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“ Every sentence I utter must be understood not as an affirmation, but as a question ”

Niels Bohr

Dedication

This thèses is dédicated to:

To my loving parents, **Mohamed** and **Zekhroufa BEN KHARFIA**, Who Learned to Live on Their Hands, who was give me the best basic education possible that they could provide, who first taught me the value of education and critical thought.

To my brothers **Haidare** and **Aymen**, and my sisters **Messouda**, **Hiba**, **Aya** and the little **Batoul** for the support and motivation when I needed

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To all my friends at **SM** department and **IRTQA** Association.

To my dear students in IRTQA Association, although you called me teacher, I was the one who was learning, I wish you success.

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Preface

This thesis is the result of months of full-time studies and homework in the isolation at quarantine form the pandemic "COVID19" year (2019/2020 school year). The work is submitted as the Master Program at the department of material Sciences, faculty of sciences, UNIVERSITY AMAR TELIDJI LAGHOUAT, has been carried out under the supervision of Mr. KHENCHOUL Salah.

Outline

In this thesis we shall derive a unified SU(2)×U(1) Ginzburg-Landau equations. To facilitate its reading and understanding, we have introduced some important elements through the first and second chapters, where the first one included a history of superconductivity and some distinctive characteristics. The second one, Include an explanation of the breaking symmetry notion and a details of the derivation of GL equations in SU(2) and U(1) breaking symmetry cases. The third chapter presents the results of our analysis for the unified SU(2)×U(1) breaking symmetry case.

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General Introduction

General Introduction

The phenomenological Ginzburg-Landau (GL) theory [1] is a system of ideas intended to explain the phase transitions in superconductors. This theory stipulates that the transition from the normal state to a superconducting state can be described by an order parameter. At low temperature, the system exhibits an disordered phase associated with a non-zero order parameter. Above a certain critical temperature, the system becomes ordered corresponding to zero order parameter. A few years after the formulation of the GL theory, Gorkov [2] has shown that the last one can be inferred from the Bardeen, Cooper and Schrieffer (BCS) theory [3]. He has reported that the condensed electrons in the GL theory are actually the Cooper pairs.

The Brout-Englert-Higgs (BEH) mechanism [4,5] is quite similar to another well-known as the abelian U(1) spontaneous symmetry breaking (SSB) mechanism in condensed matter physics that was discovered earlier. Anderson [6] was among the first to make an analogy between superconducting state and particle physics. The inconsistency between equations describing the particle's behavior and the matter properties under certain constrain has been the center of attention of the scientific community in recent years. He has used the concept of massive photons to explain several phenomena appearing in superconductors and superfluids. The BCS theory shows that the Fermi surface is unstable during the formation of Cooper pairs. Unlike fermions that are subject to the Pauli Exclusion Principle, these pairs are bosons that can occupy the same state and form Bose condensates. The Cooper pairs formation corresponds to U(1) spontaneous broken symmetry. Despite the Anderson's insistence on the importance of the familiarization between superconductivity and BEH mechanism, this aspect is little discussed in the literature because of the existence of a realists philosophy in the context of the ad-hoc hypothesis.

Frequently, gauge theory was used to discuss the physical costs of some interactions, such as spin-orbit interaction [7,8]. Yang and Mills [9] are the first to give a theoretical framework to the gauge symmetrization in the case of a Lie group of dimension higher than U(1) group. It turned out that non-abelian SU(2) symmetry can be an effective tool for describing some fundamental interactions. Zhi-qiang al [10,11] have tried to drive the Ginzburg-Landau equations in the SU(2)-breaking symmetry background. Although there are some deficiencies in the GL equation of free energy provided, especially in the description of the radiation part, where the given expression in these posts designates the U(1) symmetry. Apart from that, these studies offered a good picture can portray the spin dynamic in superconductors which is missed in the conventional GL equations.

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Chapter 01: Fundamentals and Basics of the Superconductivity.

1.1. Introduction

Superconductivity is a phenomenon going on in certain materials at set temperatures, characterized by the complete absence of electrical resistance and the damping of the inside magnetic field. Unlike an ordinary metallic conductor, whose resistance decreases gradually as its temperature is soften even down to near absolute zero, a superconductor has a characteristic critical temperature below which the resistance drops abruptly to zero. An electric current through a loop of superconducting wire can persist indefinitely with no power source. In this Chapter, we will display the most important milestones that the growth of these materials has gone through. Over this chapter, we will learn about the aspirations of researchers and industrialists in this field.

1.2. Briefe history

The first inspection of this phenomenon was in 1911 by Dutch physicist HEIKE KAMERLINGH ONNES who was studying the resistance of solid mercury at cryogenic temperatures using the recently produced liquid helium as a refrigerant. At the temperature of 4.2 K, he observed that the resistance unexpectedly left [1]. In the same experiment, he also observed the superfluid transition of helium at 2.2 K, without recognizing its significance. The precise date and circumstances of the discovery were only reconstructed a century later, when Onnes's notebook was found. In subsequent decades, superconductivity was observed in several other materials. In 1913, lead was found a superconductor at 7 K, and in 1941 niobium nitride was found to superconduct at 16 K [2].

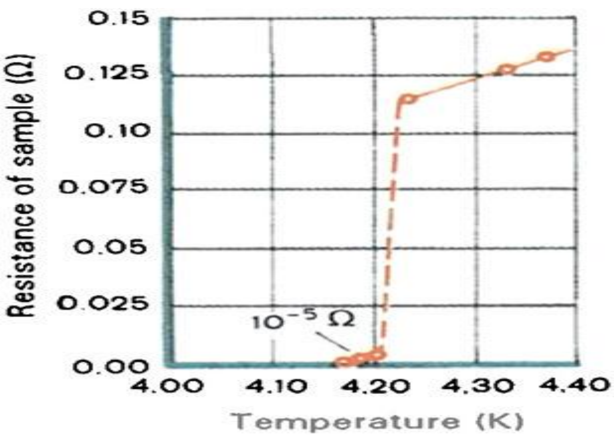

Fig.1. 1. *Illustration of Kamerlingh Onnes's discovery of superconductivity and vanishing of the electrical resistivity* [1].

When a superconductor is placed in a weak external magnetic field H, the field penetrates the superconductor for only a little distance from the edge, called the penetration depth λ, after which it degrade rapidly to zero. This is called the Meissner effect. For most superconductors, the penetration depth is on the order of 100 nm. The Meissner effect is sometimes confused with the "perfect diamagnetism" one would expect in a perfect electrical conductor: according to Lenz's law, when a changing magnetic field is applied to a conductor, it will induce an electrical current in the conductor that creates an opposing magnetic field. In a perfect conductor, an arbitrarily large current can be induced, and the resulting magnetic field exactly cancels the applied field [3].

Fig.1. 2. *A comparison between how a superconductor acts when it is above and below the Tc* [3].

When the external field is applied in parallel with the boundary, the field that is suddenly applied to zero on the surface of the superconductor is not reduced, but it degenerated mainly under the relationship:

$$H(x) = H(0)\exp\left(-{x}/{\lambda}\right) \tag{1.1}$$

The first successful set of phenomenological equations for superconducting metals, were given by F. London in 1935 [4]. In 1950 almost 40 years after the discovery of the phenomenon, there was no any adequate microscopic theory of superconductivity. In this year Ginzburg and Landau formulate a phenomenological theory of superconductivity [5] based on intuitive considerations which we shall detailed in the second chapter. At the time, the theory they proposed could not be justified in terms of a microscopic Hamiltonian.

However, by 1935, single elements necessary to a successful theory to explain superconductivity was known to theorists. The peculiar of condensation of Bose-Einstein gas was predicted by Einstein in 1925 [6]. The idea is that pairs of fermions can be combined to form bosons has been known since 1931. In 1950 the most relevant ideas of superconductivity have been summarized by F. London in his famous book "superfluids" [7]. At last, Bardeen, Cooper and Schrieffer (BCS) theory (1957) [8], was the first successful theory to explain the mechanism of superconductivity in metals and alloys.

In 1959, Gorkov [9] showed that the Ginzburg-Landau equations can be obtained from the BCS theory when the temperature is close enough to the critical temperature T_c. Since then the Ginzburg-Landau theory has been systematically used to describe macroscopic phenomena in superconductors.

1.3. Critical factors of superconductors

To identify the superconductor properties through three important factors: critical temperature (T_c), critical magnetic field (H_c), and critical current density (I_c). Each of the three factors depends heavily on the characteristics of the other two factors [10].

1.3.1. Critical temperature (T_c)

The superconductivity phenomenon is characterized by the loss of the major resistance to the sample when it is cooled to a certain temperature. Overhead this temperature, the resistance is small, but it exists, while the resistance under this point is basically zero. Critical temperature T_c is indicated to this degree that the samples are supposed to be in transition from the normal state to the superconductor state, the critical temperature varies from material to material, and It should be noted that T_c for highly accessed materials reductions with augmented current density applied, as all Superconductor materials are inspected through the use of a particularly critical current feature for that material.

1.3.2. Critical magnetic field (Hc)

It has been reported that the superconductor state will be destroyed by a magnetic field when the flow density becomes superior to the critical magnetic field H_c. The critical magnetic values decrease from that magnitude with increasing temperature, reaching zero at the critical temperature of superconductivity. The critical magnetic field at any temperature below the T_c is given by the empirical relationship:

$$H_c = H_{c0}\left[1 - \left({}^{T}/_{T_c}\right)^2\right], \tag{1.2}$$

Where H_{c0} it's the critical magnetic field at absolute zero temperature.

1.3.3. Critical current density (Ic)

It is the maximum current intensity that can be passed in a sample without destroying the Superconductor state. Superconducting properties disappear as the passage of a highly proportion current intensity through it. When the flow of current intensity through a superconductor, it will create a magnetic field that can destroy the state of superconductor. The critical current I_c is related to the critical magnetic field by the follow expression:

$$I_c = 2\pi r H_c \quad (1.2)$$

Typically, superconductivity and normal states are related with three critical factors: the critical temperature, critical magnetic field and critical current density as in Figure 1.3.

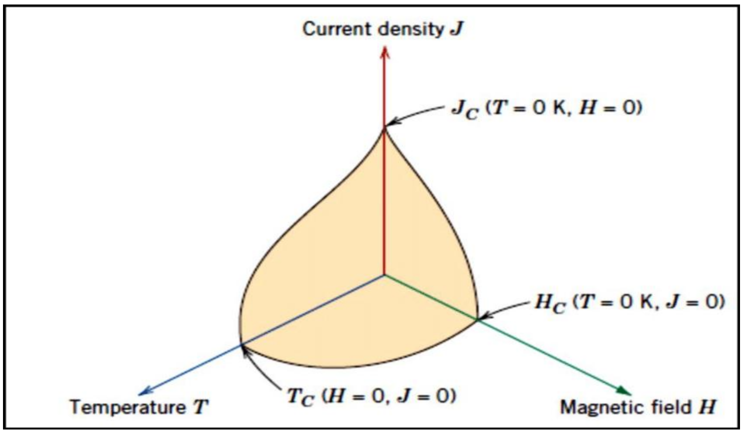

Fig.1. 3. *Approximate phase diagram of the superconductor explains the relation between Tc, Hc and Jc* [10].

1.4. Superconducting energy gap

For superconductors the energy gap is a area of suppressed density of states approximately the Fermi energy, with the size of the energy gap much smaller than the energy scale of the band structure. The superconducting energy gap is a key aspect in the theoretical description of superconductivity and thus features importantly in BCS theory. The range of the gap indicates the energy gain for two electrons upon formation of a Cooper pair. If a conventional superconducting material is cooled from its metallic state into the superconducting state, then the superconducting gap is absent above the critical temperature and it grows upon further cooling.

Table.1. 1. *Superconducting gap for some elements at T=0K* [11].

Element	Superconducting gap (meV)
Aluminium	0.34
Cadmium	0.15
Gallium	0.33
Indium	0.105

1.5. Superconductor types

In 1952, Abrikosov [12] showed that the Ginzburg-Landau theory predicted two types of superconductors (type I and type II). For the superconductors of the first group (type I) there is a single critical magnetization H_c while for those of the second group, there are two H_{c1} and H_{c2}. The phase diagrams of the superconductors of the two groups are summarized in Fig. 1.4. For the superconductors of the first group, there are only two phases corresponding to the superconducting state or to the normal state. The diagram of the superconductors of the second group highlights a mixed phase where the two states coexist by creating the vortex state.

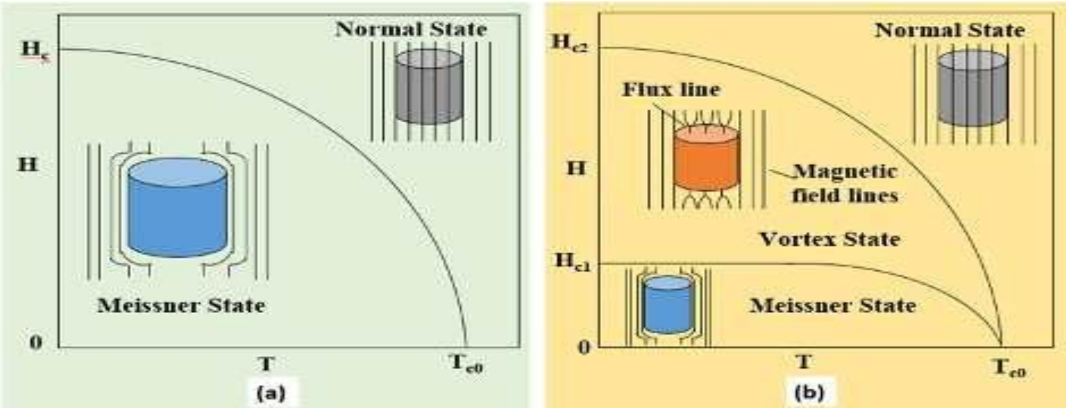

Fig.1. 4. *Phase diagrams of the superconductors. (a) Superconductor type I. (b) Superconductor type II.*

1.6. BCS theory

BCS theory or Bardeen–Cooper–Schrieffer theory [8] is the first microscopic formulation of superconductivity phenomenon since Onnes's discovery. The theory describes superconductivity as a microscopic effect caused by a condensation of Cooper pairs. A pair of electrons bound together at low temperatures in a certain manner. Cooper showed that an arbitrarily small attraction between electrons in a metal can cause a paired state of electrons to have a lower energy than the Fermi energy, which implies that the pair is bound. In conventional superconductors, this attraction is due to the electron-phonon interaction. Electrons have spin 1/2, so they are fermions, but the total spin of a Cooper pair is integer (0 or 1) so it is a composite boson. This means the wave functions are symmetric under particle interchange. Therefore, unlike electrons, multiple Cooper pairs are allowed to be in the same quantum state, which is responsible for the phenomena of superconductivity. The wave function describing these pairs is in fact the complex order parameter of Ginzburg-Landau theory.

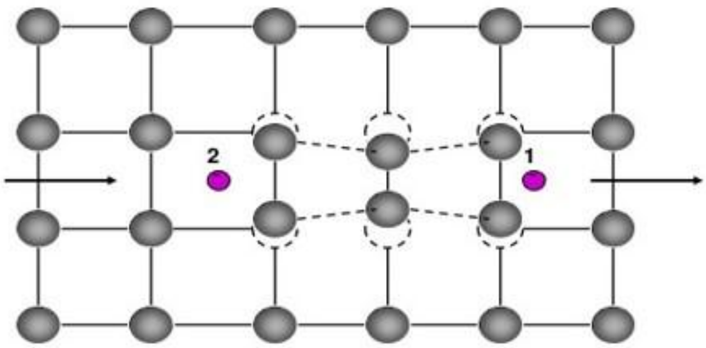

Fig.1. 5. *Electron-phonon coupling and Cooper pairs formation.*

Note, however, that the BCS theory fails to describe certain types of so-called unconventional superconductivity. Therefore, specialists in this field began to think about adopting other approaches to explain the superconductivity, including the adoption of spin as a distinct degree of freedom of electrons to quantify some properties of these materials and this is precisely the idea that we are interested on in this thesis.

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Chapter 02: Theory and Background

2.1. Introduction

In this chapter, we will begin by introducing the Ginzburg-Landau theory [1] and establishing the basic equations correspond to the U(1) symmetry breaking. The Ginzburg-Landau (GL) theory naturally leads to certain phenomenon such as the quantification of the flux or the expulsion of the magnetic field, also known as the Meissner effect [2]. Starting from the definition of symmetry in physics, we introduce the U(1) electromagnetic symmetry breaking and its consequences. Thus, we presented an overview of the study of Zhi-qiang & al. [3, 4], that addressed the effect of the SU(2) breaking symmetry on the GL equations.

2.2. Symmetry and breaking symmetry

2.2.1. Definition

Symmetry is one of the great themes in physics. From cosmology to nuclear physics and from soft matter to quantum materials, symmetries determine which shapes, interactions, and evolutions occur in nature. Perhaps the most important aspect of symmetry in theories of physics is the idea that the states of a system do not need to have the same symmetries as the theory that describes them. Such spontaneous breakdown of symmetries governs the dynamics of phase transitions, the emergence of new particles and excitations, the rigidity of collective states of matter, and is one of the main ways classical physics emerges in a quantum world.

Before talking about the breaking of symmetry, we have to define and understand what it meant by symmetry itself is. We need to differentiate between, on the one hand, the symmetries of the laws of nature, equations of motion, and the action or Hamiltonian, and on the other hand, the symmetries of states, objects, and solutions to the equations of motion. Intuitively, we would say an object possess a symmetry, if it looks identical from different viewpoints. For example, a sphere looks identical from any angle, and is therefore concluded to be rotationally symmetric. Likewise, the concept of the symmetry, which is intimately associated to the notion of the invariance, refers to the possibility of considering the same physical system from several viewpoints that are distinct in terms of description but equivalent in terms of the predictions made on its evolution [5].

We say that the symmetry is broken when, after changing certain characteristics of the system, the latter or the laws which administrate its behavior are no longer invariant under the transformation associated with this symmetry.

2.2.2. Symmetry breaking in Landau theory

The essential suggestion of Landau theory is that the free energy of any system undergoing a phase transition can be expressed as a functional of the order parameter φ, which itself depends on temperature. In a incessant phase transition, the order parameter $\varphi(T)$ is guaranteed to be small close to the critical temperature, and one can carry out a Taylor expansion for small $\varphi(T)$ in that region. Which types of terms appear in such an expansion depend strongly on the nature of the order parameter and the symmetries of the system. In the simplest case of a real-valued scalar field φ, the Taylor expansion of the Landau functional reads [6]:

$$F(\varphi,T) = F(0,T) + \frac{1}{2}\alpha(T)\varphi^2 + \frac{1}{4}\beta(T)\varphi^4 + \cdots. \tag{2.1}$$

Here α and β, are two phenomenological parameters. Landau theory can describe both continuous and discontinuous phase transitions in terms of an 'order parameter', whose value changes from zero to non-zero at the critical temperature Tc. see how this description of phase transitions is related to spontaneous symmetry breaking, consider the shape of the free energies in Figs. 2.1. At the lowest temperatures, there are two minima with equal energies, related by the transformation $\varphi \rightarrow - \varphi$.

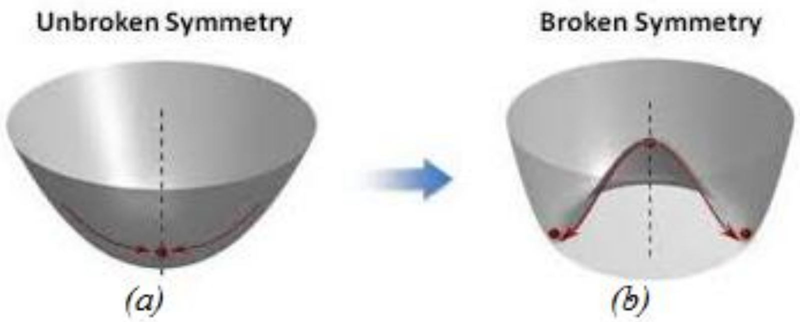

Fig.2. 1. *Landau free energy as a functional of the order parameter φ. (a) for* $\alpha > 0$ *and (b) for* $\alpha < 0$ [6].

If $\alpha < 0$, so F has a ring of minima with equal amplitude (modulus) $|\varphi|$ but arbitrary phase. We easily see:

$$\frac{dF}{d\varphi} = \alpha(T)\varphi + \beta(T)\varphi^3 = 0 \Rightarrow |\varphi| = 0 \; or \; |\varphi| = \sqrt{\frac{-\alpha}{\beta}}. \tag{2.2}$$

2.3. Ginzburg-Landau theory

Ginzburg-Landau theory [1] is a mathematical theory used to describe superconductivity. Initially, it was proposed by Landau as a phenomenological model which could describe type-I superconductor without examining their microscopic properties. Ginzburg was very impressed by this Landau's work on phase transitions and had been thinking about how to apply it to the phase transitions inside superconductors, based on Landau's established theory of second-order phase transitions, both Landau and Ginzburg suggested that the free energy, for a superconductor near the superconducting transition can be expressed in terms of a complex order parameter field ψ, which magnitude describes how deep in superconducting phase.

2.3.1. U(1) electromagnetic breaking symmetry and Ginzburg-Landau equation

The GL theory supposes the existence of a complex order parameter ψ representing the superconducting particles (Cooper pairs) and that the local density of these particles is given by its squared modulus $|\psi|^2$. The order parameter is coupled with electromagnetic field in a minimal manner the operator of the $(-i\hbar\nabla)$ pulse is thus replaced by the operator of the canonical $(-i\hbar\nabla + e^*\boldsymbol{A})$, where e^* is the charge associated with superconducting particles. In fact, this is simply a replacement of the usual derivative operator by the operator representing the covariant derivative. Ginzburg and Landau postulated that, near the critical temperature T_c, Gibbs free energy can be developed in series of powers of the order parameter ψ. Thus, for a complex order parameter the Landau expansions of the Gibbs free energy for small$|\psi|$ would be, expressed in units of Gauss (CGS), is given by:

$$G_s(\psi\,;\nabla\psi) = G_n + \int_V \left[\alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left(-i\hbar\nabla\psi + \frac{e^*}{c}A\psi\right)^2 + \frac{B^2}{8\pi}\right]dV \tag{2.3}$$

The quantity m^* is the mass of a superconducting particle. The first term represents the Gibbs free energy of the superconductor in the normal state. The first two terms in the integral come from the Taylor series development of Gibbs free energy detailed in the previous section and the third term gives the kinetic energy of the electrons superconducting coupled to the magnetic field. The fourth term gives the energy density of the magnetic field. Ginzburg and Landau postulated that α changes sign at the critical temperature T_c, (α cancels out at the critical temperature and is negative below it), $\alpha\,(T) \propto (T - T_c)$, while $\beta\,(T) > 0$ for all temperatures. For $T > T_c$, we obtain a stable solution at $\psi = 0$, which means that the disordered phase, with high symmetry, is the most stable phase. The disordered phase corresponds to the normal state, since there is no condensation and therefore no correlation between the particles. For $T < Tc$, we obtain two stable solutions for ψ, different from zero. This means that below the critical temperature, the low symmetry ordered phase is the most stable. The ordered phase corresponds to the superconducting state of the material, since a condensate has formed and the electrons are therefore perfectly correlated.

The sought-after ψ function is that which minimizes Gibbs' free energy, and to determine it, we can therefore apply the variational principle. it consists to cancel the variation in Gibbs energy, by considering ψ and $\nabla\psi$ independent variables [1]:

$$\delta G = 0 \Leftrightarrow \delta \int_V \; g(\psi\,; \nabla\psi) dV = 0 \tag{2.4}$$

$$\Leftrightarrow \int_V \; \left(\frac{\partial g}{\partial \psi}\,\delta\psi + \frac{\partial g}{\partial \nabla\psi}\,\delta\nabla\psi\right) dV = 0 \tag{2.5}$$

With:

$$g = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left(-i\hbar\nabla\psi + \frac{e^*}{c}A\psi\right)^2 + \frac{B^2}{8\pi} \tag{2.6}$$

After that we us the following relation:

$$\nabla\cdot\left(\frac{\partial g}{\partial\nabla\psi}\,\delta\psi\right) = \delta\psi\nabla\cdot\left(\frac{\partial g}{\partial\nabla\psi}\right) + \frac{\partial g}{\partial\nabla\psi}\cdot\nabla\delta\psi \tag{2.7}$$

And the fact that the variational principle allows us to limit the variations such that the relation $\delta\nabla\psi = \nabla\delta\psi$ is right, equation (2.6) becomes:

$$\int_V \; \left[\frac{\partial g}{\partial\psi}\,\delta\psi + \nabla\cdot\left(\frac{\partial g}{\partial\nabla\psi}\,\delta\psi\right) - \nabla\cdot\left(\frac{\partial g}{\partial\nabla\psi}\right)\delta\psi\right] dV = 0 \tag{2.8}$$

The second term integral can be transformed to surface integral by the Gauss theorem:

$$\int_V \; \nabla\cdot\left(\frac{\partial g}{\partial\nabla\psi}\,\delta\psi\right) dV = \int_S \; \left(\frac{\partial g}{\partial\nabla\psi}\right)\delta\psi\mathbf{n}\; dS \tag{2.9}$$

Where $\boldsymbol{n}$ is the unit vector perpendicular to the surface of the superconductor. If we replace (2.8) in (2.7) and require this equation to remain true for any variation $\delta\psi$, we find the following equations:

$$\frac{\partial g}{\partial \psi} - \nabla \cdot \left(\frac{\partial g}{\partial \nabla \psi}\right) = 0 \tag{2.10}$$

$$\left(\frac{\partial g}{\partial \nabla \psi}\right) \cdot \mathbf{n} = 0 \tag{2.11}$$

After replacing $\boldsymbol{g}$ formula in these equations, we find the first Ginzburg-Landau equation and the boundary condition:

$$\alpha\psi^* + \beta\psi^*|\psi^*|^2 + \frac{1}{2m^*}\left[\frac{ie^*\hbar}{c}A\nabla\psi^* + \left(\frac{e^*}{c}\right)^2 A^2\psi^*\right] - \frac{1}{2m^*}\nabla\left(\hbar^2\nabla\psi^* - \frac{ie^*\hbar}{c}A\psi^*\right) = 0 \tag{2.12}$$

$$\Leftrightarrow \alpha\psi^* + \beta\psi^*|\psi^*|^2 + \frac{1}{2m^*}\left(i\hbar\nabla + \frac{e^*}{c}A\right)^2 \psi^* = 0 \tag{2.13}$$

And the boundary condition is

$$\left(i\hbar\nabla\psi^* + \frac{e^*}{c}A\psi^*\right) \cdot \mathbf{n} = 0 \tag{2.14}$$

Specifically, it is the conjugated equation of the Ginzburg-Landau equation. The boundary condition simply ensuring that the component of the current normal to the surface of the superconductor is zero, so there is no current crossing the surface of a superconductor, which is a natural hypothesis. Of course, we can do the same if you use ψ^*, you will find the usual Ginzburg-Landau equation:

$$\alpha\psi + \beta\psi|\psi|^2 + \frac{1}{2m^*}\left(i\hbar\nabla + \frac{e^*}{c}A\right)^2 \psi = 0 \tag{2.15}$$

By considering that $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$ we observe that Gibbs' free energy is also a functional for $\boldsymbol{A}$, we can so also apply the variational principle per relation to $\boldsymbol{A}$. The elimination of the variation of $\boldsymbol{G}$ for a variation of $\boldsymbol{A}$ gives us:

$$\delta G = 0 \Leftrightarrow \delta \int_V g(A\,; \nabla \times \mathrm{A})dV = 0 \tag{2.16}$$

$$\Leftrightarrow \int_V \left(\frac{\partial g}{\partial A}\,\delta A + \frac{\partial g}{\partial(\nabla \times A)}\delta(\nabla \times A)\right) dV = 0 \tag{2.17}$$

With:

$$g = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left(-i\hbar\nabla\psi + \frac{e^*}{c}A\psi\right)^2 + \frac{(\nabla \times A)^2}{8\pi} \tag{2.18}$$

Limited to variations such as $\delta(\nabla \times A) = \nabla \times \delta A$.After that we use the following relation:

$$\left(\frac{\partial g}{\partial(\nabla \times A)} \cdot (\nabla \times \delta A)\right) = \delta A \cdot \left(\nabla \times \frac{\partial g}{\partial(\nabla \times A)}\right) - \nabla \cdot \left(\frac{\partial g}{\partial(\nabla \times A)} \times \delta A\right) \tag{2.19}$$

The integral takes the following form:

$$\int_V \left(\frac{\partial g}{\partial A}\,\delta A += \delta A \cdot \left(\nabla \times \frac{\partial g}{\partial(\nabla \times A)}\right) - \nabla \cdot \left(\frac{\partial g}{\partial(\nabla \times A)} \times \delta A\right)\right) dV = 0 \tag{2.20}$$

Again, we can use Gauss's theorem to transform the volume integral of the third term into a surface integral:

$$\int_V \nabla \cdot \left(\frac{\partial g}{\partial(\nabla \times A)} \times \delta A\right) dV = \int_S \mathbf{n} \cdot \left(\frac{\partial g}{\partial(\nabla \times A)} \times \delta A\right) dS \tag{2.21}$$

The integral on the surface equals zero because the magnetic field is fixed to it, so we have that $\delta A|_S = 0$ [2]. By presupposing of equation (2.18) we find the following equation:

$$\frac{\partial g}{\partial A} + \left(\nabla \times \frac{\partial g}{\partial(\nabla \times A)}\right) = 0 \tag{2.22}$$

After the replacing of $\boldsymbol{g}$, we find the second Ginzburg-Landau equation:

$$\mathbf{J} = \frac{c}{4\pi} \nabla \times (\nabla \times A) = -\frac{i\hbar e^*}{2m^*}\,(\psi^* \nabla \psi - \psi \nabla \psi^*) - \frac{e^{*2}}{m^* c}\,\mathrm{A}|\psi|^2 \tag{2.23}$$

Where $\boldsymbol{J}$ is the super current density given by Maxwell's equation $\mathbf{J} = \frac{c}{4\pi} \boldsymbol{\nabla} \times \boldsymbol{B}$.

2.3.2. SU(2) breaking symmetry and Ginzburg-Landau equation

Frequently, gauge theory was used to discuss the physical consequences of some interactions, such as spin-orbit coupling [7,8]. Yang and Mills [9] are the first to give a theoretical framework to the gauge symmetrization in the case of a Lie group of dimension higher than U(1) group. It turned out that non-abelian SU(2) symmetry can be an effective tool for describing some fundamental interactions. Zhi-qiang & al. [3,4] have tried to drive the Ginzburg-Landau equations in the SU(2)-breaking symmetry background. Although there are some insufficiencies in the GL equation of free energy provided, especially in the description of the radiation part, where the given expression in these posts designates the U(1) symmetry.

For the spin superconductor in the presence of an external electric field $\boldsymbol{E}$, the free energy can be written as:

$$F_s = F_n + \alpha(T)|\psi(\boldsymbol{r})|^2 + \frac{1}{2}\beta(T)|\psi(\boldsymbol{r})|^4 + \frac{1}{2m^*}|(-i\hbar\nabla + \alpha_0 \boldsymbol{s} \times \boldsymbol{\nabla}\varphi)\psi(\boldsymbol{r})|^2 + \frac{1}{2}\epsilon_0(\nabla\varphi)^2 \tag{2.24}$$

The fourth term can be seen as the kinetic energy and the spin–orbit coupling term where α_0 is the coefficient of the spin–orbit coupling. The last term in equation (2.24) is theenergy of the electric field.

So we use the variational method to obtain the GL-type equations. The minimization of free energy with respect to the complex conjugate of the wave function gives:

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m^*}[-i\hbar\nabla + \alpha_0(\boldsymbol{s} \times \nabla\varphi)]^2\psi = 0 \tag{2.25}$$

For the boundary condition:

$$[-i\hbar\boldsymbol{\nabla} + \alpha_0(\boldsymbol{s} \times \boldsymbol{\nabla}\varphi)]_n\psi(\boldsymbol{r}) = 0 \tag{2.26}$$

The super-current formula is written as:

$$\boldsymbol{j}_s = Re(\psi^*\hat{v}\psi) = \frac{i\hbar}{2m^*}(\psi^*\nabla\psi - \psi\nabla\psi^*) + \frac{\alpha_0}{m^*}|\psi|^2(\boldsymbol{s} \times \nabla\varphi) \tag{2.27}$$

With $\hat{v}$ is the velocity operator. By substituting$\psi(r) = \sqrt{n_s(r)}e^{i\theta(r)}$ into equation (2.27), we obtain

$$\boldsymbol{j}_s = \frac{n_s(r)}{m^*}[\hbar\nabla\theta + \alpha_0(\boldsymbol{s} \times \nabla\varphi)] \tag{2.28}$$

Here, $n_s(r) = |\psi(r)|^2$ is the spin superfluid density. Equation (2.28) can be seen as the generalized London-type equation. This is manifested as follows. If $\boldsymbol{n_s}$ is independent of $\boldsymbol{r}$, and take the curl of equation (2.28), we can get $\nabla \times \boldsymbol{j}_s = -\frac{\alpha_0 n_s}{m^*}[(\nabla \cdot \boldsymbol{E})\boldsymbol{s} - (\boldsymbol{s} \cdot \nabla)\boldsymbol{E}]$. In addition, the derivation of the second London-type equation does not consider the effect of electric charge in the system. Thus, we can take $\nabla \cdot \boldsymbol{E} = 0$ then we obtain $\nabla \times \boldsymbol{j}_s = \frac{\alpha_0 n_s}{m^*}(\boldsymbol{s} \cdot \nabla)\boldsymbol{E}$. It is the same as the second London-type equation.

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Chapter 03: Dynamic Breaking Of The SU(2)×U(1) Symmetry and G-L Equations

3.1. Introduction

The idea on which our study rests is that the superconductor-normal phase transition scenario at a critical temperature T_c is accompanied by a breaking symmetry of the SU(2)×U(1) group to the U(1) subgroup. Since the Cooper pair is composed of two electrons of opposite spins, the total spin of the particle is zero. We have inspired our idea of the isospin concept, introduced in 1932 by W. Heisenberg [1], translates the fact that the proton and the neutron which are two distinct states can form a state of a single particle.

3.2. SU(2)×U(1) symmetry and G-L equations

. Given that, these bosons are scalar particles, so we can use the Lagrangian of the complex scalar field coupled to the vector potential of the Maxwell electromagnetic field $\boldsymbol{A}_i$ and the vector potential of the Yang-Mills field $\boldsymbol{W}_\mu$ [2].

$$\mathcal{L} = -|\boldsymbol{\mathcal{D}}_u|^2 - \frac{1}{4}\mathcal{F}_{uv}\mathcal{F}^{uv} + \frac{1}{4}\mathcal{G}^a_{uv}\mathcal{G}^{auv} + V(\varphi^+\varphi) \tag{3.1}$$

Where the complex scalar field (the order parameter) is a Pauli spinor $\varphi = \binom{\varphi_\uparrow}{\varphi_\downarrow}$ The space-time covariant derivative is given by $\boldsymbol{\mathcal{D}}_u = \boldsymbol{\partial}_u - ie^*\boldsymbol{A}_u - i\eta\mathcal{W}^{\boldsymbol{a}}_{\boldsymbol{u}}\tau^a$ for the derivative.

$\boldsymbol{\mathcal{F}}_{uv} = \partial_u A_v - \partial_v A_u$ and $\mathcal{G}^a_{uv} = \partial_u\mathcal{W}^a_v - \partial_v\mathcal{W}^a_u - \eta\varepsilon_{abc}\mathcal{W}^b_v\mathcal{W}^c_u$ are the U(1) and SU(2) field strength tensors, respectively. The Greek superscripts/subscripts stand for 0,1,2,3 and the Latin ones for 1, 2, 3. For the potential $V\,(\varphi^+\varphi)$ is defined by the following formula:

$$V(\varphi^+\varphi) = \lambda(\varphi^+\varphi) + \frac{\beta}{2}(\varphi^+\varphi)^2 \tag{3.2}$$

λ and β are arbitrary parameters and the shape of the potential depends on the sign of λ. For a value of $\lambda < 0$, the potential curve as a function of φ shows a shape of a Mexican-hat. This situation corresponds to the temperatures of the system below than T_c. For $\lambda > 0$, the potential curve has a parabolic form.

In the following study we considered the stationary electronic state, i.e. the order parameter is time independent.

The quantification of Gibbs energy passes through calculating the equations of motion with respect to the non-abelian and abelian gauge fields components, which makes us face some constraints summarized in the equation (3.3):

$$\begin{cases} \partial_u \boldsymbol{\mathcal{G}}^{uva} + \boldsymbol{\mathcal{J}}_u^a \varepsilon_{abc} \boldsymbol{\mathcal{G}}^{uva} \mathcal{W}\,_u^b = 0 \\ \partial_{mu} \boldsymbol{\mathcal{F}}^{uv} + j_u = 0 \end{cases} \tag{3.3}$$

Or the non-abelian and abelain current density take the following expression:

$$\begin{cases} \boldsymbol{\mathcal{J}}_u^a = i\eta(\varphi^+ \tau^a \partial_u \varphi - \varphi \tau^a \partial_u \varphi^+) + 2\eta e^* \boldsymbol{A}_u(\varphi^+ \tau^a \varphi) + 2\eta \mathcal{W}_u^a(\varphi^+ \varphi) \\ j_u = ie(\varphi^+ \partial_u \varphi - \varphi \partial_u \varphi^+) + 2e^{*^2} \boldsymbol{A}_u(\varphi^+ \varphi) + 2\eta \mathcal{W}_u^a(\varphi^+ \tau^a \varphi) \end{cases} \tag{3.4}$$

In order to obtain the Gibbs free energy, it is requisite to determine the Hamiltonian of the system which is given by the Legendre transformation of the Lagrangian density (3.1).

$$\mathcal{H} = p^+ \partial_0 \varphi^+ + p \partial_0 \varphi + \boldsymbol{\mathcal{P}}^u \partial_0 \boldsymbol{A}_u + \Pi_u^a \partial_0 \mathcal{W}_u^a - \boldsymbol{\mathcal{L}} \tag{3.5}$$

Where p^+,p,$\boldsymbol{\mathcal{P}}^u$and Π_u^a are the conjugate momentums of the canonical variables φ^+, φ, $\boldsymbol{A}_u$and $\mathcal{W}_u^a$, respectively. They are defined by the following relations:

$$p^+ = \frac{\partial \boldsymbol{\mathcal{L}}}{\partial \partial_0 \varphi^+} \;;\; p = \frac{\partial \boldsymbol{\mathcal{L}}}{\partial \partial_0 \varphi} \;;\; \boldsymbol{\mathcal{P}}^u = \frac{\partial \boldsymbol{\mathcal{L}}}{\partial \partial_0 \boldsymbol{A}_u} \;;\; \Pi_u^a = \frac{\partial \boldsymbol{\mathcal{L}}}{\partial \partial_0 \mathcal{W}_u^a} \tag{3.6}$$

A direct calculation leads to collide with a second group of constraints which are:

$$\begin{cases} \boldsymbol{\mathcal{P}}^0 = \dfrac{\partial \boldsymbol{\mathcal{L}}}{\partial \partial_0 \boldsymbol{A}_i} = 0 \\ \Pi_0^a = \dfrac{\partial \boldsymbol{\mathcal{L}}}{\partial \partial_0 \mathcal{W}_0^a} = 0 \end{cases} \tag{3.7}$$

To cope with these constraints, it is necessary to fix the appropriate gauge as in electrodynamics case. In the theory of non-abelian gauges, problems of gauge-fixing are often encountered. The Coulomb gauge espoused in U(1) breaking symmetry leads to annoying complications represented in Gribov ambiguity [3]. To avoid these obstacles, we relied on what is known as axial gauge where$\mathcal{W}_3^a = 0$.

Then, the obtained Hamiltonian density can be expressed as:

$$\mathcal{H} = \alpha(\varphi^+ \varphi) + \frac{\beta}{2}(\varphi^+ \varphi)^2 + |\partial_0 \varphi|^2 + 2\eta e^* A_0 \mathcal{W}_0^a \varphi^+ \tau^a \varphi + \frac{1}{2m^*}|(\partial_i - ie^* A_i - i\eta \mathcal{W}_m^a \tau^a)\varphi|^2 - \frac{1}{2}\left(\mathcal{F}_{0i}\mathcal{F}^{0i} + \frac{1}{2}\mathcal{F}_{ij}\mathcal{F}^{ij}\right) - \mathcal{F}_{0i}\partial_i A_0 - \frac{1}{2}\left(\mathcal{G}_{m0}^a \mathcal{G}^{am0} + \mathcal{G}_{mn}^a \mathcal{G}^{amn} + \mathcal{G}_{m3}^a \mathcal{G}^{am3} + \mathcal{G}_{03}^a \mathcal{G}^{a03}\right) - \mathcal{G}_{m0}^a \partial_m \mathcal{W}_0^a + \frac{1}{2}\partial_3 \mathcal{W}_m^a \partial_3 \mathcal{W}_m^a, \tag{3.8}$$

where $\alpha = \left(2e^{*2}{A_0}^2 + 2\eta^2 \mathcal{W}_0^a\, \mathcal{W}^{a0} - \lambda\right)$. The Latin superscripts and subscripts *m* and *n* now running over the values 1 and 2. The presence of the temporal components in the expression of α indicates the critical temperature can be manipulated by applying an electric potential. It is an impressive result. This indicates that the *measured transition temperature* differs from the material's *proper transition temperature.*

For the fifth and seventh terms we can rewrite them, basing on the equation (3.3), as follow:

$$\begin{cases} \mathcal{G}_{m0}^a \partial_m \mathcal{W}\,_0^a = \partial_m\left(\mathcal{G}_{m0}^a \mathcal{W}\,_0^a\right) - \mathcal{J}_0^a \mathcal{W}\,_0^a - \varepsilon_{abc} \mathcal{G}^{uva} \mathcal{W}\,_u^b \mathcal{W}\,_0^a \\ \mathcal{F}_{i0} \partial_i A_0 = \partial_i(\mathcal{F}_{i0} A_0) - j_0 A_0 \end{cases} \tag{3.9}$$

To obtain the Hamiltonian, we integrate equation (3.8) with respect to the volume of the system and time, what will make the fifth term disappear. Indeed, this term is a divergence which can therefore be transformed into a surface integral by applying the Gauss theorem. We replaced the formulas of $\mathcal{J}_0^a$ and j_0 and we found the final shape of the Hamiltonian:

$$H = \int dt \int \mathcal{H} dx^3 = \int dt \int \Big[\alpha(\varphi^+\varphi) + \frac{\beta}{2}(\varphi^+\varphi)^2 - \frac{1}{2m^*}|(\partial_i - ie^*A_i - i\eta \mathcal{W}_m^a \tau^a)\varphi|^2 + \frac{1}{2}(E^2 + B^2) + \frac{1}{2}\left(\mathcal{E}^{a^2} + \mathcal{B}^{a^2}\right)\Big]\, dx^3 \tag{3.10}$$

Where the electric and magnetic fields are given by: $E^2 = -\mathcal{F}_{0i}\mathcal{F}^{0i}$ and $B^2 = -\frac{1}{2}\mathcal{F}_{ij}\mathcal{F}^{ij}$.

For the non-abelian radiation part we put $\mathcal{E}^{a^2} = \left[-\frac{1}{2}\left(\mathcal{G}_{m0}^a \mathcal{G}^{am0} + \mathcal{G}_{m3}^a \mathcal{G}^{am3} + \mathcal{G}_{03}^a \mathcal{G}^{a03}\right) - \partial_m\left(\mathcal{G}_{m0}^a \mathcal{W}_0^a\right)\right]$ and $\mathcal{B}^{a^2} = -\frac{1}{2}(\mathcal{G}_{mn}^a \mathcal{G}^{amn} - \partial_3 \mathcal{W}_m^a \partial_3 \mathcal{W}_m^a)$. Here, we suppose that the order parameter depends only on its position in space. The equation (3.10) corresponds well to the internal energy U. The passage from internal energy to free energy is then done by admitting that the coefficient α depends on the temperature and this assumption justifies the use of Hamiltonian to describe Gibbs energy. Also, we note that the starting Lagrangian density was expressed in natural units, in which *ħ = c = 1,* while the Gibbs energy is expressed in (CGS) units. Through the previous equation, it is clear that, it is not possible to directly insert non-abelian potential in GL equation. Because we will not get a complete equation and we will drop important terms.

In order to obtain the first Landau-Ginzburg equation known as Schrodinger-like equation, it is enough to minimize the free energy with respect to the conjugate complex of the order parameter within the variational method framework we get the equation (3.11):

$$\alpha\varphi + \beta|\varphi|^2\varphi - \frac{1}{2m^*}|(\partial_i - ie^*\boldsymbol{A}_i - i\eta\mathcal{W}_m^a\tau^a)|^2\varphi = 0 \tag{3.11}$$

With a boundary condition which ensures that the normal component of the current at the surface of the superconductor is zero:

$$(\partial_i\varphi - ie^*\boldsymbol{A}_i\varphi - i\eta\mathcal{W}_m^a\tau^a\varphi)\cdot\mathbf{n} = 0 \tag{3.12}$$

The minimization over the abelian and non-abelian fields allows getting the second of GL equation which describes the spin-charge current density:

$$\mathbf{J}_{\mathrm{SU(2)\times U(1)}} = -\frac{ie^*}{2m^*}\left[(\varphi^+\partial_i\varphi - \varphi\partial_i\varphi^+) + \frac{\eta}{e^*}(\varphi^+\tau^a\partial_i\varphi - \varphi\tau^a\partial_i\varphi^+)\right] + \frac{e^{*2}}{m^*}\boldsymbol{A}_i(\varphi^+\varphi) + \frac{\eta^2}{m^*}\mathbf{W}_m^a(\varphi^+\varphi) - i\varphi^+Q\varphi \tag{3.13}$$

Where $Q = i\left[\left(\frac{\eta e^*}{m^*}(\boldsymbol{A}_i\mathbf{W}_m^a) + 2\eta e^*\boldsymbol{A}_0\mathcal{W}_0^a\right)\tau^a\right]$ is an hypercomplex number, known as quaternion. This quadruplet provides a convenient mathematical notation to represent the rotation of electrons in 3 dimensions. Despite the undeniable concrete performance of the quaternions in different fields, they are little used by physicists [4]. We will study the effect of this operator on spin rotations in future works.

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Conclusion

Conclusion

In this work we have derived the GL-type equations in the SU(2)×U(1) breaking symmetry case. To close our study, we will briefly present our findings in this section.

- By calculating the Gibbs energy from the Lagrangian density, it was found that it is not possible to directly insert or change an abelian electromagnetic potential with a non-abelian other one as the spin-orbit coupling or the Yang Milles potential.

- The existence of the temporal components in the expression of breaking symmetry potential constant α, indicates that the critical temperature can be manipulated by applying an electric potential. It is an impressive result. This indicates that the *measured transition temperature* differs from the material's *proper transition temperature.*

We hope to follow up this study in other research work by introducing direct applications to the obtained results which considered as a generalization of the Ginzburg-Landau equations.

ANNEX (1)

Electromagnetic tensor

In electromagnetism, the electromagnetic tensor or electromagnetic field tensor (sometimes called the field strength tensor, Faraday tensor or Maxwell bivector) is a mathematical object that describes the electromagnetic field in spacetime. The field tensor was first used after the four-dimensional tensor formulation of special relativity was introduced by Hermann Minkowski. The tensor allows related physical laws to be written very concisely, where the conventionally labelled F, is defined as the exterior derivative of the electromagnetic four-potential, A, a differential 1-form:[1][2]

$$F = dA$$

Therefore, F is a differential 2-form that is, an antisymmetric rank-2 tensor field on Minkowski space. In component form,

$$F_{uv} = \partial_u A_v - \partial_v A_u$$

Where ∂ is the four-gradient and A is the four-potential.

The electric and magnetic fields can be obtained from the components of the electromagnetic tensor. The relationship is simplest in Cartesian coordinates:

$$E_i = cF_{0i}$$

Where c is the speed of light, and

$$B_i = -\frac{1}{2}\epsilon_{ijk}F^{jk}$$

Where ϵ_{ijk} is the Levi-Civita tensor. This gives the fields in a particular reference frame; if the reference frame is changed, the components of the electromagnetic tensor will transform covariantly, and the fields in the new frame will be given by the new components. In contravariant matrix form

$$F^{uv} = \begin{bmatrix} 0 & -E_y/c & E_y/c & E_y/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_y/c & -B_x & B_x & 0 \end{bmatrix}$$

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ANNEX (2)

Non-abelian group

In mathematics, and specifically in group theory, a non-abelian group, sometimes called a non-commutative group, is a group $(G, *)$ in which there exists at least one pair of elements a and b of G, such that $a * b \neq b * a$.[1][2] This class of groups contrasts with the abelian groups. (In an abelian group, all pairs of group elements commute).

Non-abelian groups are pervasive in mathematics and physics. One of the simplest examples of a non-abelian group is the dihedral group of order 6. It is the smallest finite non-abelian group. A common example from physics is the rotation group SO(3) in three dimensions (for example, rotating something 90 degrees along one axis and then 90 degrees along a different axis is not the same as doing them the other way round).

Both discrete groups and continuous groups may be non-abelian. Most of the interesting Lie groups are non-abelian, and these play an important role in gauge theory.

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Abstract

Spin superconductivity is analogue of conventional charge superconductivity. Here, we derive a framework for describing the SU(2)×U(1) breaking symmetry in Ginzburg-Landau equations. We have found that it is not possible to directly insert or change an abelian electromagnetic potential with a non-abelian other one. The existence of the temporal components in the expression of breaking symmetry potential constant, indicates that the critical temperature can be controlled by applying an electric potential.

Key words: Ginzburg-Landau equations, breaking symmetry, Spin superconductivity, abelian and non-abelian electromagnetic potential

ملخص

الموصلية الفائقة للسبين مماثلة للموصلية الفائقة للشحنة التقليدية. في هذه المذكرة، لقد قمنا باشتقاق إطارًا لوصف كسر التناظر SU (2) × U (1) في معادلات Ginzburg-Landau. و وجدنا أنه من غير الممكن إدخال أو تغيير جهد كهرومغناطيسي تبديلي بشكل مباشر مع كمونات أخرى غير تبديلية. يشير وجود مركبات زمنية في عبارة ثابت الكمون الذي يصف إنكسار التناظر، إلى أنه يمكن التحكم في درجة الحرارة الحرجة عن طريق تطبيق جهد كهربائي.

الكلمات المفتاحية: معادلات Ginzburg-Landau، كسر التناظر، الموصلية الفائقة للسبين، جهد كهرومغناطيسي تبديلي و غير تبديلي.

Résumé

La supraconductivité de spin est analogue à la supraconductivité de charge conventionnelle. Ici, nous dérivons un cadre pour décrire la brisure de symétrie SU (2) × U (1) dans les équations de Ginzburg-Landau. Nous avons trouvé qu'il n'est pas possible d'insérer ou de changer directement un potentiel électromagnétique abélien avec un autre non-abélien. L'existence des composantes temporelles dans l'expression de la constante du potentiel décrivant la brisure symétrie, indique que la température critique peut être contrôlée en appliquant un potentiel électrique.

Mots clés : Equations de Ginzburg-Landau, brisure de symétrie, supraconductivité de spin, potentiel électromagnétique abélien et non-abélien,