begin{pmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{pmatrix}$
Mode q_3	$u_1 = 0, u_2 = 1$	$A_{q_3} = \begin{pmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{pmatrix}$
Mode q_4	$u_1 = 1, u_2 = 1$	$A_{q_4} = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{R}{L} \end{pmatrix}$

A- Observability Analysis with PSDM

Since the control signal is periodic (PWM), the observability is synthesized only for one period [Gazzam and Benalia, 2018] and for $\alpha_{ref} < \frac{1}{3}$. However, one can obtain the same conclusion in the other cases $\alpha_{ref} > \frac{1}{3}$. In this case, the sequence of modes is given by the $PSDM$ is

$$S^1 = S^1(S^1_1, 1, 3) \text{ where } S^1_1 = (0.1)(0.6, 2)$$

$$S^2 = S^2(S^2_1, 1, 3) \text{ where } S^2_2 = (0.2)(0.3, 3)(0.7, 1)$$

1- Observability analysis of S^1_1

the analysis of the observability of this sequence is restricted to the two periods.
The unobservable subspaces of the sequence S^1_1 and the matrix are :

$$ker(Q_{S^1_1}) = ker(\begin{bmatrix} Q_1 \\ Q_2 e^{0.4A_1} \end{bmatrix}) = span\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right\}$$

$$M^1 = e^{0.4\times A_2} e^{0.6\times A_1} = \begin{bmatrix} 1.23 & 0.058 \\ -0.001 & 0.1487 \end{bmatrix}$$

$$J_1 = ker(Q_{S^1_1}) \cap (M^1)^{-1} J_0 = span\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right\} \cap span\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right\} = span\begin{pmatrix} 1 \\ 0 \end{pmatrix} = J_0$$

2- Observability analysis of S^1_1

The unobservable subspaces of the sequence S^2_1 and the matrix are :

$$ker(Q_{S_2^1}) = ker(\begin{bmatrix} Q_2 \\ Q_3 e^{0.8A_3} \end{bmatrix}) = span\left\{\begin{pmatrix} 0 \\ 0 \end{pmatrix}\right\}$$

$$M^2 = e^{0.4\times A_3} e^{0.6\times A_1} = \begin{bmatrix} 0.254 & 0.48 \\ -1.59 & 0.28 \end{bmatrix}$$

$$J_2 = ker(Q_{S_1^2}) \cap (M^2)^{-1} J_0 = span\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right\} \cap span\left\{\begin{pmatrix} 0.654 \\ 1.784 \end{pmatrix}\right\} = span\left\{\begin{pmatrix} -0.091 \\ 0.05 \end{pmatrix}\right\} =$$

$\{0\}$ as $J_1 = \{0\}$, than the sequence S^2 is observable.

4.3.3 Simulation Results

The proposed hybrid algebraic observer is validated through simulations for a typical two-cells converter with the parameters given in (Tab.4.2). In (Fig.4.2),(Fig.4.3) the evolution of active mode is presented, the used discrete modes are (q_2, q_3, q_4).

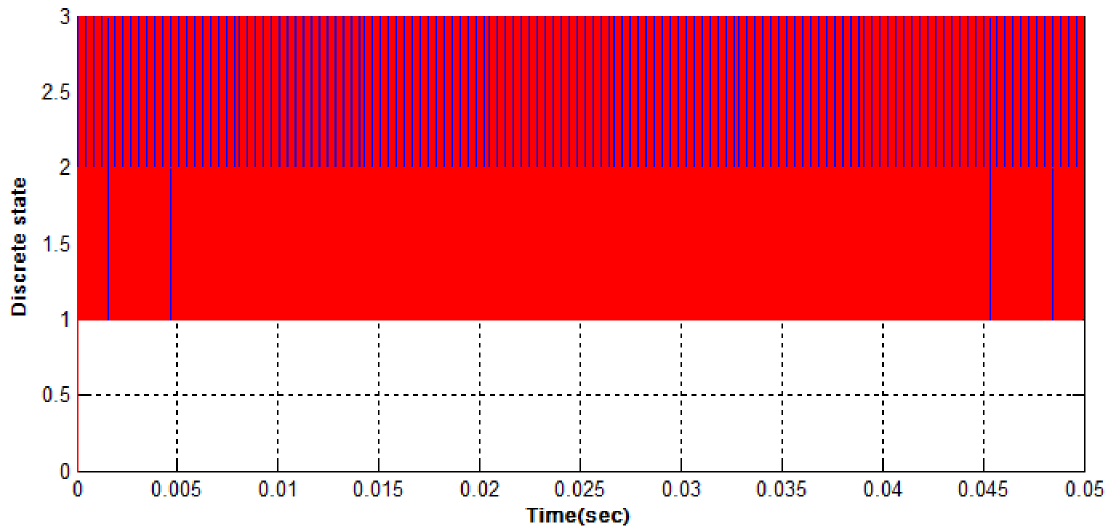

Figure 4.2: Estimation of active mode.

(Fig.4.4),(Fig.4.5) shows the floating capacitor voltage and the load current voltage evolution for a variable in the resistance value at :$t = 0.03s$ to $t = 0.04$. We notice the natural balancing around the desired floating voltage is :$v_c = \frac{E}{2} = 750V$. It should be noted that these results are completely satisfactory and in conformity with the forecasts: robust and time finite convergence (Fig.4.6).

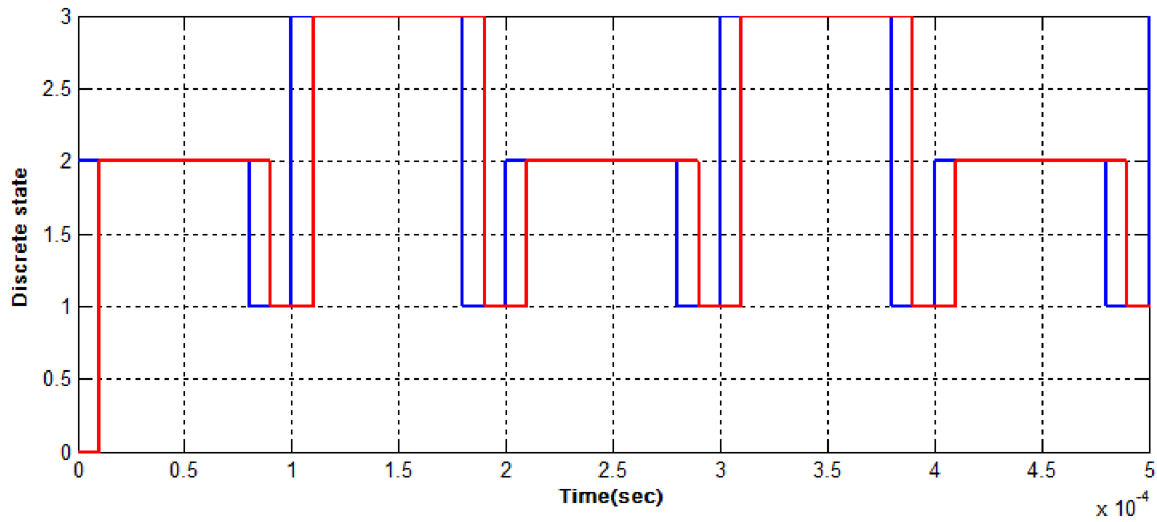

Figure 4.3: Estimation of active mode.

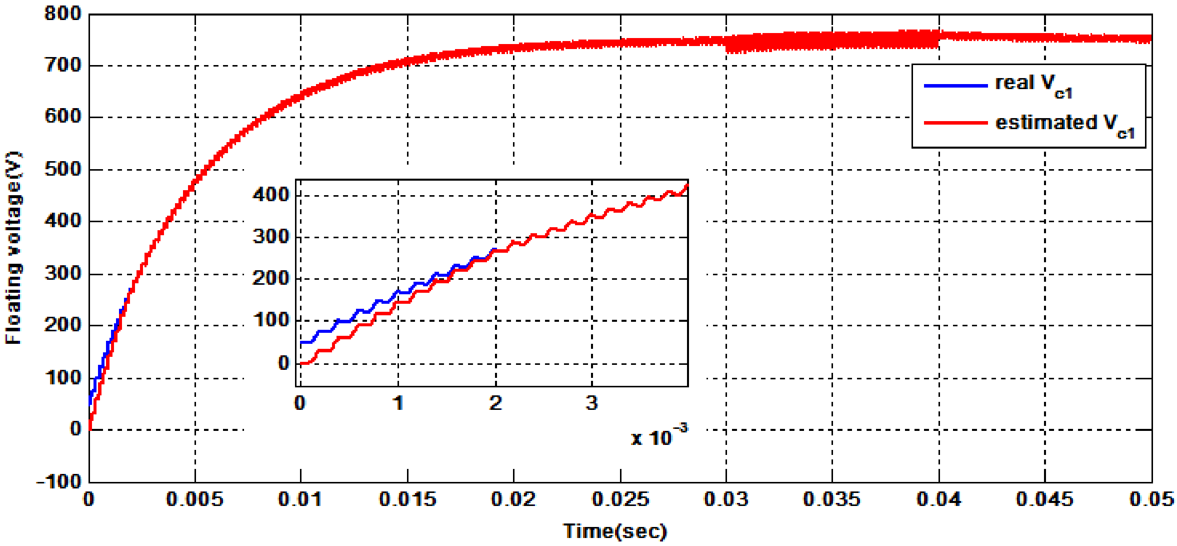

Figure 4.4: Estimation of the capacitor voltage.

Table 4.2: Two-cells converter parameters.

Parameter	E	L	C	R
Value	30V	$1mH$	$40\mu F$	100 Ω

4.4 Application to Three-cells Converter

In this section, we use the previous results of the observability analysis, control and observer design for the three-cell multicellullar converter(Fig.4.7). Depending on the cell states, one can distinguish eight different discrete modes(see Table.4.3), which offer four

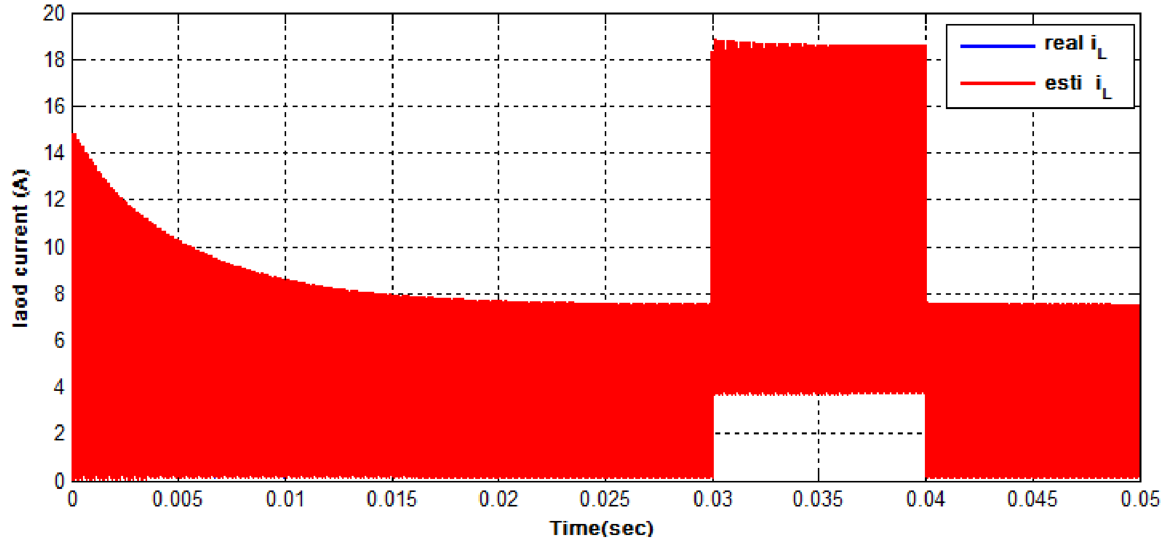

Figure 4.5: Estimation of the load current.

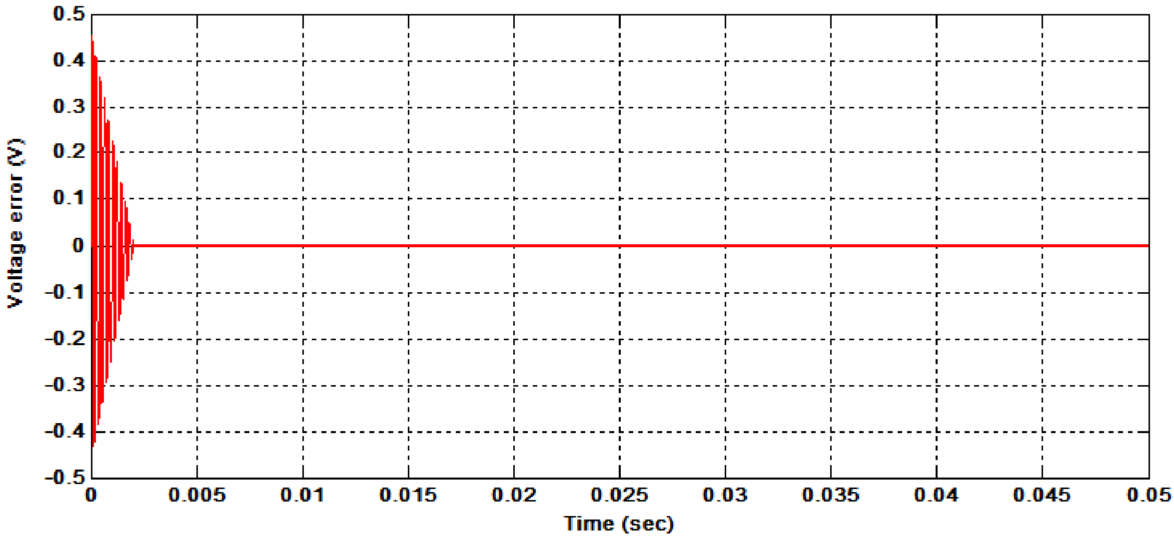

Figure 4.6: Capacitor voltage error.

output voltage levels at the converter equilibrium $\{0, \frac{E}{3}, 2\frac{E}{3}, E\}$.

4.4.1 Observability of to Three-cells Converter

The PWM strategy is a kind of control that guarantees the natural balancing of the floating capacitors crossing the cells. The switching control defined by this strategy is based on two parameters, a period T and a duty cycle α. For the simulation, we define the switching sequence by choosing the parameters of the PWM strategy in order to get the desired states. These parameters are given as follows: $T = 20ms, \alpha = 0.45$. The re-

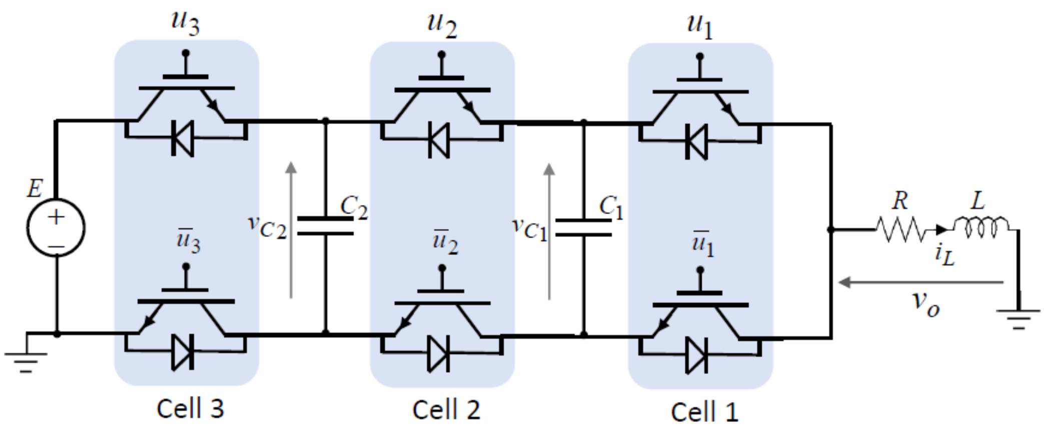

Figure 4.7: Three-cells converter associated with an inductive load.

lated cell states of the choosing parameters for a period time are shown in Figure(Fig.4.8).

A- ***Observability analysis using unobservable space***

With the $SS = (0, q_6)(2.33ms, q_2)(9ms, q_3)(13.33ms, q_7)(15ms, q_5)$ First of all, it is necessary to calculate the unobservable subspace of all modes in the sequence SS. In fact:

$$N_{q_6}^{q_6} = Ker\begin{pmatrix} C \\ CA_6 \end{pmatrix} = \{x \in \mathbb{R}^3 : x_1 = x_2 = 0\} = span\left\{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right\}$$

$$N_{q_2}^{q_2} = Ker\begin{pmatrix} C \\ CA_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ \frac{1}{L} & 0 & -\frac{R}{L} \end{pmatrix} = \{x \in \mathbb{R}^3 : x_1 = x_3 = 0\} = span\left\{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right\}$$

$$N_{q_3}^{q_3} = Ker\begin{pmatrix} C \\ CA_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ -\frac{1}{L} & \frac{1}{L} & -\frac{R}{L} \end{pmatrix} = \{x \in \mathbb{R}^3 : x_1 = x_2, x_3 = 0\} = span\left\{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right\}$$

$$N_{q_7}^{q_7} = Ker\begin{pmatrix} C \\ CA_7 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ -\frac{1}{L} & \frac{1}{L} & -\frac{R}{L} \end{pmatrix} = \{x \in \mathbb{R}^3 : x_1 = x_2, x_3 = 0\} = span\left\{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right\}$$

$$N_{q_5}^{q_5} = Ker\begin{pmatrix} C \\ CA_5 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -\frac{1}{L} & -\frac{R}{L} \end{pmatrix} = \{x \in \mathbb{R}^3 : x_1 = x_2, x_3 = 0\} = span\left\{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right\}$$

The unobservable subspace of the SS is

$$Ker\begin{pmatrix} 0 & 0 & 1 \\ \frac{1}{L} & 0 & -\frac{R}{L} \end{pmatrix} = \{0\} \tag{4.11}$$

Also, using (4.19), we can easily check the observability of the converter for the sequence . The control inputs that compose the sequence SS during one period are:

Table 4.3: Three-cells Discrete modes.

Discrete modes	Cell states	Dynamics
Mode q_1	$u_1 = 0, u_2 = 0, u_3 = 0$	$A_{q_1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{R}{L} \end{pmatrix}$
Mode q_2	$u_1 = 1, u_2 = 0, u_3 = 0$	$A_{q_2} = \begin{pmatrix} 0 & 0 & -\frac{1}{C_1} \\ 0 & 0 & 0 \\ \frac{1}{L} & 0 & -\frac{R}{L} \end{pmatrix}$
Mode q_3	$u_1 = 0, u_2 = 1, u_3 = 0$	$A_{q_3} = \begin{pmatrix} 0 & 0 & \frac{1}{C_1} \\ 0 & 0 & -\frac{1}{C_2} \\ -\frac{1}{L} & \frac{1}{L} & -\frac{R}{L} \end{pmatrix}$
Mode q_4	$u_1 = 1, u_2 = 1, u_3 = 0$	$A_{q_4} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & -\frac{R}{L} \end{pmatrix}$
Mode q_5	$u_1 = 0, u_2 = 0, u_3 = 1$	$A_{q_5} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & -\frac{R}{L} \end{pmatrix}$
Mode q_6	$u_1 = 1, u_2 = 0, u_3 = 1$	$A_{q_6} = \begin{pmatrix} 0 & 0 & -\frac{1}{C_1} \\ 0 & 0 & \frac{1}{C_2} \\ \frac{1}{L} & -\frac{1}{L} & -\frac{R}{L} \end{pmatrix}$
Mode q_7	$u_1 = 0, u_2 = 1, u_3 = 1$	$A_{q_7} = \begin{pmatrix} 0 & 0 & \frac{1}{C_1} \\ 0 & 0 & 0 \\ -\frac{1}{L} & 0 & -\frac{R}{L} \end{pmatrix}$
Mode q_8	$u_1 = 1, u_2 = 1, u_3 = 1$	$A_{q_8} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{R}{L} \end{pmatrix}$

$$Rank = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = 3$$

.

Thus the converter is observable for the sequence SS.

Table 4.4: Three-cells converter parameters.

Parameter	E	L	C	R
Value	$1500V$	$1mH$	$720\mu F$	$150\ \Omega$

4.5 Simulation Results

In order to illustrate the performance and efficiency of the suggested observer (we restrict to the estimation of the continuous state only), some simulations with/without robustness test are done. The parameters of the converter are reported in Table (Tab.4.4). It is worth noticing that the previous parameters were chosen in order to ensure a good time convergence.

Table 4.5: Three-cells observable modes.

Discrete modes	Cell states	Observable state
Mode q_1	$u_1 = 0, u_2 = 0, u_3 = 0$	$v_{q_1}(t) = 0$
Mode q_2	$u_1 = 1, u_2 = 0, u_3 = 0$	$v_{q_2}(t) = v_{c_1}(t)$
Mode q_3	$u_1 = 0, u_2 = 1, u_3 = 0$	$v_{q_3}(t) = v_{c_1}(t) - v_{c_2}(t)$
Mode q_4	$u_1 = 1, u_2 = 1, u_3 = 0$	$v_{q_4}(t) = v_{c_2}(t)$
Mode q_5	$u_1 = 0, u_2 = 0, u_3 = 1$	$v_{q_5}(t) = v_{c_2}(t)$
Mode q_6	$u_1 = 1, u_2 = 0, u_3 = 1$	$v_{q_6}(t) = v_{c_1}(t) - v_{c_2}(t)$
Mode q_7	$u_1 = 0, u_2 = 1, u_3 = 1$	$v_{q_1}(t) = 0$
Mode q_8	$u_1 = 1, u_2 = 1, u_3 = 1$	$v_{q_8}(t) = 0$

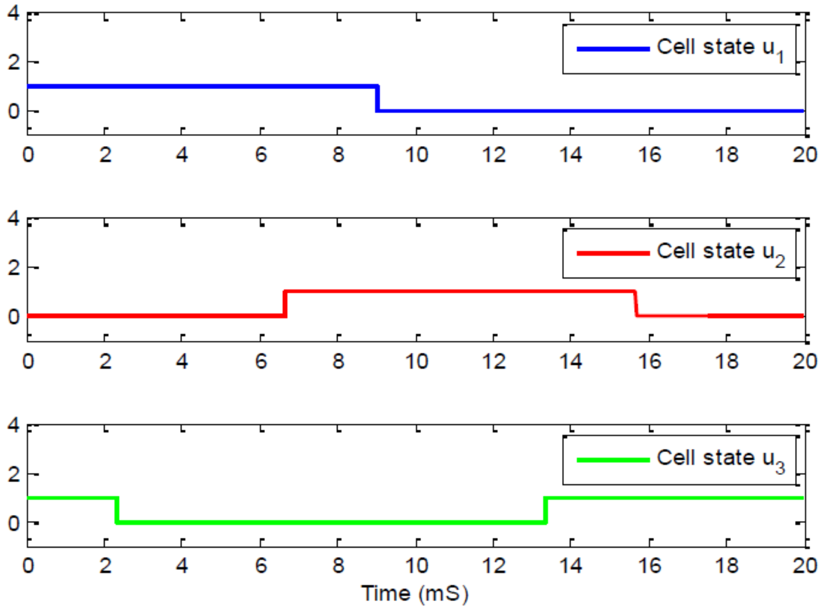

Figure 4.8: Cell states during one period.

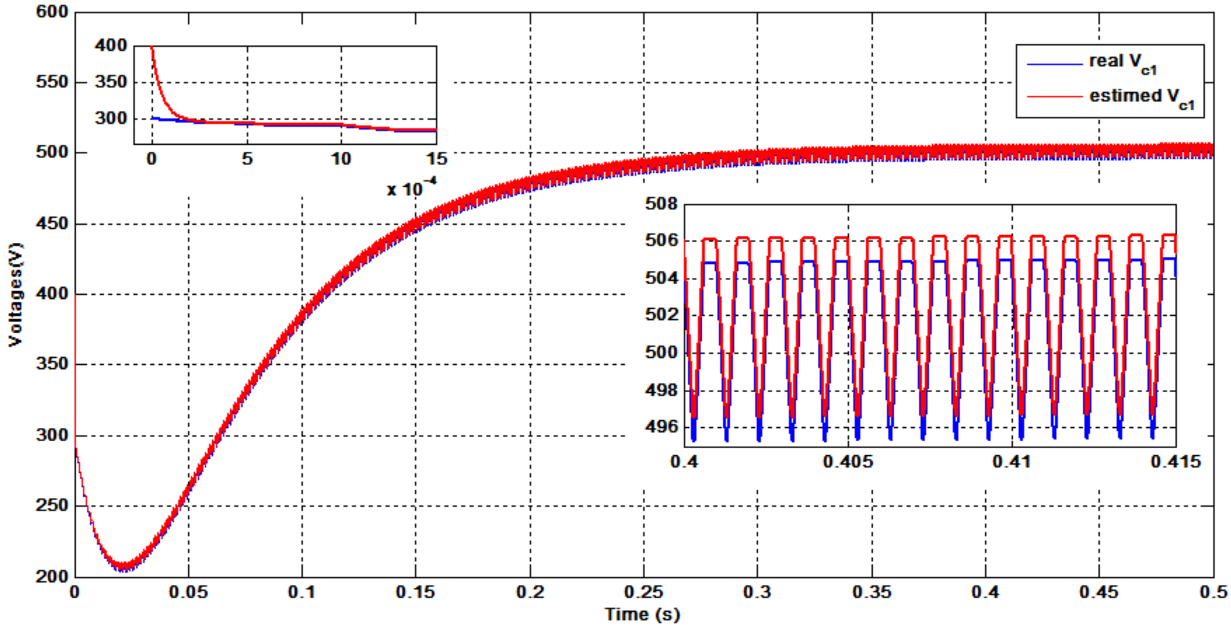

Figure 4.9: Real capacitor voltagesV_{c_1}, and its estimated $\hat{V}_{c_1}$.

4.5.1 Simulation without Robustness Test

Figures(Fig.4.9)and (Fig.4.10) show the real and estimated evolution of the capacitors voltages v_{c_1}, v_{c_2} respectively. From the Figures(Fig.4.9) and (Fig.4.10), we notice the natural balancing around the following desired states: $I_{L_{ref}} = 10A, v_{c_{1ref}} = \frac{E}{3} = 500V, v_{c_{2ref}} =$

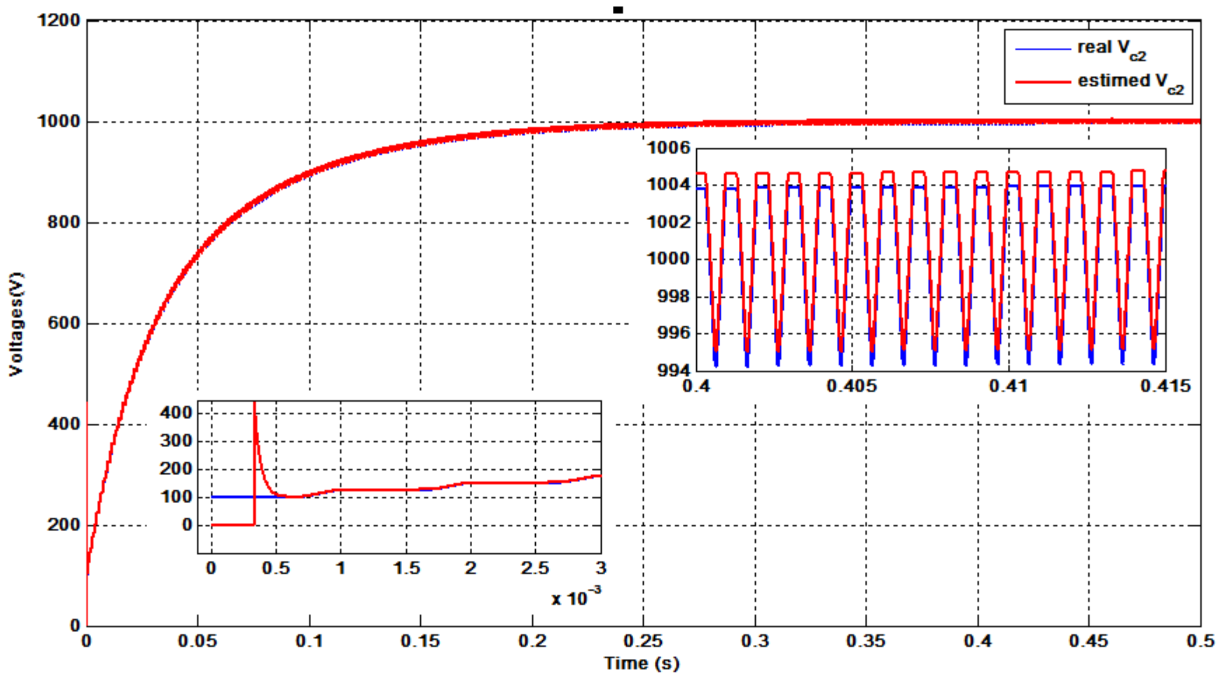

Figure 4.10: Real capacitor voltages $V_{c_2}, andits$ estimated $\hat{V}_{c_2}$.

$\frac{2E}{3} = 1000V$ The proposed observer not show only the finite-time convergence of the estimated capacitors voltage to the real ones, but also its convergence speed. These results prove the efficiency of the observer as well as the estimation error converge to the zero see Figures(Fig.4.11 and Fig.4.12).

4.5.2 Comparative study and Robustness Test

In order to illustrate the performance of proposed observer, comparative study with the super twisting observer eq(3.34) (see **section 3**) are done. The super twisting observer parameters are $\alpha = 200000, \lambda = 26922$.

also, we adopt a load resistance variation is equal to $R = 40$, which is decreased at $t = 0.4s$ by 20 and increased by 10 at $t = 0.6s$. The obtained simulation results are set out in figures (Fig.4.13) and(Fig.4.14). We notice the natural balancing around the desired states, and the algebraic observer converge rapidly to the real values at $t = 2e^{-4}s$ compared to the super-twisting observer at $t = 0.04s$ for the capacitor voltage v_{c1}. From the v_{c_2}, the algebraic observer converge at $t = 5e^{-4}s$ compared to the super-twisting observer in $t = 0.045s$. In addition, one can see clearly from the Figures, this influence does not affect the estimation of the observable states (voltage estimation) in term of convergence speed and finite-time convergence, which shows the robustness of the

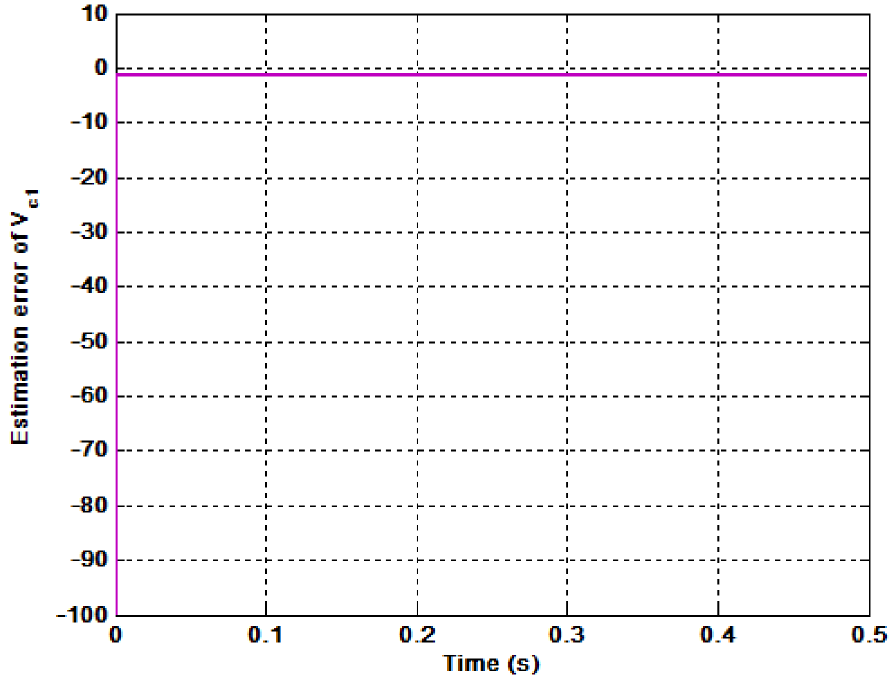

Figure 4.11: Estimation error of V_{c_1}.

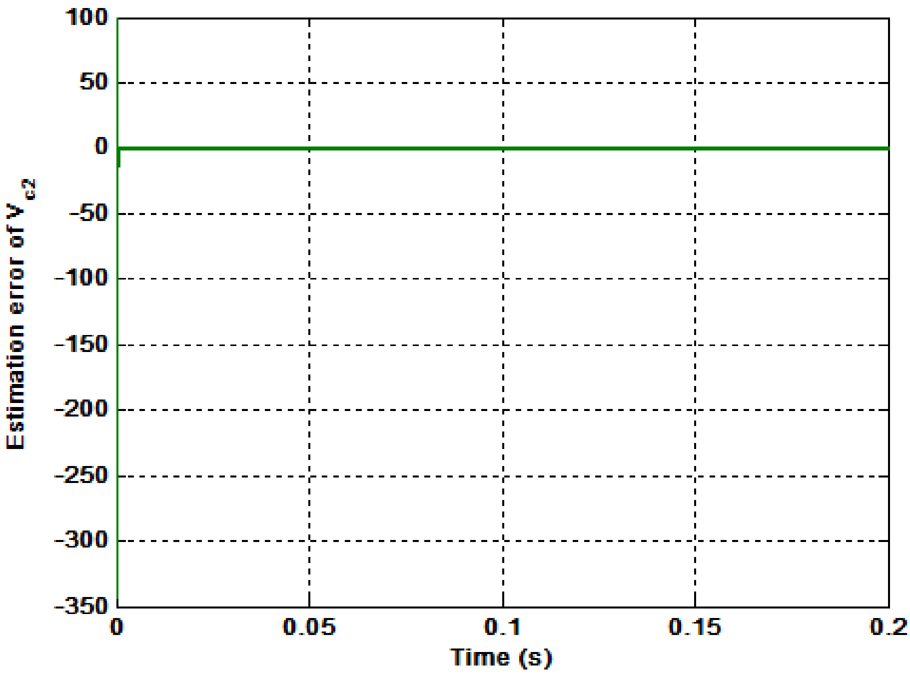

Figure 4.12: Estimation error of V_{c_2}.

proposed observer.

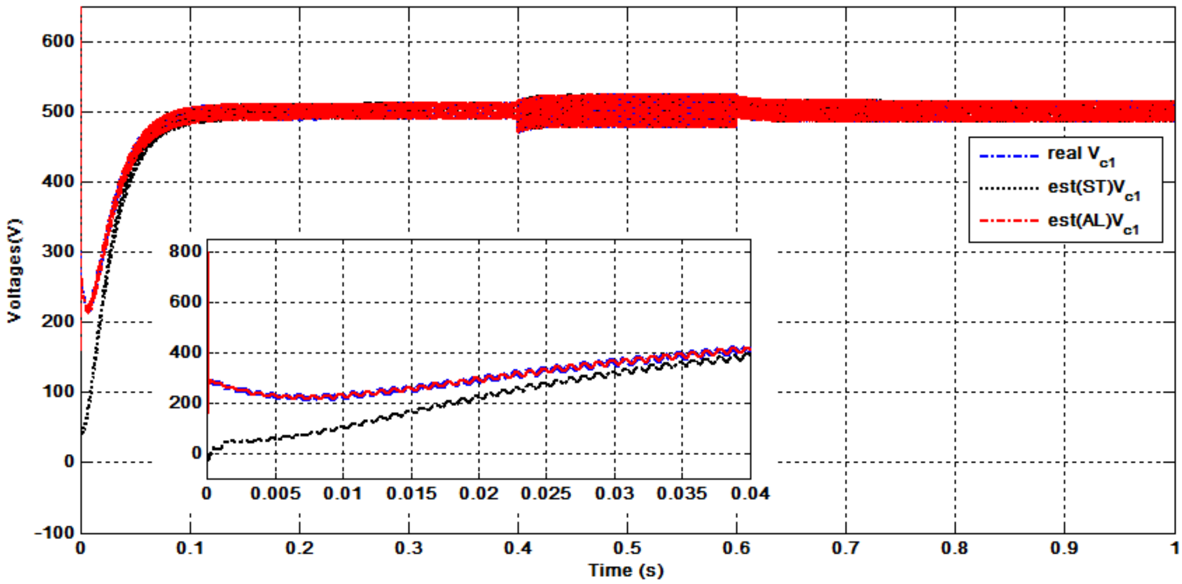

Figure 4.13: Estimation of V_{c_1} (comparative study between Super-twisting and algebraic observer with load resistance variation).

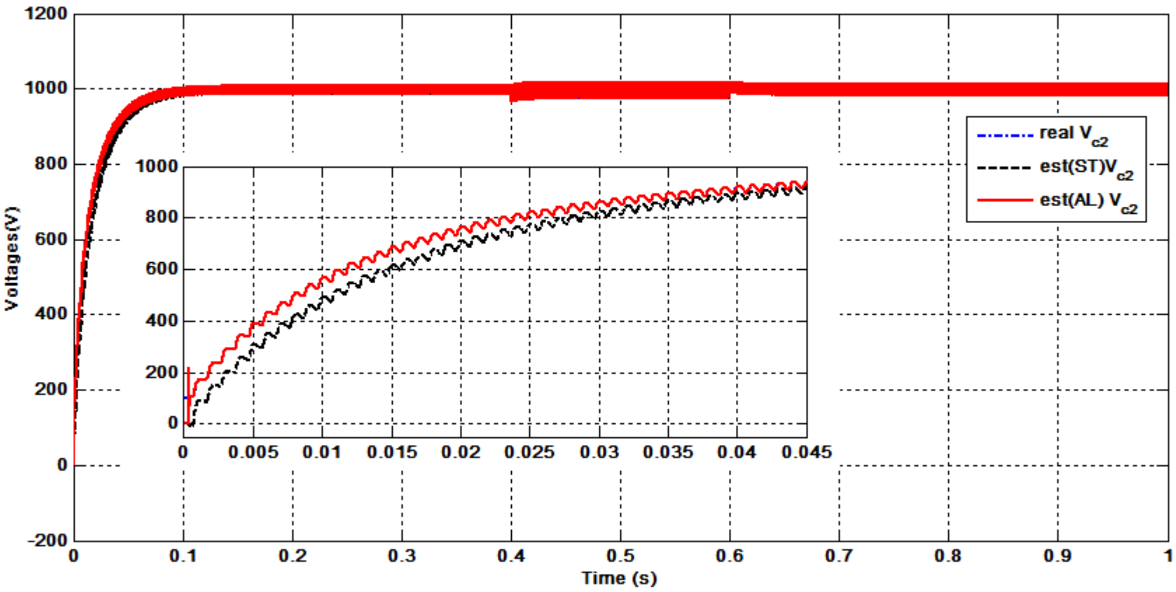

Figure 4.14: Estimation of V_{c_2} (comparative study between Super-twisting and algebraic observer with load resistance variation).

4.6 Conclusion

In this chapter, we have studied the observability of the multicellular converter using some dynamical properties. We have given a necessary condition for the observability

for a given switching sequence. We have used these results to design a hybrid algebraic observer. To test the finding results, we have applied them to the two-cells converter. For the three- cells converter, comparative study between the algebraic and super-twisting observer when the switching sequences is determined demonstrate the efficiency and the robustness of the suggested observer to achived to the real floating voltages in finite time
.

General Conclusion

This thesis work has been interested in the observability and observer desgin of switched linear system continuous case.

The analysis of switched linear systems and more precisely the two most important structural properties i.e. distiguishability and observability of such class of systems. The study has done using the geometrical and algebrical tools where we used the notions of subspace, nullspace…etc. Our study allows us to provide some significant results which are smmarized as follows:

- In chapter 2,For the distiguishability, we gave a simple condition when the system satisfy some algebraic tools relations. We have provided some results about the observability of switched linear systems. We have proven that the observability is strongly depended to switching times. We focused our study on the switching sequences where we have provided a simple necessary and sufficient condition that ensures the uniform observability respect to switching times. also, we introduce a notion of periodic sequence of discrete modes observability.

The most important contribution in this chapter is to developed a hybrid algebraic obsever with unobservable modes for estimated both discrete and contiunous states .The observer design is based on the diffentiation calcul. By an algebraic analysis of the derivatives of the switched system, we proposed a feasible algorithm able to estimate the hybrid state in fast and robust way. Illustrative examples are presented to highlight the developed ideas.

- In chapter 4, we have studied in details the switched affine model of the multicellular converter and we gave lots of important relations either for one discrete mode or between modes. Besides, we have determined some dynamical properties and characteristics of the converter. We give a necessary condition for the observability of the converter. Following the same steps in the previous chapter (chapter2)we developed a hybrid observer, which guarantees a fast convergence of the estimated floating capacitor voltage to their real value .The performance of the converter under the proposed estimator are tested through simulations by an application on the two-cells and three-cells converter. The simulation results shown the importance of our results whether for the analysis or for the design.

For the future works, we seek for the generalization of the finding results either for the observability, observer design of switched linear systems or those of the multicellular converter. In addition, we will look forward to studying the structural properties of nonlinear switched systems and apply the results to real physical systems.

Appendix A

Geometrical Tools for Linear Systems

The aim of this appendix is to provide a quick reference to basics and existing results. The geometrical approach, for the linear case, is mainly based on the geometrical tools such that subspace, invariance of subspace, image and null space of matrices. This approach offers another way to understand the basic structural properties because it can express these latters in term of some easy and simple algorithms. This approach has been used for the class of linear systems to check its controllability and observability using some easy and simple algorithms and tools. For more details of the geometrical approach for the linear systems, the reader can refer to [Bay, 1999]. In what follows, we will give some definitions, properties, remarks and algorithms for the controllability and the observability of linear systems. Before that, consider the following linear system

$$\begin{cases} \dot{x} & = Ax + Bu \\ y & = Cx \end{cases} \tag{A.1}$$

where $x \in \mathbb{R}^n$,$u \in \mathbb{R}^m$ and $y \in \mathbb{R}^m$ are the state, the input and the output matrices respectively.

Definition 1 (**controlled invariant**) A subspace $V \in \mathbb{R}^n$ is said to be an (A, B)-controlled invariant if

$AV \subseteq V + \beta$ where $\beta = Im(B)$

Definition 2 **(conditioned invariant)** A subspace $S \in \mathbb{R}^n$ is said to be an (A,C) -conditioned invariant if

$A(S \cap \varsigma) \subseteq \varsigma$ where $\varsigma = ker(C)$

Remarks

- The dual of the controlled invariant is the conditioned invariant;
- any A -invariant is also an (A,B) -controlled invariant for any B ;
- any A -invariant is also an (A,C) -conditioned invariant for any C ;

Properties

- The sum of any two (A,B) -controlled invariants is an (A,B) -controlled invariant.
- The intersection of any two (A,C) -conditioned invariants is an (A,C) -conditioned invariant.
- The orthogonal complement of an$(A, \varGamma)$-controlled (conditioned) invariant is an $(A^T, \varGamma^{)}$-conditioned (controlled) invariant.

Theorem.1

The controllable subspace of the system 1.5 is the A-invariant minimal subspace that contains $Im(B)$.

Theorem 2

The unobservable subspace of the system 1.5 is the A-invariant maximal subspace that belongs $Im(B)$.

The two subspaces given in the last theorems can be calculated by the following geometrical algorithms.

Algorithm .1 (controllable subspace)

$$J_0 = Im(B)$$

$$J_{i+1} = AJ_i + Im(B)$$

where the last step $k \leq n-1$ is determined by the condition $J_k = J_{k+1}$.

Algorithm .2 (unobservable subspace)

$$Z_0 = ker(C)$$

$$A^-1 Z_i \cap ker(C)$$

where the last step $k \leq n-1$ is determined by the condition $Z_k = Z_{k+1}$.

Remarks

- The linear system (1.2) is completely controllable if and only if the Algorithm.1 has n steps;

- the linear system (1.2) is completely observable if and only if: the Algorithm.2 has n steps.

In order to show the importance and the efficiency of these algorithms, we will use them for check the controllability and the observability of the following linear system.

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