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THEME

Backstepping and Sliding mode control for a Quadcopter

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ABSTRACT

The objective of this work is to present different methods for quadcopter control and their application , what are it's advantages and disadvantages , we designed a complete model of quadrotor that we used to study its behavior in different operating regimes and under different circumstances , then we presented an open loop and closed loop approach . after that we presented two leading theoretical approaches of two control techniques: backstepping control and sliding mode control.

Key words: Drone – Quadrotor – Quadcopter – State – space - Euler angle – Roll – Pitch – Yaw – PID – Controller – Stable – Input – Output - Sliding mode - Backstepping.

ملخص

الهدف من هذا العمل هو تقديم طرق مختلفة للتحكم طائر رباعي المحركات، بعد التعرف على رباعي المحركات الذي لدينا، ما هية مزاياه وعيوبه ، قمنا بتصميم نموذج كامل لرباعي المحركات الذي استخدمناه لدراسة سلوكه في أنظمة تشغيل مختلفة و في ظل ظروف مختلفة ، قدمنا نهج النمط الإنزلاقي والتحكم عن طريق الرجوع المرحلي. بعد ذلك قدمنا منهجين نظريين رائدين لتقنيتي تحكم: التحكم في الخلف والتحكم في وضع الانزلاق.

الكلمات الرئيسية:

طائرة بدون طيار - كوادروتر - كوادكوبتر - دولة - فضاء - زاوية أويلر - لفة - خطوة – ياو - مستقر - إدخال - إخراج - وضع منزلق - خطوة للخلف - PID تحكم

Résumé

L'objectif de ce travail est de présenter différentes méthodes de contrôle du quadcopter et leur application, nous sommes passés à en savoir plus sur ses avantages et ses inconvénients, nous avons conçu un modèle complet de quadrotor que nous avons utilisé pour étudier son comportement en différents régimes d'exploitation et dans différentes circonstances, nous avons ensuite présenté une approche en boucle ouverte et en boucle fermée. après cela, nous avons présenté deux principales approches théoriques de deux techniques de contrôle: la commande de Backstepping et la commande de mode glissant.

Mots clés: Drone – Quadrotor – Quadcopter – State – Space - Angle d'Euler – Roll – Pitch – Yaw - PID – Controller – Stable – Input – Output - Sliding mode - Backstepping.

DEDICATION

First of all we thank Allah for the guidance, strength, power of mind, protection and skills and for giving us a healthy life. This study is wholeheartedly dedicated to our beloved parents, who have been our source of inspiration and gave us strength ,and who continually provide the moral, spiritual, emotional, and financial support

APPRECIATION

We would like to give a special thanks to our mentors and teachers for their help and support throughout the years we spent in Amar Telidji university and specially throughout this work . And to all our friends, and classmates who shared their words of advice and encouragement to us finish this study.

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Summary

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NOMENCLATURE

CW	Clockwise.
CCW	Counter clockwise.
UAV	Unmanned Aerial Vehicles.
GPS	the Global Positioning System.
FAA	The Federal Aviation Administration.
ACLU	American Civil Liberties Union.
BLDC	Brushless DC motor.
PID	Proportional Integral Derivative.
UAS	Unmanned Aircraft System.
NED	North East Down.
ABC	Aircraft Body Center.
SMC	Sliding Mode Control.
DSMC	Dynamic Sliding Mode Controller.
HOSMC	High Order Sliding Mode Control.

General introduction

GENERAL INTRODUCTION

Recent advances in sensor, microcomputer technology control and in aerodynamics theory have made small Unmanned Aerial Vehicles (UAV) a reality. The vehicle consists of four rotors in total, with two pairs of counter-rotating, fixed-pitch blades located at the four corners of the aircraft. Due to its specific capabilities, the use of autonomous quadrotor vehicles has been envisaged for a variety of applications both as individual vehicles and in multiple vehicle teams, including surveillance, search and rescue and mobile sensor networks.

The small size, low cost and maneuverability of these systems have made them potential solutions in a large class of applications. However, the small size of these vehicles poses significant challenges. The small sensors used on these systems are much noisier than their larger counterparts. The compact structure of these vehicles also makes them more vulnerable to environmental effects. In this work, control strategies for an sUAV platform are developed. Simulation studies and experimental results are provided.

Since the number and complexity of applications for such systems grows daily, the control techniques involved must also improve in order to provide better performance and increased versatility. Historically, simplistic linear control techniques were employed for computational ease and stable hover flight. However, with better modelling techniques and faster on board computational capabilities, comprehensive nonlinear techniques to be run on real-time have become an achievable goal. Nonlinear methodologies promise to rapidly increase the performances for these systems and make them more robust. This work presents several approaches to the automatic control of a quadrotor.

The particular interest of the research community in the quadrotor design can be linked to two main advantages over comparable vertical take off and landing (VTOL) UAVs, such as helicopters. First, quadrotors do not require complex mechanical control linkages for rotor actuation, relying instead on fixed pitch rotors and using variation in motor speed for vehicle control. This simplifies both the design and maintenance of the vehicle. Second, the use of four rotors ensures that individual rotors are smaller in diameter than the equivalent main rotor on a helicopter, relative to the airframe size. The individual rotors, therefore, store less kinetic energy during flight, mitigating the risk posed by the rotors should they entrain any objects. Furthermore, by enclosing the rotors within a frame, the rotors can be protected from breaking during collisions, permitting flights indoors and in obstacle-dense environments, with low risk of damaging the vehicle, its operators, or its surroundings. These added safety benefits greatly accelerate the design and test flight process by allowing testing to take place indoors, by inexperienced pilots, with a short turnaround time for recovery from incidents.

Previous treatments of quadrotor vehicle dynamics have often ignored known aerodynamic effects of rotorcraft vehicles. At slow velocities, such as while hovering, this is indeed a reasonable assumption. However, even at moderate velocities, the impact of the aerodynamic effects resulting from variation in air speed is significant.

General introduction

Objective

the objective of this study is to identify the quadcopter from the outside to the inside starting with the types and advantages and disadvantages present some of it's components to the inside where we will provide the mathematical modeling of the quadcopter then build a control loop using the PID both in open loop and closed loop , in the end two theoretical approaches will be presented for future work.

Chapter I

Background and mathematical modeling of Quadcopter

CHAPTER I: BACKGROUND AND MATHEMATICAL MODELING OF QUADCOPTER

I.1. INTRODUCTION

A quadcopter, also called a quad rotor helicopter or quad rotor, is a multirotor helicopter that is lifted and propelled by four rotors. Quadcopters are classified as rotorcraft, as opposed to fix-edwing aircraft, because their lift is generated by a set of rotors (vertically oriented propellers).
Unlike most helicopters, quadcopters use two sets of identical fixed pitched propellers; two clockwise (CW) and two counter-clockwise (CCW). These use variation of RPM to control lift and torque. Control of vehicle motion is achieved by altering the rotation rate of one or more rotor discs, thereby changing its torque load and thrust/lift characteristics.

The quadrotor, an aircraft made up of four engines, holds the electronic board in the middle and the engines at four extremities. Before describing the mathematical model of a quadrotor, it is necessary to introduce the reference coordinates in which we describe the structure and the position. For the quadrotor, it is possible to use two reference systems. The first is fixed and the second is mobile. The fixed coordinate system, called also inertial, is a system where the first Newton's law is considered valid. As fixed coordinate system, we use the O_{NED} systems, where NED is for *North-East-Down*. As we can observe from the following Figure II.1, its vectors are directed to Nord, East and to the center of the Earth.

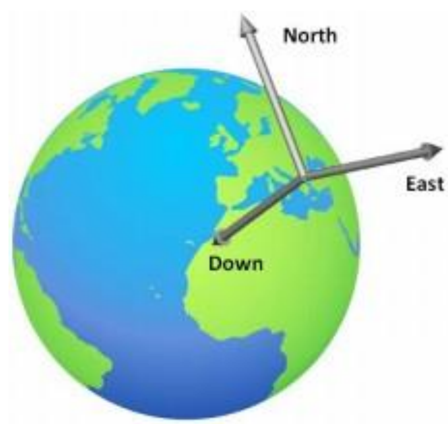


Figure I. 1: ONED fixed reference system[3].

The mobile reference system that we have previously mentioned is united with the barycenter of the quadrotor. In the scientific literature it is called O_{ABC} system, where ABC is for *Aircraft BodyCenter*. Figure II.1 illustrates underlines the two coordinate systems.

The attitude and position of the quadrotor can be controlled to desired values by changing the speeds of the four motors. The following forces and moments can be performed on the quadrotor: the thrust caused by rotors rotation, the pitching moment and rolling moment caused by the difference of four rotors thrust, the gravity, the gyroscopic effect, and the yawing moment. The gyroscopic effect only appears in the lightweight construction quadrotor. The yawing moment is caused by the unbalanced of the four rotors rotational speeds. The yawing moment can be cancelled out when two rotors rotate in the opposite direction. So, the propellers are divided in two groups.

I.2. QUADROTOR HISTORY

Etienne Oehmichen was the first scientist who experimented with rotorcraft designs in 1920[1]. Among the six designs he tried, his second multicopter had four rotors and eight propellers, all driven by a single engine. The Oehmichen used a steel-tube frame, with two-bladed rotors at the ends of the four arms. The angle of these blades could be varied by warping. Five of the propellers, spinning in the horizontal plane, stabilized the machine laterally. Another propeller was mounted at the nose for steering. The remaining pair of propellers was for forward propulsion. The aircraft exhibited a considerable d egree of stability and controllability for its time, and made more than a thousand test flights during the middle 1920. By 1923 it was able to remain airborne for several minutes at a time, and on April 14, 1924 it established the first-ever International Aeronautical Federation (IAF)[2] distance record for helicopters of 360 m. Later, it completed the first 1 kilometer closed-circuit flight by a rotorcraft.

After Oehmichen, Dr. George de Bothezat and Ivan Jerome developed this aircraft , with six bladed rotors at the end of an X-shaped structure. Two small propellers with variable pitch were used for thrust and yaw control. The vehicle used collective pitch control. It made its first flight in October 1922. About 100 flights were made by the end of 1923. The highest it ever reached was about 5 m. Although demonstrating feasibility, it was underpowered, unresponsive, mechanically complex and susceptible to reliability problems. Pilot workload was too high during hover to attempt lateral motion.

Convertawings Model A Quadrotor (1956) was intended to be the prototype for a line of much larger civil and military quadrotor helicopters[1]. The design featured two engines driving four rotors with wings added for additional lift in forward flight. No tail rotor was needed and control was obtained by varying the thrust between rotors. Flown successfully many times in the mid-1950s, this helicopter proved the quadrotor design and it was also the first four-rotor helicopter to demonstrate successful forward flight. However, due to the lack of orders for commercial or military versions however, the project was terminated. Convertawings proposed a Model E that would have a maximum weight of 19,000 kg with a payload of 4,900 kg.

I.3. QUADCOPTER FRAME TYPES

There are many different styles of frame, all related to the stance of the arms and the size and shape of the drones. Below, each frame type is explained along with a graphical example.

a. Type (X):

The true X is shaped as it sounds, an X geometry to which a motor is mounted to each end of the arms. The perpendicular distance between the center of each motor is equal, therefore giving the quadcopter the same level of stability on all axis.

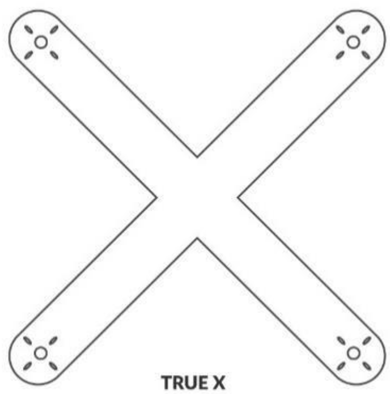


Figure I. 2: Type(X) drone [4].

b. Dead Cat

The dead cat style is typically favored by larger quadcopter designs. Its purpose is to remove the propellers from the sight of the on-board HD camera, this is achieved by increasing the perpendicular distance between the two frontal motors. The popularity of the dead cat design has sagged along with the increasing interest in smaller miniguides. Although, there are some mini and micro quads that continue to utilize the dead cat design, typically as a means of accommodating uniquely shaped center carriages.

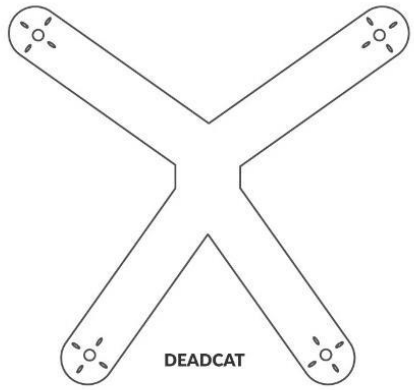


Figure I. 3: Dead cat drone [4].

a. H Shape

The H style is another archaic style of quadcopter design. In a H quad, the arms are positioned at the front of a long “bus” style carriage. Recently, the H quad has lost favor due to its bulky size and awkward configuration.

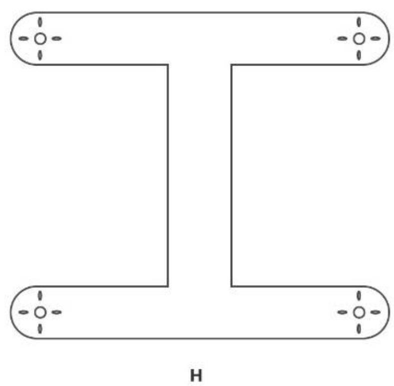


Figure I. 4: H shape drone [4].

b. HX Shape

The HX is a newer variant of the H. Instead of placing the arms at the tip and tail of the carriage, a true X, wide X or stretch X configuration is applied, most often wide or true X.

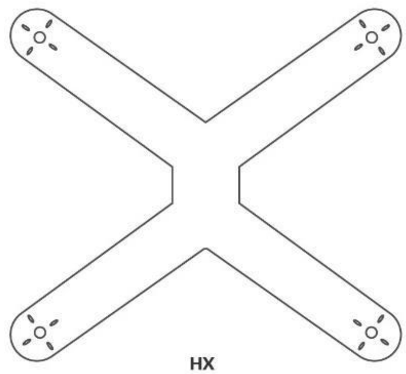


Figure I. 5: HX shape drone[4].

c. Plus Shape

A plus frame has the same footprint as a X frame that has been turned 45°. A plus frame can be seen as advantageous in that each motor is responsible for rotational movement in only one axis, theoretically meaning finer control is possible. Although, plus frames are more prone to breakage due to most impacts involving a forceful strike to the front arm only.

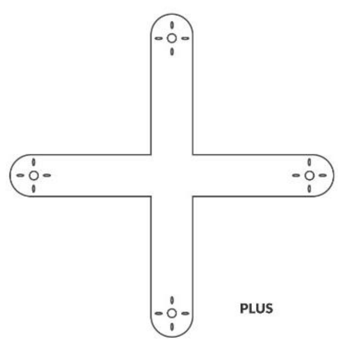


Figure I. 6: Plus shape drone [4].

d. Vertical Arms

Vertical arms rotate the orientation of the arms to produce as small of a surface area as possible to minimize drag. Durability is not usually compromised as the arm may still maintain width; however, construction of the frame is often more complex than standard horizontal frames.

Chapter I: background and mathematical modeling of Quadcopter

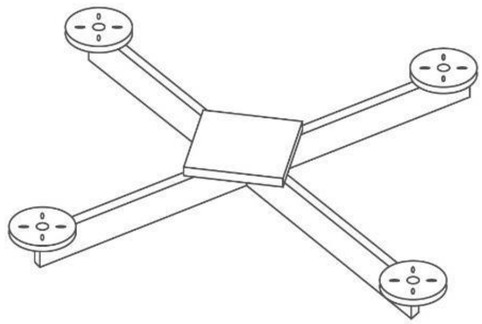


Figure I. 7: Vertical arms drone [4].

I.4. ADVANTAGES AND DISADVANTAGES OF UAVS

Unmanned Aerial Vehicles (UAVs), or drones, are aircraft that can be controlled remotely by a pilot, or by preprogrammed plans or automation systems that enable them to fly autonomously. A large number of industries and organizations are adopting this technology, including military, government, commercial, and recreational users.

As drone technology advances, these aircraft are becoming more common and affordable, giving rise to debates that weigh their benefits against new ethical and legal concerns. As such, the decisions derived from discussing the pros and cons of drones and UAVs is poised to carry a substantial impact on the private and public sector[6].

I.4.1. ADVANTAGES OF DRONES

To properly weigh the pros and cons of drones and their use, it's important to examine both sides of the debate on their own merits. For instance, there are plenty of positive reasons to use drones.

A. Quality Aerial Imaging

Drones are excellent for taking high-quality aerial photographs and video, and collecting vast amounts of imaging data. These high-resolution images can be used to create 3-D maps and interactive 3-D models, which have many beneficial uses. For example, 3-D mapping of disaster areas can enable rescue teams to be better prepared before entering hazardous situations.

B. Precision

Since unmanned aerial vehicles use GPS (the Global Positioning System), they can be programmed and maneuvered accurately to precise locations. This is especially helpful in a variety of situations. In precision agriculture, for example, UAVs are used for a variety of farming needs such as spraying fertilizer and insecticide, identifying weed infestations, and monitoring crop health. The precision of UAVs saves farmers both time and cost.

C. Easily Deployable

With advances in control technology, most drones can be deployed and operated with relatively minimal experience. Combined with the relatively low cost of most models, drones are becoming accessible to a wide range of operators. UAVs also have a greater range of movement than manned aircraft. They are able to fly lower and in more directions, allowing them to easily navigate traditionally hard-to-access areas.

D. Security

Another plus to drone use centers on security. With the appropriate license, operators can use unmanned aerial vehicles to provide security and surveillance to private companies, sporting events, public gatherings, and other venues. Drones can also gather valuable data during and after natural disasters to aid in security and recovery efforts.

I.2.2. DISADVANTAGES OF DRONES

While there are numerous pros to using drones, there are also several perceived challenges to their deployment. These concerns are important to consider, particularly given the wide range of circumstances in which drones can be used[6].

a. Legislative Uncertainty

Since the widespread use of unmanned aerial vehicles is relatively new, legislation is still catching up. The Federal Aviation Administration (FAA) has established certain rules for small, unmanned aircraft that apply to commercial and recreational use, but there are still ambiguities. Questions include how best to determine airspace property rights and protect landowners from aerial trespassing. Further adding to the confusion are conflicts between federal regulations and some state and local laws.

b. Safety

Safety is a primary concern when dealing with unmanned aerial vehicles. To avoid mid-air collisions, UAVs must be programmed with “sense and avoid” capabilities that match those of manned aircraft. This means that drones must be able to detect a potential collision and maneuver to safety. In the event of system failures, falling drones are another serious danger, especially when they are used near large crowds.

c. Privacy

One of the most common concerns from the public about UAVs is privacy. Drones can collect data and images without drawing attention, leading many Americans to fear their Fourth Amendment right to privacy may be in jeopardy. This can occur if government entities were to use drones to monitor the public. The way in which the Fourth Amendment is interpreted, and the efforts of privacy rights organizations such as the American Civil Liberties Union (ACLU), continue to influence how this issue of privacy is regulated.

I.5. TYPICAL USES

Drones have been around for many years, they are used for several different purposes and can be very helpful in many areas. However, drones have become much more popular in recent times, and their application has increased rapidly in various fields.

I.6. PRINCIPAL FUNCTIONING OF DRONES

In order to understand better how drones fly and work, we need to know some basic information about the construction of a typical drone. unmanned aerial vehicles (UAVs) are made from different light composite materials in order to increase maneuverability while flying and reduce weight. they can be equipped with a variety of additional equipment, including cameras, GPS guided missiles, Global Positioning Systems (GPS), navigation systems, sensors, and various other drone software and hardware[7].

Drones can come in a broad range of shapes, sizes, and with various functions. The vast majority of today's models can be launched by hand, and they can be operated with a remote control or from special ground cockpits. The commercial models come in small sizes and have simplified construction. These drones are suitable even for kids because they are very easy to control.

There are different variations in the frame and construction of drones, but the essential components that every drone must have is a waterproof motor frame, flight and motor controllers, motors, transmitter and receiver, propellers, and a battery or any other source of energy.

Yes, these small aerial acrobats are able to pack a lot of accessories into their metal or plastic frames. Most drones are ready to fly upon purchase. However, some people like building a drone themselves using kits

What makes these unmanned aerial vehicles remarkable is their great flight capability. Drones have ultra-stable flight, and they can hover and perform different acrobatics in the air. How far you can fly your drone depends on the space you are flying it, as well as the line of sight.

When it comes to commercial drones for entertainment, they have a short control distance and can fly for just a few minutes. On the contrary, the advanced drones which are used in the military or mapping are able to hover for hours in the air and be controlled from a great distance. Military drones are often the most advanced, and in fact are a large source of innovation in drone technology[7].

And today the quadcopters are used in so many domains such as :

- Aerial photography and video aerial inspection, leisure, agriculture, construction, security;
- Aerial Photography & Videography;
- Real estate photography;
- Mapping & Surveying;
- Asset Inspection;
- Payload carrying;
- Agriculture;
- Bird Control;
- Crop spraying;
- Crop monitoring;
- Multispectral/thermal/NIR cameras;
- Live streaming events;
- Roof inspections;
- Emergency Response;
- Search and Rescue;
- Marine Rescue;
- Disaster zone mapping;
- Disaster Relief;
- Forensics;
- Mining;
- Firefighting;
- Monitoring Poachers;
- Insurance;
- Aviation;
- Meteorology;
- Product Delivery;

I.7. CLASSIFICATION DIFFERENT TYPES OF DRONES

I.7.1. NANO AND MINI DRONES

The smallest drones on the planet are members of these categories. They can be divided into two groups: Nano and Mini drones.

Nano drones are the smallest and they usually have the same dimensions as insects. On the other side, mini drones can reach up to 50cm in length and they have more powerful electric motors and more advanced features than Nano drones.

In general, models from both categories are used by the military, in spying and smaller tasks, due to the fact they can be easily maneuvered and they can reach remote locations. On the other side, there are several models on the market that are available to ordinary consumers[4].

I.7.2. SMALL SIZES DRONES

Drones from this group have dimensions between 50 cm and 2 m. They don't have powerful motors so they must be thrown into the air, in order to start flying.

These products are also very popular on the market because they have great features and they are more affordable than bigger drones.

These drones are also the most common type of drones available to average customers. Simply said, most drones that you can see on the market are members of this group. All of them have a radius of 5 km and they can fly between 20 and 40 minutes.

I.7.3. MEDIUM SIZES DRONES

Every drone that has a wingspan between 5 and 10 m falls into the medium category. These drones can carry up to 200 kg of weight and they have powerful motors.

These drones are usually used for transporting goods, to remote locations and by the military. Keep in mind that these drones are still smaller and lighter than other light aircraft. They can fly up to 50 km and their flying time can be long as 6 hours.

I.7.4. BIG DRONES

A drone with a wingspan bigger than 10m is a member of this group. In general, these drones have the same dimensions as smaller aircraft and they are used by the army primarily. A civilian cannot buy this drone, due to the fact it is treated as an aircraft.

Most drones are equipped with weapons and missiles, so they are used in tactical attacks or when sending human pilots is a high risk. It is believed that big drones will completely replace the aircraft that require a pilot in the near future.

Of course, this type of aircraft comes with its own host of political and moral concerns. Primarily, the fact that it removes people from the consequences of their actions, so the potential for carelessness and collateral damage is high. This article will not dive into these issues, as we are just focusing on drone technology, but it is important to be aware of the amount of concerns and debates concerning military drone usage.

Chapter I: background and mathematical modeling of Quadcopter

Due to the fact that there are a lot of different drones from this group, they can be classified in these smaller groups for further specification:

a) **Short range models:** They can fly up to 150 km and more than 11 hours.
b) **Drones withy medium range:** These drones can have a range of 650 km and they are commonly used in weather forecasting.
c) **Drones with the longest radius:** Drones from this group can fly up to 9 km. above the sea level and they can stay in the air more than 36 hours. The main goal of these drones is to reach remote distances.

I.8. MOTOR AND CONTROLLER CHOICE

After getting to know the quadcopter now we have to know it's main part , the rotors which allow it to hover and move , these rotors are DC motors that are controlled by a controller (PID).

I.9. DC MOTORS

After getting to know the quadcopter now we have to know it's main part , the rotors which allow it to hover and move , these rotors are DC motors that are controlled by a controller (PID).
The DC motor is a machine that transforms electric energy into mechanical energy in form of rotation. Its movement is produced by the physical behavior of electromagnetism. DC motors have inductors inside, which produce the magnetic field used to generate movement.

I.10. THE DRONE USE BRUSHLESS MOTOR

A brushless DC motor (also known as a BLDC motor or BL motor) is an electronically commuted DC motor which does not have brushes. The controller provides pulses of current to the motor windings which control the speed and torque of the synchronous motor.
These types of motors are highly efficient in producing a large amount of torque over a vast speed range. In brushless motors, permanent magnets rotate around a fixed armature and overcome the problem of connecting current to the armature. Commutation with electronics has a large scope of capabilities and flexibility. They are known for smooth operation and holding torque when stationary.

a. Advantages of brushless DC motor

The advantages of a BLDC motor are:

1. Brushless motors are more efficient as its velocity is determined by the frequency at which current is supplied, not the voltage.
2. As brushes are absent, the mechanical energy loss due to friction is less which enhanced efficiency.
3. BLDC motor can operate at high-speed under any condition.
4. There is no sparking and much less noise during operation.
5. More electromagnets could be used on the stator for more precise control.
6. BLDC motors accelerate and decelerate easily as they are having low rotor inertia.
7. It is high performance motor that provides large torque per cubic inch over a vast sped rang.
8. BLDC motors do not have brushes which make it more reliable, high life expectancies, and maintenance free operation.
9. There is no ionizing sparks from the commutator, and electromagnetic interference is also get reduced.
10. Such motors cooled by conduction and no air flow are required for inside cooling.

b. Disadvantages of brushless DC motors

The disadvantages of a BLDC motor are:

1. BLDC motor cost more than a brushed DC motor.
2. The limited high power could be supplied to BLDC motor, otherwise, too much heat weakens the magnets and the insulation of winding may get damaged.

I.11. QUADROTOR DYNAMIC MODEL

two groups. In each group there are two diametrically opposite motors that we can easily observe thanks to their direction of rotation. Namely, we distinguish:

- Front and rear propellers (numbers 2 and 4 in Figure II.3), rotating coun- terclockwise;
- Right and left propellers (numbers 1 and 3 in Figure II.3), rotating clockwise.

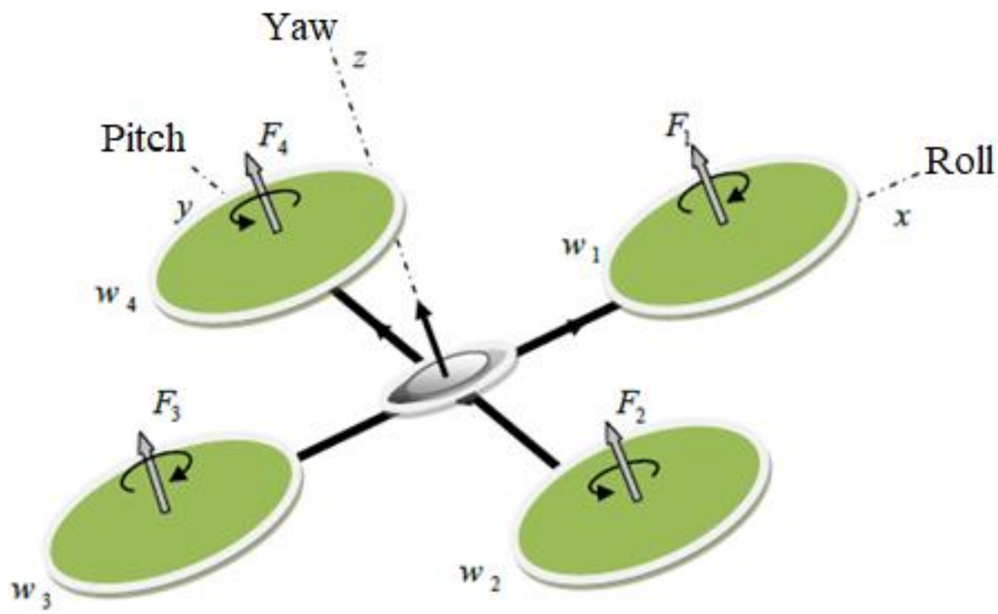


Figure I. 8: General structure of a quadrotor [5].

I.11.1. THE MOVEMENTS OF THE QUADROTOR

In conventional helicopters, when the main rotor turns, it produces a reactive torque which would cause the body of the helicopter to turn in the opposite direction if this torque is not countered. This is usually done by adding a tail rotor which produces thrust in a lateral direction. However, this rotor with its associated power supply makes no contribution to the thrust. On the other hand, in the case of a quadcopter, the right rotor and the left rotor turn clockwise and the rotors in the opposite direction.

front and rear, this effectively neutralizes the unwanted reactive torque and allows the vehicle to hover without turning out of control. Besides, differently to conventional helicopters, all the energy expended to counteract the rotational movement contributes to the pushing force [8].

The basic quadrotor movements are achieved by varying the speed of each rotor thereby changing the thrust produced. The quadrotor tilts towards the direction of the slower rotor, which then allows for translation along that axis. Therefore, as in a conventional helicopter, the movements are coupled, meaning that the quadrotor cannot translate without rolling or pitching, which means that a change in the speed of a rotor results in a movement in at least three degrees of freedom. For example, increasing the speed of the left thruster will result in a rolling motion (the quadrotor tilts towards the slower rotor, to the right), a yawing motion (the balance between the rotors turning clockwise). 'a watch and the rotors which turns in the opposite direction is disturbed resulting in a horizontal rotational movement), and a translation (the rolling movement inclines the armature and with it, the orientation of the pushing force). This coupling is the reason we can control the six degrees of freedom of the quadrotor with just four controls (the torque applied by the motors to each thruster).

The quadrotor has five main movements:

- Vertical movement
- Roll rotation
- Pitch rotation
- Yaw rotation
- Horizontal translations

I.11.2. Vertical movement

In order to hover, the entire lift force should only be along the z axis with a magnitude exactly opposite to the force of gravity. Besides, the lift force created by each rotor must be equal to prevent the vehicle from overturning more. Consequently, the thrust produced by each rotor must be identical.

The upward and downward movement is obtained by the variation of the rotation speed of the motors (consequently the thrust produced), if the lift force is greater than the weight of the quadrotor the movement is upward, and if the lift force is lower than the weight of the quadrotor the movement is descending.

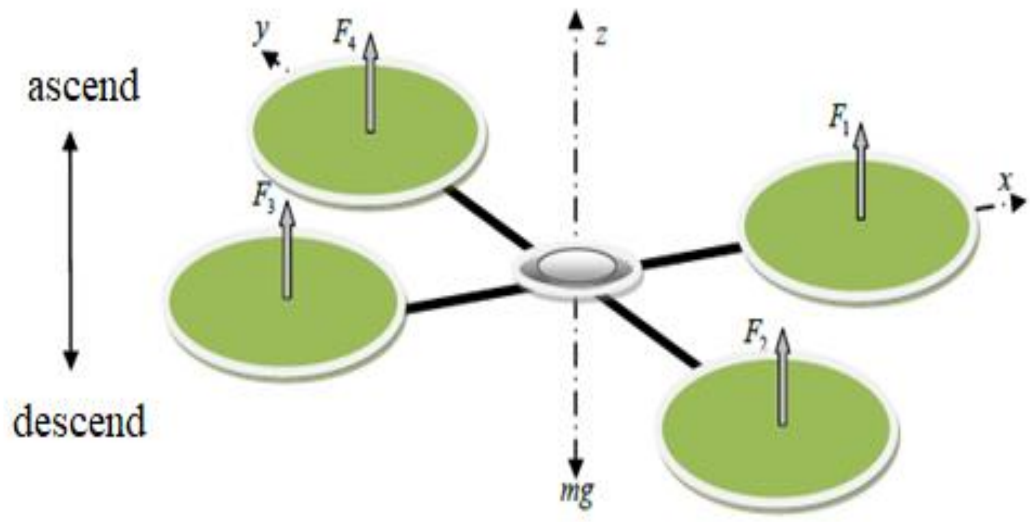


Figure I. 9: Vertical mouvement [5].

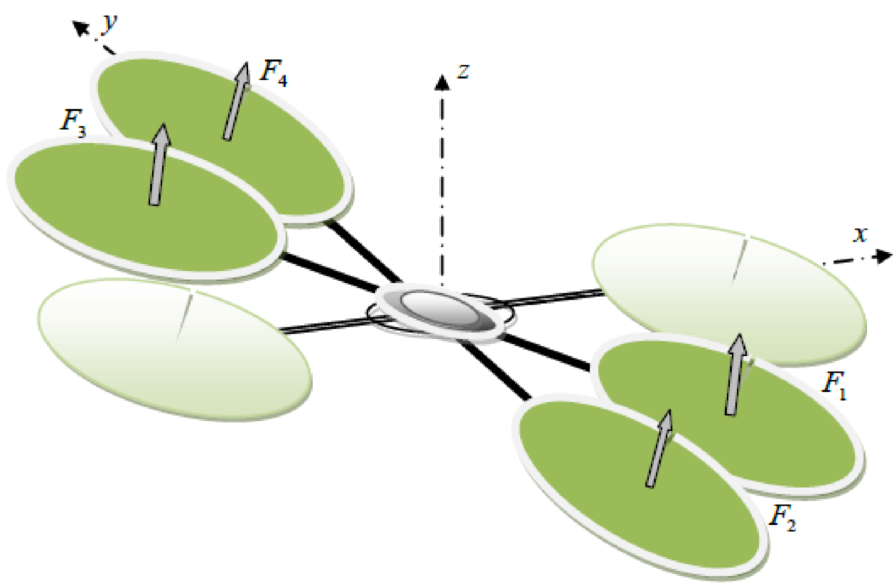


Figure I. 10: Roll rotation [5].

I.11.3. ROLL ROTATION

In this case, a torque is applied around the x axis, i.e. by applying a difference in thrust between the rotor 2 and the rotor 4. This movement (rotation around the x axis) is coupled with a translational movement along the y axis.

I.11.4. PITCH ROTATION

In this case, a torque is applied around the y axis, that is to say by applying a thrust difference between the rotor 1 and the rotor 3. This movement (rotation around y) is coupled with a movement. Of translation along the x axis.

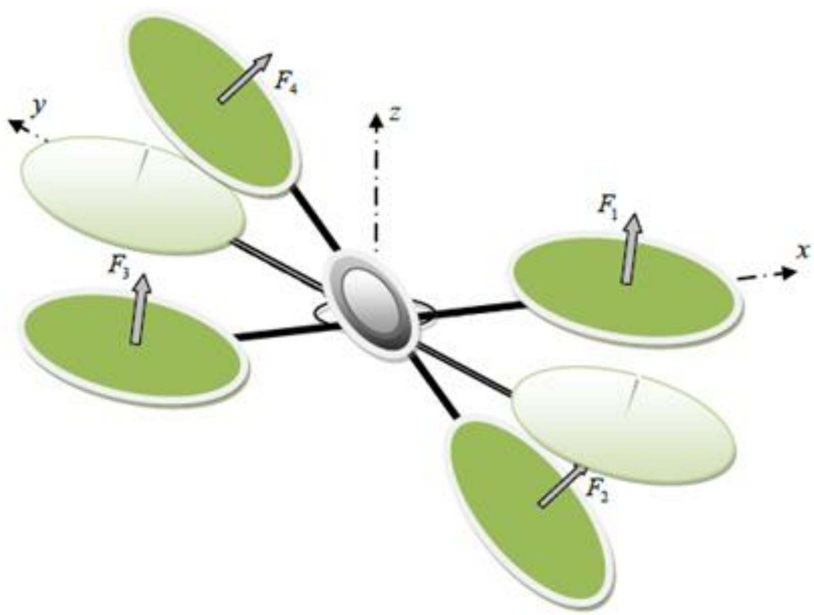


Figure I. 11: Pitch rotation [5].

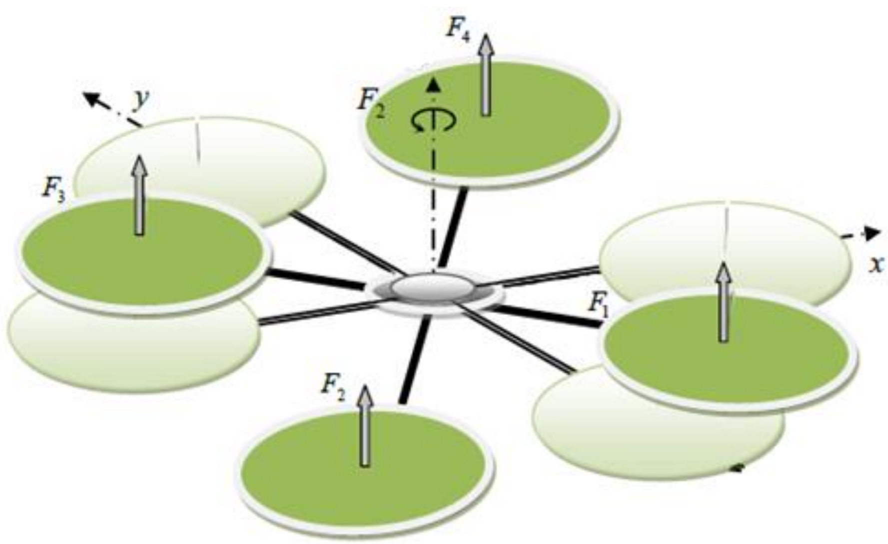


Figure I. 12: Yaw rotation [5].

I.11.5. YAW ROTATION

In this case, we want to apply a torque around the z axis, which is done by applying a speed difference between the rotors {1,3} and {2,4}. This movement is not a direct result of the thrust produced by the thrusters but by the reactive torques produced by the rotation of the rotors. The direction of the pushing force does not shift during movement, but the increase in the lift force in one pair of rotors must be equal to the decrease in the other pairs to ensure that all the pushing force remains the same.

I.11.6. TRANSLATION MOVEMENTS

In this case, we want to apply a force along x or y which is done by tilting the body (by pitching or rolling) and increasing all the thrust produced to keep the importance of the z component of the thrust equal to the force of gravity.

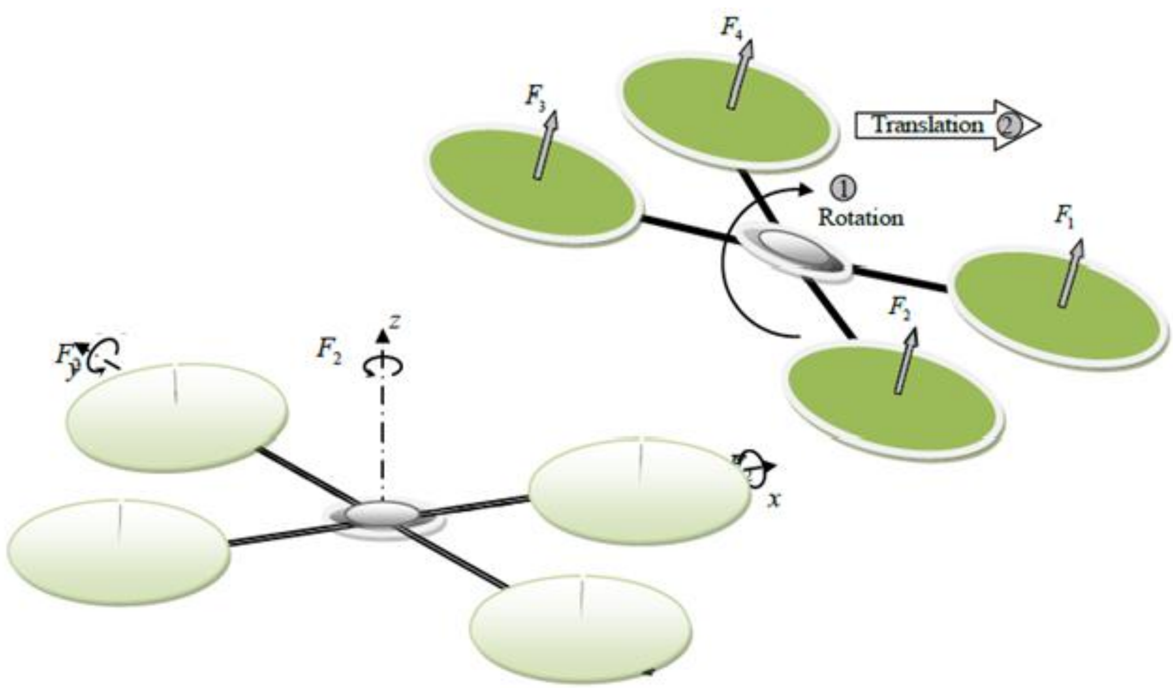


Figure I. 13: Translational movement [5].

I.12. QUADROTOR MATHEMATICAL MODEL

I.12.1. EULER ANGLES

The Euler angles are three angles introduced by Leonhard Euler to describe the orientation of a rigid body. To describe such an orientation in the 3-dimensional Euclidean space, three parameters are required. They can be given in several ways; we will use ZYX Euler angles [9]. They are also used to describe the orientation of a frame of reference relative to another and they transform the coordinates of a point in a reference frame in the coordinates of the same point in another reference frame. The Euler angles are typically denoted as $\phi \in]-\pi, \pi], \theta \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $\psi \in]-\pi, \pi]$ Euler angles represent a sequence of three elemental rotations, i.e. rotations about the axes of a coordinate system, since any orientation can be achieved by composing three elemental rotations. These rotations start from a known standard orientation. This combination used is described by the following rotation matrices [10]:

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c(\phi) & -s(\phi) \\ 0 & s(\phi) & c(\phi) \end{bmatrix} \quad \text{(I.1)}$$

$$R_y(\theta) = \begin{bmatrix} c(\theta) & 0 & s(\theta) \\ 0 & 1 & 0 \\ -s(\theta) & 0 & c(\theta) \end{bmatrix} \quad \text{(I.2)}$$

$$R_z(\psi) = \begin{bmatrix} c(\psi) & -s(\psi) & 0 \\ s(\psi) & c(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{(I.3)}$$

Where

$c(\varphi) = cos(\varphi), s(\varphi) = sin(\varphi), c(\theta) = cos(\theta), s(\theta) = sin(\theta), c(\psi) = cos(\psi), s(\psi) = sin(\psi)$.

So, the inertial position coordinates and the body reference coordinates are related by the rotation matrix $\boldsymbol{R}_{zyx}(\varphi, \theta, \psi) \in SO(3)$:

$$R_{xyz}(\phi, \theta, \psi) = R_x(\phi) \cdot R_y(\theta) \cdot R_z(\psi)$$

$$= \begin{bmatrix} c(\theta)c(\psi) & s(\phi)s(\theta)c(\psi) - c(\phi)s(\psi) & c(\phi)s(\theta)c(\psi) + s(\phi)s(\psi) \\ c(\theta)s(\psi) & s(\phi)s(\theta)c(\psi) + c(\phi)s(\psi) & c(\phi)s(\theta)s(\psi) - s(\phi)c(\psi) \\ -s(\theta) & s(\phi)c(\theta) & c(\phi)c(\theta) \end{bmatrix} \quad \text{(I.4)}$$

This matrix describe the rotation from the body reference system to the inertial reference.

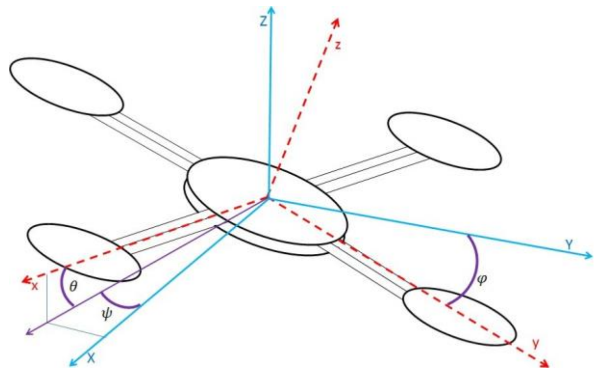


Figure I. 14: Euler Angles.

I.12.2. MATHEMATICAL MODELING

We're going to provide a mathematical model of the quadrotor, exploiting Newton and Euler equations for the 3D motion of a rigid body. The goal of this section is to obtain a deeper understanding of the dynamics of the quadrotor and to provide a model that is sufficiently reliable for simulating and controlling its behavior. Let us call $[x \ \ y \ \ z \ \ \varphi \ \ \theta \ \ \psi]^T$ the vector containing the linear and angular position of the quadrotor in the earth frame and $[u \ \ v \ \ w \ \ p \ \ q \ \ r]^T$ the vector containing the linear and angular velocities in the body frame. From 3D body dynamics, it follows that the two reference frames are linked by the following relations:

$$v = R \cdot \mathrm{v}_B, \tag{I.5}$$

$$\omega = \mathrm{T} \cdot \omega_{\mathrm{B}}, \tag{I.6}$$

Where $\mathrm{v} = [\dot{\mathrm{x}} \ \ \dot{\mathrm{y}} \ \ \dot{\mathrm{z}}]^{\mathrm{T}} \in \mathbb{R}^3, \boldsymbol{\omega} = \left[\dot{\phi} \ \ \dot{\theta} \ \ \dot{\psi}\right]^{\mathrm{T}} \in \mathbb{R}^3, \mathrm{v}_{\mathrm{B}} = [\mathrm{u} \ \ \mathrm{v} \ \ \mathrm{w}]^{\mathrm{T}} \in \mathbb{R}^3, \omega_{\mathrm{B}} = [\mathrm{p} \ \ \mathrm{q} \ \ \mathrm{r}]^{\mathrm{T}} \in \mathbb{R}^3$,and T is a matrix for angular transformation [12].

$$\mathrm{T} = \begin{bmatrix} 1 & s(\phi)t(\theta) & c(\phi)t(\theta) \\ 0 & c(\phi) & -s(\phi) \\ 0 & \frac{s(\phi)}{c(\theta)} & \frac{c(\phi)}{c(\theta)} \end{bmatrix} \tag{I.7}$$

where $t(\theta) = tan(\theta)$. So, the kinematic model of the quadrotor is:

$$\begin{cases} \dot{x} = w[s(\phi)s(\psi) + c(\phi)c(\psi)s(\theta)] - v[c(\phi)s(\psi) - c(\psi)s(\phi)s(\theta)] + [c(\psi)c(\theta)] \\ \dot{y} = v[c(\phi)c(\psi) + s(v)s(\psi)s(\theta)] - w[c(\psi)s(\phi) - c(\phi)s(\psi)s(\theta)] + u[c(\theta)s(\psi)] \\ \dot{z} = w[c(\phi)c(\theta)] - u[s(\theta)] + v[c(\theta)s(\phi)] \\ \dot{\varphi} = p + r[c(\phi)t(\theta)] + q[s(\phi)t(\theta)] \\ \dot{\theta} = q[c(\phi)] - r[s(\phi)] \\ \dot{\psi} = r\frac{c(\phi)}{c(\theta)} + q\frac{s(\phi)}{c(\theta)} \end{cases} \tag{I.8}$$

Newton's law states the following matrix relation for the total force acting on the quadrotor:

$$m(\omega_B \wedge v_B + \dot{\omega}_B) = f_B \tag{I.9}$$

where m is the mass of the quadrotor, $\wedge$ is the cross prodauct and $f_B = [f_x \ f_y \ f_z]^T \in \mathbb{R}^3$ is the total force. Euler's equation gives the total torque applied to the quadrotor:

$$I \cdot \dot{\omega}_B + \omega_B \wedge (I \cdot \omega_B) = m_B \tag{I.10}$$

where $m_B = [m_x \, m_y \, m_z]^T \in \mathbb{R}^3$ is the total torque and I is the diagonal inertia matrix :

$$I = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \in \mathbb{R}^{3\times3}$$

So, the dynamic model of the quadrotor in the body frame is:

$$\begin{cases} f_x = m(\dot{u} + qw - rv) \\ f_y = m(\dot{v} - pw + ru) \\ f_z = m(\dot{w} + pv - qu) \\ m_x = \dot{p}I_x - qrI_y + qrI_z \\ m_y = \dot{q}I_x + prI_y - prI_z \\ m_z = \dot{r}I_x - pqI_y - pqI_z \end{cases} \tag{I.11}$$

The equations stand as long as we assume that the origin and the axes of the body frame coincide with the barycenter of the quadrotor and the principal axes.

I.12.3. FORCES AND MOMENTS

The external forces in the body frame, $\mathbf{f}_B$ are given by

$$f_B = mgR^T \cdot \hat{e}_z - f_t\hat{e}_3 + f_w \tag{I.12}$$

where $\hat{e}_z$is the unit vector in the inertial z axis, $\hat{e}_3$ is the unit vector in the body z axis, g is the gravitational acceleration, f_t is the total thrust generated by rotors and $\mathbf{f}_w = [f_{wx} \, f_{wy} \, f_{wz}]^T \in \mathbb{R}^3$ are the forces produced by wind on the quadrotors. The external moments in the body frame, m_B are given by

$$m_B = \tau_B - g_a + \tau_w \tag{I.13}$$

where g_a represents the gyroscopic moments caused by the combined rotation of the four rotors and the vehicle body, $\boldsymbol{T_B} = \left[T_x\ T_y\ T_z\right]^{\boldsymbol{T}} \in \mathbb{R}^{\mathbf{3}}$ are the control torques generated by differences in the rotor speeds and $\boldsymbol{T_w} = \left[T_{wx}\ T_{wy}\ T_{wz}\right]^{\boldsymbol{T}} \in \mathbb{R}^{\mathbf{3}}$ are the torques produced by wind on the quadrotors. g_a is given by

$$g_a = \sum_{i=1}^{4} J_p(\omega_B \wedge \hat{e}_3)(-1)^{i+1}\Omega_i \tag{I.14}$$

where J_p is the inertia of each rotor and i is the angular speed of the i_{th} rotor. According to [3], the J_p term is found to be small and, for this reason, the gyroscopic moments are removed in the controller formulation. In addition, there are numerous aerodynamic and aeroelastic phenomenon that affect the flight of the quadrotor, such as the ground effects: when flying close to the ground (or during the landing stage), the air flow generated by the propellers disturbs the dynamics of the quadrotors. So, the complete dynamic model of the quadrotor in the body frame is obtained substituting the force expression in (I.11):

$$\begin{cases} -mg[s(\theta)] + f_{wx} = m(\dot{u} + qw - rv) \\ mg[c(\theta)s(\phi)] + f_{wy} = m(\dot{v} - pw + ru) \\ mg[c(\theta)c(\phi)]f_{wz} - f_t = m(\dot{w} + pv - qu) \\ \tau_x + \tau_{wx} = \dot{p}I_x - qrI_y + qrI_z \\ \tau_y + \tau_{wy} = \dot{q}I_x + prI_y - prI_z \\ \tau_z + \tau_{wz} = \dot{r}I_x - pqI_y - pqI_z \end{cases} \tag{I.15}$$

I.12.4. ACTUATOR DYNAMICS

Here we consider the inputs that can be applied to the system in order to control the behavior of the quadrotor. The rotors are four and the degrees of freedom we control are as many: commonly, the control inputs that are considered are one for the vertical thrust and one for each of the angular motions. Let us consider the values of the input forces and torques proportional to the squared speeds of the rotors [11]; their values are the following:

$$\begin{cases} f_t = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ \tau_x = b(\Omega_3^2 - \Omega_1^2) \\ \tau_y = b(\Omega_4^2 - \Omega_2^2) \\ \tau_z = b(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) \end{cases} \tag{I.16}$$

where l is the distance between any rotor and the center of the drone, b is the thrust factor and d is the drag factor. Substituting (I.16) in (I.15), we have: So, the dynamic model of the quadrotor in the body frame is:

$$\begin{cases} -mg[s(\theta)] + f_{wx} = m(\dot{u} + qw - rv) \\ mg[c(\theta)s(\phi)] + f_{wy} = m(\dot{v} - pw + ru) \\ mg[c(\theta)c(\phi)]f_{wz} - b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) = m(\dot{w} + pv - qu) \\ b(\Omega_3^2 - \Omega_1^2) + \tau_{wx} = \dot{p}I_x - qrI_y + qrI_z \\ b(\Omega_4^2 - \Omega_2^2) + \tau_{wy} = \dot{q}I_x + prI_y - prI_z \\ b(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) + \tau_{wz} = \dot{r}I_x - pqI_y - pqI_z \end{cases} \tag{I.17}$$

I.12.5. STATE-SPACE MODEL

Organizing the state's vector in the following way:

$$\mathrm{x} = [\phi \quad \theta \quad \psi \quad p \quad q \quad r \quad u \quad v \quad w \quad x \quad y \quad z]^T \in \mathbb{R}^{12} \tag{I.18}$$

it is possible to rewrite the equations of the dynamics of the quadrotor in the state-space from (I.8) and (I.15):

$$\begin{cases} \dot{\varphi} = p + r[c(\phi)t(\theta)] + q[s(\phi)t(\theta)] \\ \dot{\theta} = q[c(\phi)] - r[s(\phi)] \\ \dot{\psi} = r\frac{c(\phi)}{c(\theta)} + q\frac{s(\phi)}{c(\theta)} \\ \dot{p} = \frac{I_y - I_z}{I_x}rq + \frac{\tau_x + \tau_{wx}}{I_x} \\ \dot{q} = \frac{I_z - I_x}{I_y}rq + \frac{\tau_y + \tau_{wy}}{I_y} \\ \dot{r} = \frac{I_x - I_y}{I_z}rq + \frac{\tau_z + \tau_{wz}}{I_z} \\ \dot{u} = rv - rw - g[s(\theta)] + \frac{f_{wx}}{m} \\ \dot{v} = pw - ru + g[s(\phi)c(\theta)] + \frac{f_{wy}}{m} \\ \dot{w} = qu - pv + g[c(\theta)c(\phi)] + \frac{f_{wz} - f_t}{m} \\ \dot{x} = w[s(\phi)s(\psi) + c(\phi)c(\psi)s(\theta)] - v[c(\phi)s(\psi) - c(\psi)s(\phi)s(\theta)] + u[c(\psi)c(\theta)] \\ \dot{y} = v[c(\phi)c(\psi) + s(\phi)s(\psi)s(\theta)] - w[c(\psi)s(\phi) - c(\phi)s(\psi)s(\theta)] + u[c(\theta)s(\psi)] \\ \dot{z} = w[c(\phi)c(\theta)] - u[s(\theta)] + v[c(\theta)s(\phi)] \end{cases} \tag{I.19}$$

Below we obtain two alternative forms of the dynamical model useful for studying the control. From Newton's law we can write:

$$m\dot{v} = \mathrm{R} \cdot \mathrm{f}_{\boldsymbol{B}} = mg\hat{e}_z - f_t R \cdot \hat{e}_3 \tag{I.20}$$

therefore:

$$\begin{cases} \ddot{x} = -\frac{f_t}{m}[s(\phi)s(\psi) + c(\phi)c(\psi)s(\theta)] \\ \ddot{y} = -\frac{f_t}{m}[c(\phi)s(\psi)s(\theta) - c(\psi)s(\phi)] \\ \ddot{z} = g - \frac{f_t}{m}[c(\phi)c(\theta)] \end{cases} \tag{I.21}$$

Now a simplification is made by setting $\left[\dot{\phi}\ \dot{\theta}\ \dot{\psi}\right]^T = [p\ q\ r]^T$. This assumption holds true for small angles of movement [12]. So, the dynamic model of the quadrotor in the inertial frame is:

$$\begin{cases} \ddot{x} = -\frac{f_t}{m}[s(\phi)s(\psi) + c(\phi)c(\psi)s(\theta)] \\ \ddot{y} = -\frac{f_t}{m}[c(\phi)s(\psi)s(\theta) - c(\psi)s(\phi)] \\ \ddot{z} = g - \frac{f_t}{m}[c(\phi)c(\theta)] \\ \ddot{\phi} = \frac{I_y - I_z}{I_x}\dot{\theta}\dot{\psi} + \frac{\tau_x + \tau_{wx}}{I_x} \\ \ddot{\theta} = \frac{I_z - I_x}{I_y}\dot{\phi}\dot{\psi} + \frac{\tau_y + \tau_{wy}}{I_y} \\ \ddot{\psi} = \frac{I_x - I_y}{I_z}\dot{\phi}\dot{\theta} + \frac{\tau_z + \tau_{wz}}{I_z} \end{cases} \tag{I.22}$$

Redefining the state's vector as:

$$\mathrm{x} = \left[x\ \ y\ \ z\ \ \phi\ \ \theta\ \ \psi\ \ \dot{x}\ \ \dot{y}\ \ \dot{z}\ \ \dot{\phi}\ \ \dot{\theta}\ \ \dot{\psi}\ \right]^T \in \mathbb{R}^{12} \tag{I.23}$$

it is possible to rewrite the equations of the quadrotor in the spate-space:

$$\dot{\mathrm{x}} = \mathrm{f}(\mathrm{x}) + \textstyle\sum_{i+1}^{4} \mathrm{g}_i\,(\mathrm{x})\mathrm{u}_i \tag{I.24}$$

Where $$f(x) = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ q\frac{s(\phi)}{c(\theta)} + r\frac{c(\phi)}{c(\theta)} \\ q[c(\phi)] - r[s(\phi)] \\ p + q[c(\phi)t(\theta)] + r[s(\phi)t(\theta)] \\ 0 \\ 0 \\ g \\ \frac{I_y - I_z}{I_x} rq \\ \frac{I_z - I_x}{I_y} rq \\ \frac{I_x - I_y}{I_z} rq \end{bmatrix} \tag{I.25}$$

And

$$g_1(x) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & g_{7_1} & g_{8_1} & g_{9_1} & 0 & 0 & 0 \end{bmatrix}^T \in \mathbb{R}^{12}$$
$$g_2(x) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{I_x} & 0 & 0 \end{bmatrix}^T \in \mathbb{R}^{12}$$
$$g_3(x) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{I_y} & 0 \end{bmatrix}^T \in \mathbb{R}^{12}$$
$$g_4(x) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{I_z} \end{bmatrix}^T \in \mathbb{R}^{12}$$

With

$$g_{7_1} = -\frac{1}{m}[s(\phi)s(\psi) + c(\phi)c(\psi)s(\theta)]$$
$$g_{8_1} = -\frac{1}{m}[c(\psi)s(\phi) - c(\phi)s(\psi)s(\theta)]$$
$$g_{9_1} = -\frac{1}{m}[c(\phi)c(\theta)]$$

I.12.6. LINEAR MODEL

Set u the control vector: $\mathbf{u} = [\boldsymbol{f_t}\ T_{\boldsymbol{x}}\ T_{\boldsymbol{y}}\ T_{\boldsymbol{z}}]^{\boldsymbol{T}} \in \mathbb{R}^{\mathbf{4}}$. The linearization's procedure is developed around an equilibrium point _x, which for fixed input $\bar{u}$ is the solution of the algebric system: or rather that value of state's vector, which on fixed constant input is the solution of algebraic system:

$$\hat{\mathrm{f}}\,(\bar{x}, \bar{u}) = 0 \tag{I.26}$$

Since the function $\hat{\mathrm{f}}$ is nonlinear, problems related to the existence an uniqueness of the solution of system (I.26) arise. In particular, for the system in hand, the solution is difficult to find in closed form because of trigonometric functions related each other in no-elementary way. For this reason, the linearization is performed on a simplified model called to small oscillations. This simplification is made by approximating the sine function with its argument and the cosine function with unity. The approximation is valid if the argument is small. The resulting system is described by the following equations:

$$\begin{cases} \dot{\varphi} \approx p + r\theta + q\phi\theta \\ \dot{\theta} \approx q - r\phi \\ \dot{\psi} \approx r + q\phi \\ \dot{p} \approx \frac{I_y - I_z}{I_x} rq + \frac{\tau_x + \tau_{wx}}{I_x} \\ \dot{q} \approx \frac{I_z - I_x}{I_y} rq + \frac{\tau_y + \tau_{wy}}{I_y} \\ \dot{r} \approx \frac{I_x - I_y}{I_z} rq + \frac{\tau_z + \tau_{wz}}{I_z} \\ \dot{u} \approx rv - rw - g\theta + \frac{f_{wx}}{m} \\ \dot{v} \approx pw - ru + g\phi + \frac{f_{wy}}{m} \\ \dot{w} \approx qu - pv + g + \frac{f_{wz} - f_t}{m} \\ \dot{x} \approx w[\phi\psi + \theta] - v[\psi - \phi\theta] + u \\ \dot{y} \approx v[1 + \phi\psi\theta] - w[\phi - \psi\theta] + u\psi \\ \dot{z} \approx w - u\theta + v\phi \end{cases} \tag{I.27}$$

which can be written in the compact form

$$\dot{\mathrm{x}} = \mathrm{f}(\mathrm{x}, \mathrm{u}) \tag{I.28}$$

I.13. CONCLUSION

In this chapter we learned about quadcopters and their main part (the rotors) , after that we got know what kind of motor are we going to use and which controller are we relying on to control these four rotors.

Any system has advantage and disadvantage and Drones cannot be exceptional. Drones are aircraft devices that are capable of flying and carrying materials above the ground. The formal name of the device is an unmanned aerial vehicle (UAV). These devices are part of unmanned aircraft system (UAS). The components of these devices are communication systems, a ground-based controller, and unmanned aerial equipment. The controllers of an unmanned aerial vehicle can be both either an automated set of computers or a human operator. Drones provide useful missions to prevent injuries, accidents, and hazards when it involves navigating in an environment that is not suitable for human survival. Today, drones are now undergoing a commercialization process because it is now being manufactured and produced by technological and logistics companies around the world.

After that we learned what is a quadcopter , how it operates ,it's movements and the physical effects acting on the quadrotor and it's dynamic equations all in details , in other words we learned how our Quadcopter functions and what can effect it while operating , this drone is based on four rotors which allow us to move in any direction while manipulating the rotor's speeds.

The use of the Newton-Euler formalism allowed us to establish the dynamic model of the quadrotor, From the model obtained, we conclude that the quadcopter is an under-actuated system. In addition, the complexity of the model, the non-linearity, and the interaction between the states of the system, can be clearly seen.

In the next chapter we're going to apply a PID controller to our model and build simulation models using MATlab Simulink.

Chapter II

Control methods of quadrotor

CHAPTER II: CONTROL METHODS OF QUADROTOR

II.1. INTRODUCTION

The function of any electronic system is to automatically regulate the output and keep it within the systems desired input value or “set point”. If the systems input changes for whatever reason, the output of the system must respond accordingly and change itself to reflect the new input value.

Likewise, if something happens to disturb the systems output without any change to the input value, the output must respond by returning back to its previous set value. In the past, electrical control systems were basically manual or what is called an Open-loop system with very few automatic control or feedback features built in to regulate the process variable so as to maintain the desired output level or value[13].

An open-loop system, also referred to as non-feedback system, is a type of continuous control system in which the output has no influence or effect on the control action of the input signal. In other words, in an open-loop control system the output is neither measured nor “fed back” for comparison with the input. Therefore, an open-loop system is expected to faithfully follow its input command or set point regardless of the final result.

Also, an open-loop system has no knowledge of the output condition so cannot self-correct any errors it could make when the preset value drifts, even if this results in large deviations from the preset value.

Another disadvantage of open-loop systems is that they are poorly equipped to handle disturbances or changes in the conditions which may reduce its ability to complete the desired task.

This type of manual open-loop control which reacts before an error actually occurs is called Feed forward Control.

The objective of feed forward control, also known as predictive control, is to measure or predict any potential open-loop disturbances and compensate for them manually before the controlled variable deviates too far from the original set point.

II.2. PID INTRODUCTION

The best-known controllers used in industrial control processes are proportional-integral-derivative (PID) controllers because of their simple structure and robust performance in a wide range of operating conditions. The design of such a controller requires specification of three parameters: proportional gain, integral time constant and derivative time constant .so far, great effort has been devoted to develop methods to reduce the time spent on optimizing the choice of controller parameters .

Setting the PID parameters are called tuning ,the classical tuning method is to use the famous Ziegler-Nichols tuning formula. This tuning formula however, is not always effective in the sense that it may completely fail to tune the processes with, for example, relatively large dead time. Moreover, its tuning will have to be supplemented with purely experience-based fine-tuning to meet the specifications .

The parameters of the PID controller kp, ki and kd (or kp, Ti and Td) can be manipulated to produce various response curves from a given process as we will see later.

PID controllers are commonly used to regulate the time-domain behavior of many different types of dynamic plants. These controllers are extremely popular because they can usually provide good closed-loop response characteristics, can be tuned using relatively simple design rules, and are easy to construct using either analog or digital components.

DC motors have speed-control capability, which means that speed, torque and even direction of rotation can be changed at any time

II.2.1. PID BASED CONTROLLER

Proportional Integral Derivative (PID) is one of the techniques that can be employed on the given hardware. The plant is controlled by PID based control system for now. The hardware at hand has the potential of employing more advanced control techniques such as Sliding Mode Control and backstepping control.

II.2.2. REASONS FOR CHOOSING PID

The rationale behind the selection of a PID Controller is its simplicity and easy implementation. The PID gains can be designed upon the system parameters if they are estimated precisely. Moreover, the PID gains can be designed by treating the system as a 'Black Box' and by just tracking the error. However, one needs to balance all three parameters of PID and in an attempt to do so, one may compromise transient response of the system, such as settling time, overshoots etc.

II.2.3. FEEDBACK LOOP

The Quadcopter plant represents an under-actuated system. The system has four virtual control inputs , which are u_1, u_2, u_3 and u_4. These four input states are used to monitor twelve states. For the sake of simplicity, initially, four states were monitored using the four control inputs and it was assumed that controlling these four states will lead to control of the whole Quadcopter. PID was implemented on all four inputs with different gains. Height and the three angles (yaw, pitch and roll) were fed back and the errors served as input to the PID controllers as shown in the figure above. All the states, needed to be controlled, were fed back the variable as inputs in their differential equations. For u_1 (first step input), z (height) is fed back into it accordingly with the differential equations stated above. Similarly. for u_2, u_3 and u_4 the Euler angles, theta, psi and phi, are fed back into the feedback loop respectively.

II.3. OPEN LOOP APPROACH

II.3.1. OPEN-LOOP CONTROL SYSTEMS SUMMARY

We have seen that a controller can manipulate its inputs to obtain the desired effect on the output of a system. One type of control system in which the output has no influence or effect on the control action of the input signal is called an Open-loop system.

An "open-loop system" is defined by the fact that the output signal or condition is neither measured nor "fed back" for comparison with the input signal or system set point. Therefore open-loop systems are commonly referred to as "Non-feedback systems".

Also, as an open-loop system does not use feedback to determine if its required output was achieved, it "assumes" that the desired goal of the input was successful because it cannot correct any errors it could make, and so cannot compensate for any external disturbances to the system[13].

II.3.2. OPEN LOOP SIMULATION RESULTS

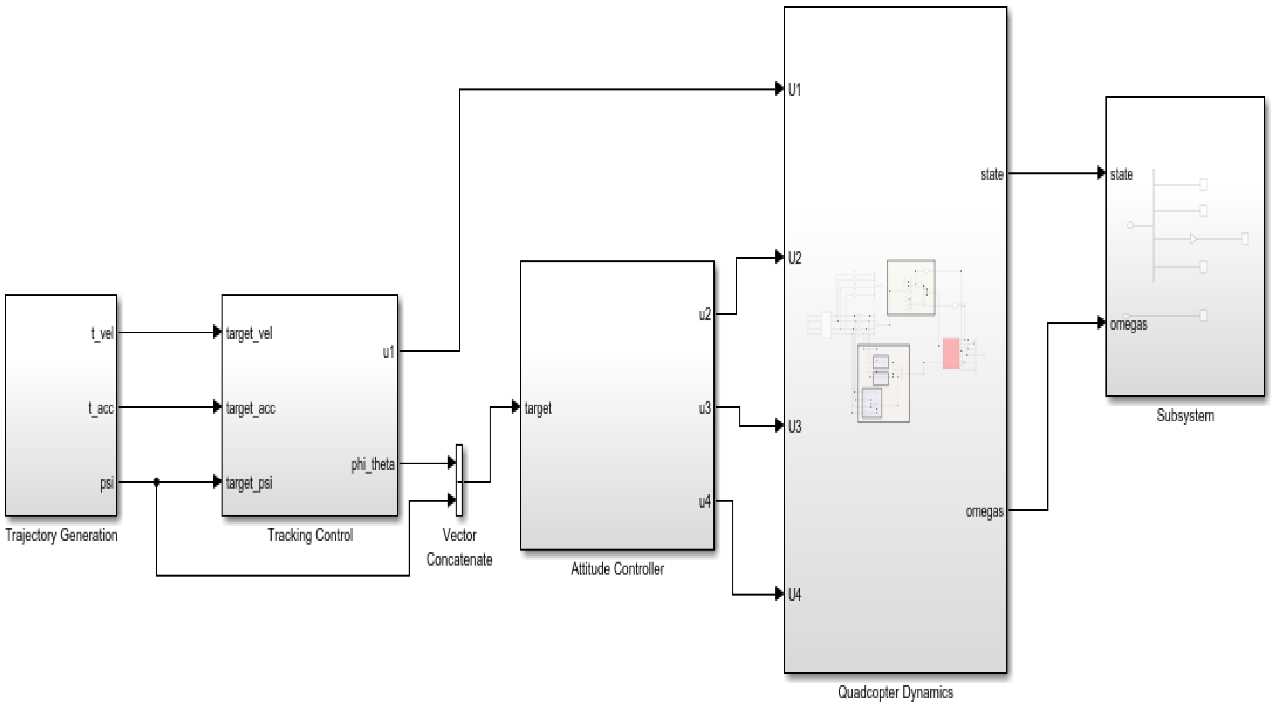


Figure II. 1: Open loop simulation block model.

This figure shows a more detailed view of the open loop simulation block model, it contains 4 principal blocks: trajectory generator ,tracking control, altitude control and the quadrotor dynamics block.

After building the simulation model based on the dynamic model above using the MATlab Simulink we achieved the following results.

a. Trajectory generator

Construct a trajectory (path + time scaling) so that. the quadrotor reaches a sequence of points in a given time. Trajectory should be sufficiently smooth and respect limits on joint variables, velocities, accelerations, or torques.

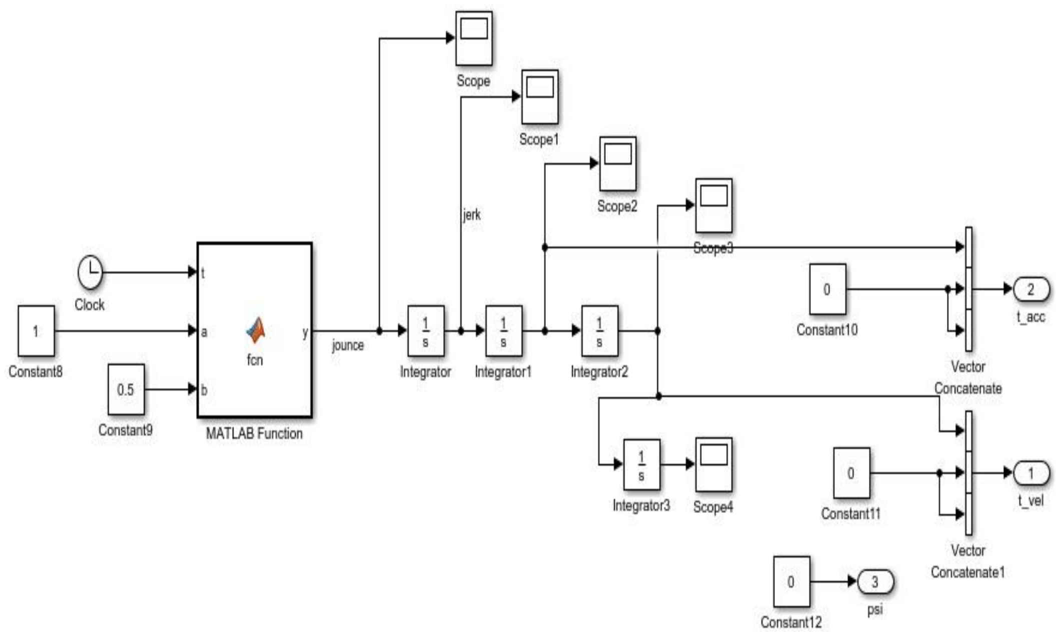


Figure II. 2: Trajectory generator for the quadrotor model.

b. Tracking control

The objective of this system is to take a mathematical model of the drone by taking into account different mechanical actions and disturbances that act on the system, thus, to simulate the behavior of the drone under Simulink, which requires the use of a cascaded PID controller to govern the manipulation of the drone for trajectory tracking to make a smooth implementation. The results of the simulations made were very satisfactory and the drone manages to follow perfectly the instructions of the user.

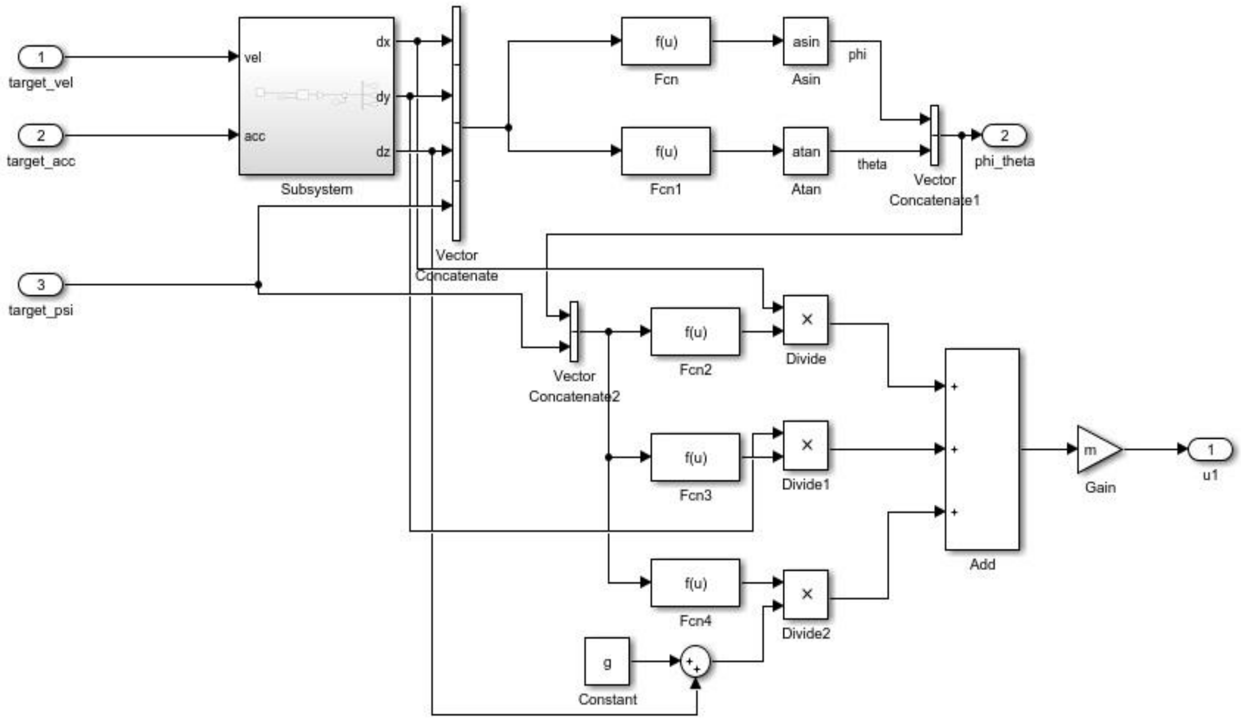


Figure II. 3: Tracking control system of the quadrotor.

c. Altitude controller

Attitude control is the process of controlling the orientation of an aerospace vehicle with respect to an inertial frame of reference or another entity such as the celestial sphere, certain fields, and nearby objects, etc.

Controlling vehicle attitude requires sensors to measure vehicle orientation, actuators to apply the torques needed to orient the vehicle to a desired attitude, and algorithms to command the actuators based on (1) sensor measurements of the current attitude and (2) specification of a desired attitude. The integrated field that studies the combination of sensors, actuators and algorithms is called guidance, navigation and control (GNC).

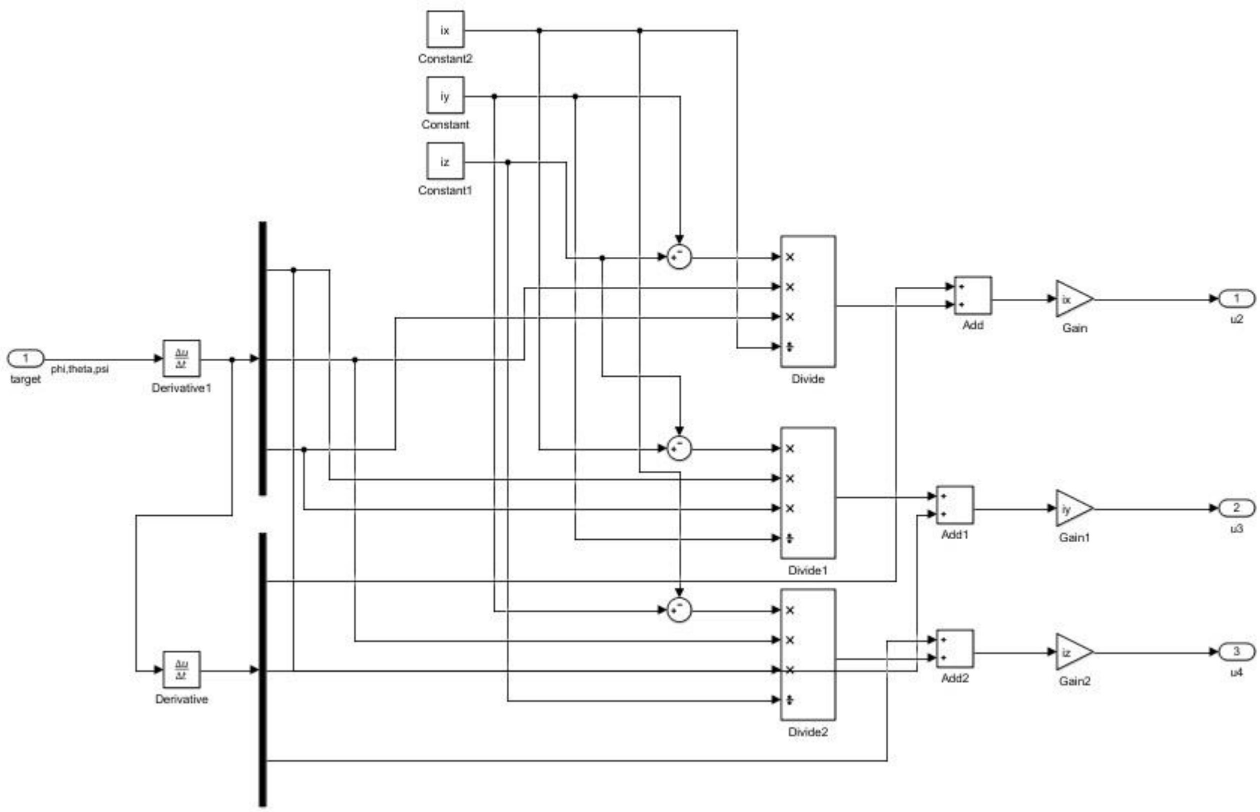


Figure II. 4: Altitude control system of the quadrotor.

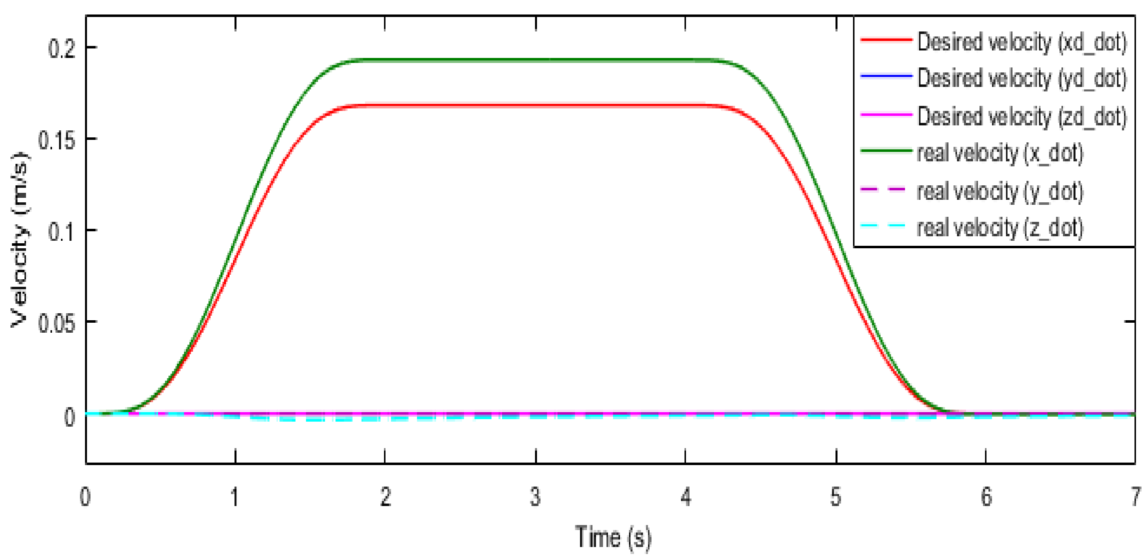


Figure II. 5: Targeted and actual Angular speed Velocity.

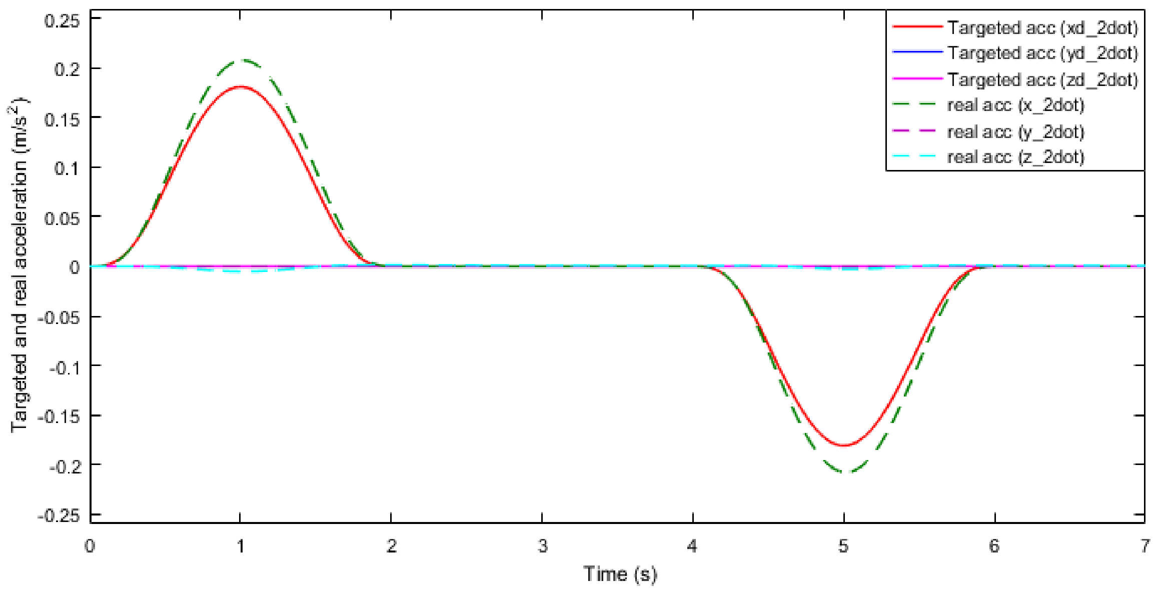


Figure II. 6: Targeted and actual Angular speed acceleration.

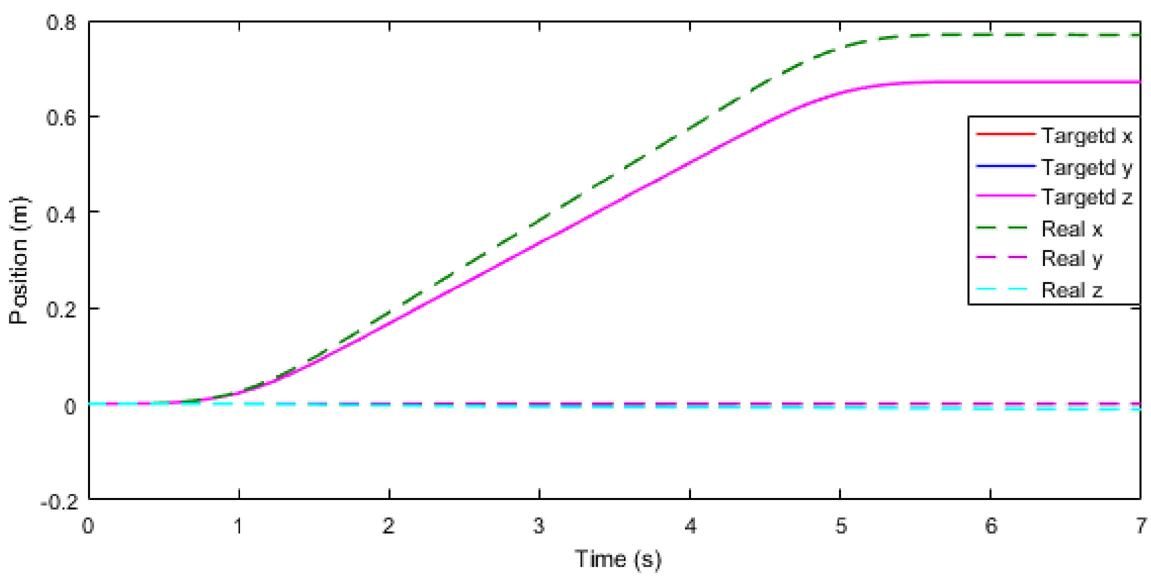


Figure II. 7: Targeted and actual Angular speed Positions.

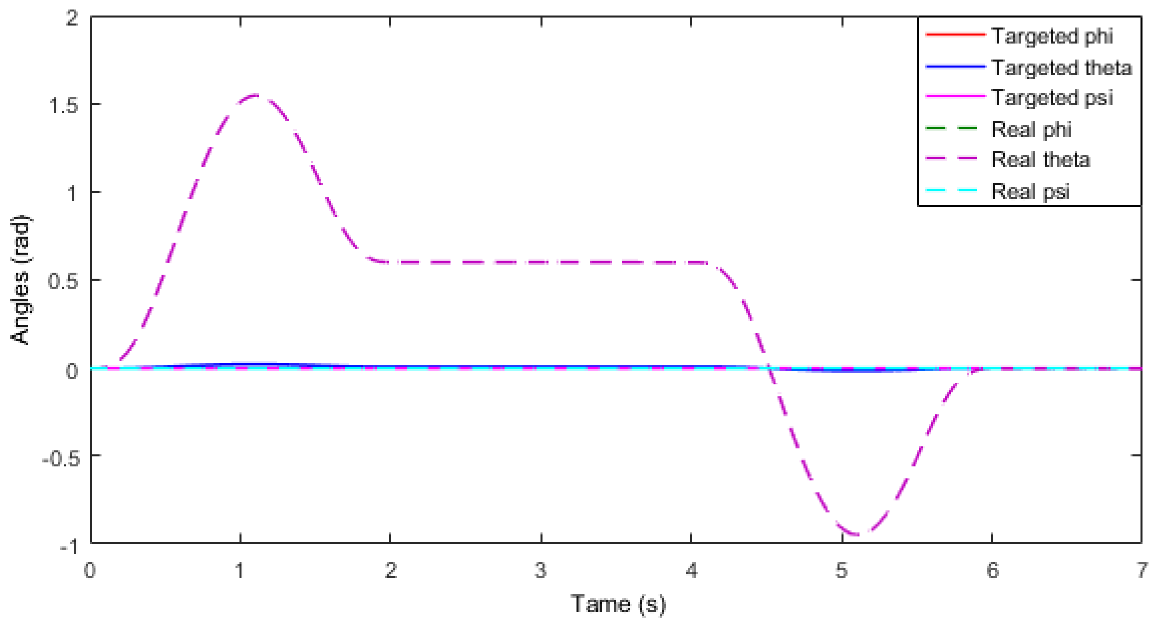


Figure II. 8: Targeted and actual angles.

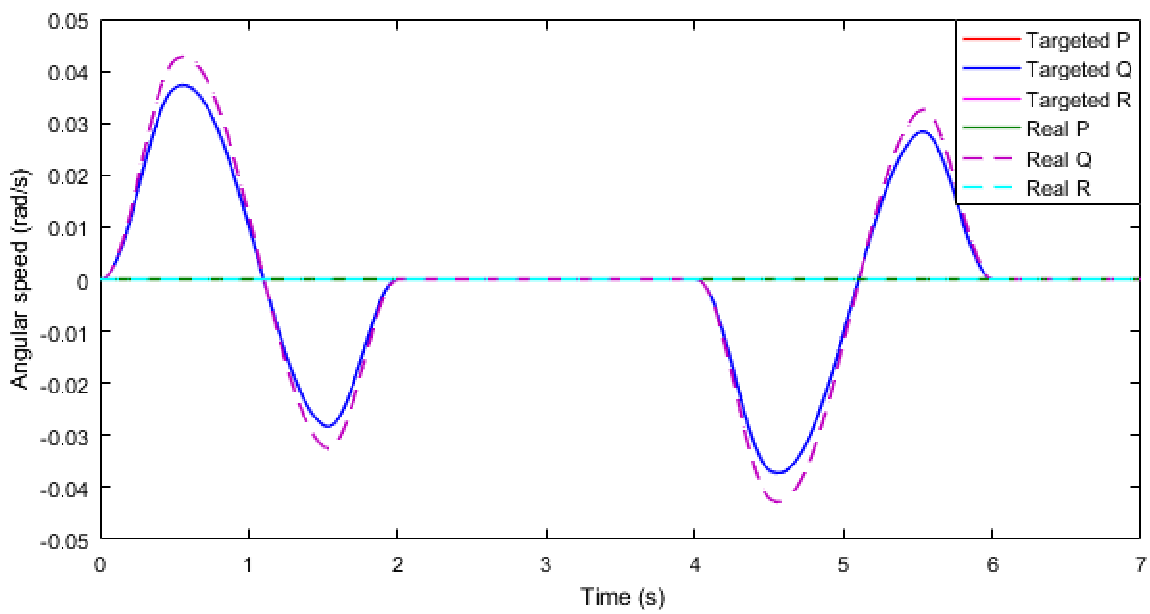


Figure II. 9: Targeted and actual Angular speed .

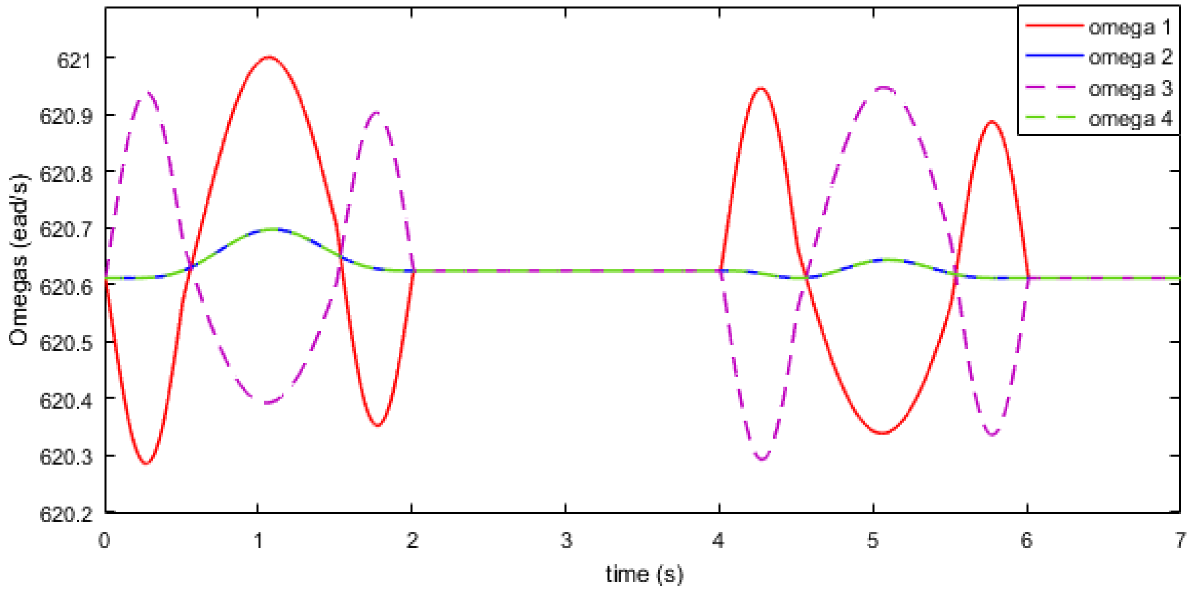


Figure II. 10: Open loop responses of the rotor speed.

Each of these results were independently tested for verifying its stability and reference tracking characteristics.

After the results shown above we can conclude that the quadrotor won't operate safely in an open loop control because it's an open loop system that follows the given input without consideration of the present state of the system which we see in figure (a) as the drone starts to drift slowly from the desired position due to absence of the feed-back .

II.3.4. ANALYZING RESULTS

In this part we have seen an open loop approach to control the quadcopter we demonstrated what is an open loop control system , how can we link it with the controller ,open loop motor control , how does it work and what do we take into consideration when it comes to this approach . We also got to demonstrate how to calculate the transfer function of any open loop system , so we define the main characteristics of an "Open-loop system" as being:

- There is no comparison between actual and desired values.
- An open-loop system has no self-regulation or control action over the output value.
- Each input setting determines a fixed operating position for the controller.
- Changes or disturbances in external conditions does not result in a direct output change (unless the controller setting is altered manually).

From what we've seen from this approach which didn't show great results because of many factors such as the constantly changing input and outside disturbances (environmental and systematic).

II.4. CLOSED LOOP APPROACH

Systems in which the output quantity has no effect upon the input to the control process are called open-loop control systems, and that open-loop systems are just that, open ended non-feedback systems[14].

But the goal of any electrical or electronic control system is to measure, monitor, and control a process and one way in which we can accurately control the process is by monitoring its output and “feeding” some of it back to compare the actual output with the desired output so as to reduce the error and if disturbed, bring the output of the system back to the original or desired response.

The quantity of the output being measured is called the “feedback signal”, and the type of control system which uses feedback signals to both control and adjust itself is called a Close-loop System**.**

A Closed-loop Control System, also known as a feedback control system is a control system which uses the concept of an open loop system as its forward path but has one or more feedback loops (hence its name) or paths between its output and its input. The reference to “feedback”, simply means that some portion of the output is returned “back” to the input to form part of the systems excitation.

Closed-loop systems are designed to automatically achieve and maintain the desired output condition by comparing it with the actual condition. It does this by generating an error signal which is the difference between the output and the reference input. In other words, a “closed-loop system” is a fully automatic control system in which its control action being dependent on the output in some way.

III.4.1. CLOSED-LOOP CONTROL

The closed-loop configuration is characterized by the feedback signal, derived from the sensor in our system. The magnitude and polarity of the resulting error signal, would be directly related to the difference between the required value and actual value of the output[14].

Also, because a closed-loop system has some knowledge of the output condition, (via the sensor) it is better equipped to handle any system disturbances or changes in the conditions which may reduce its ability to complete the desired task.

As we can see, in a closed-loop control system the error signal, which is the difference between the input signal and the feedback signal (which may be the output signal itself or a function of the output signal), is fed to the controller so as to reduce the systems error and bring the output of the system back to a desired value. In our case the dryness of the clothes. Clearly, when the error is zero the clothes are dry.

The term Closed-loop control always implies the use of a feedback control action in order to reduce any errors within the system, and its “feedback” which distinguishes the main differences between an open-loop and a closed-loop system. The accuracy of the output thus depends on the feedback path, which in general can be made very accurate and within electronic control systems and circuits, feedback control is more commonly used than open-loop or feed forward control.

Closed-loop systems have many advantages over open-loop systems. The primary advantage of a closed-loop feedback control system is its ability to reduce a system's sensitivity to external disturbances, for example opening of the dryer door, giving the system a more robust control as any changes in the feedback signal will result in compensation by the controller[14].

Then we can define the main characteristics of Closed-loop control as being:

- To reduce errors by automatically adjusting the systems input.
- To improve stability of an unstable system.
- To increase or reduce the systems sensitivity.
- To enhance robustness against external disturbances to the process.
- To produce a reliable and repeatable performance.

While a good closed-loop system can have many advantages over an open-loop control system, its main disadvantage is that in order to provide the required amount of control, a closed-loop system must be more complex by having one or more feedback paths. Also, if the gain of the controller is too sensitive to changes in its input commands or signals it can become unstable and start to oscillate as the controller tries to over-correct itself, and eventually something would break. So we need to "tell" the system how we want it to behave within some pre-defined limits.

II.4.2. CLOSED LOOP

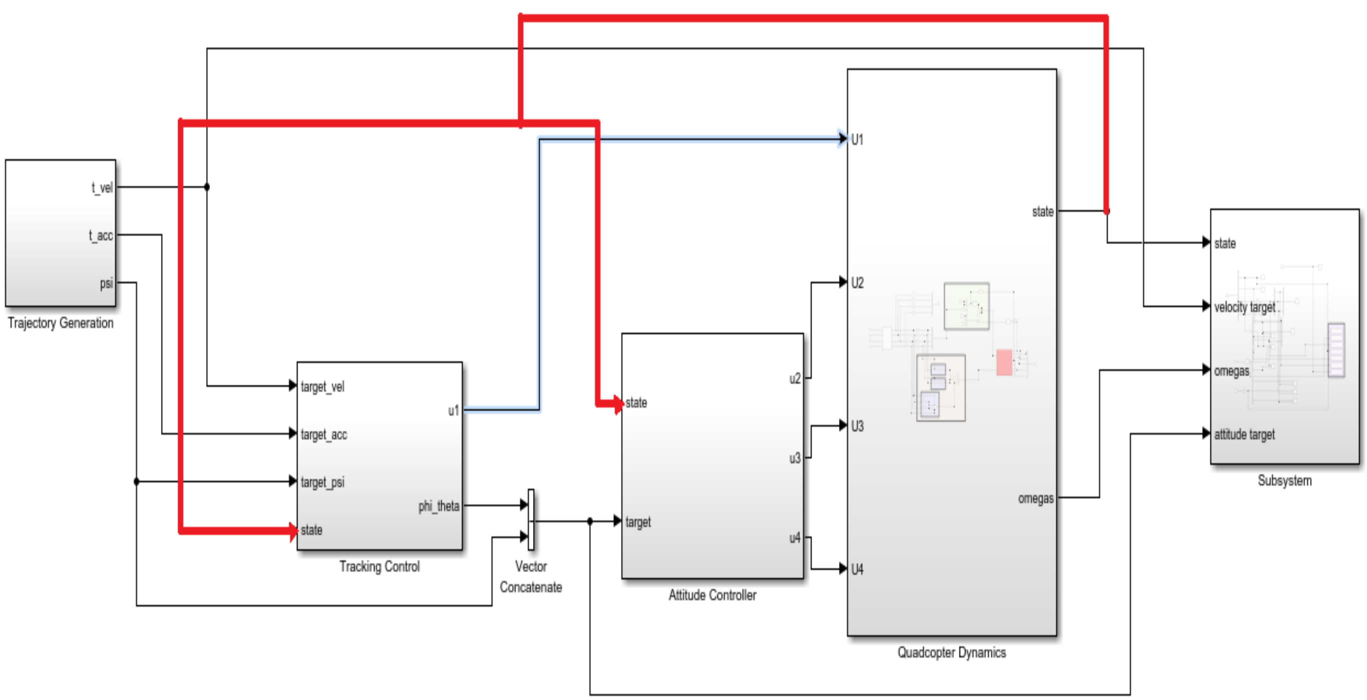


Figure II. 11: Closed loop simulation block model.

The figure III.11 shows the general view of the closed loop control system of a quadritor for this model we used a trajectory generator which allow us to generate the trajectory we desire for the quadretor to follow , we also used a tracking control system that tracks the quadretor's position in real-time , for the altitude control we designed an altitude control system which allows us to manipulate the altitude of the quadritor at all time , then we have our main model built with the use of the equations in chapter two, and finally the subsystems block which contains the results graphics (positions , velocity , angles …)

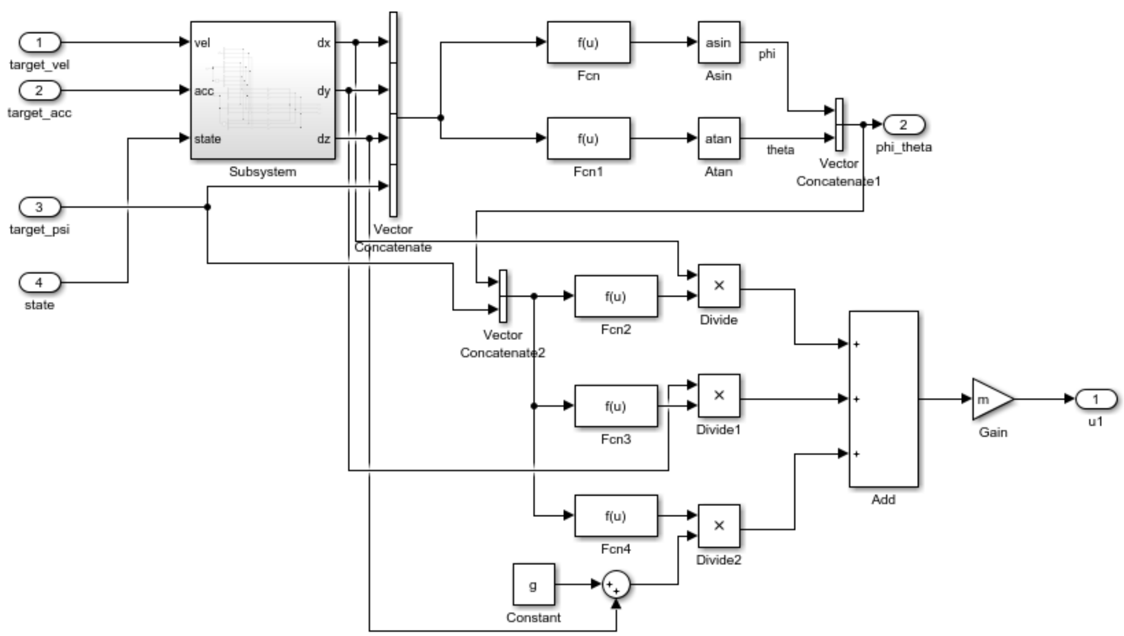


Figure II. 12: tracking control system of the quadrotor (close lope systems).

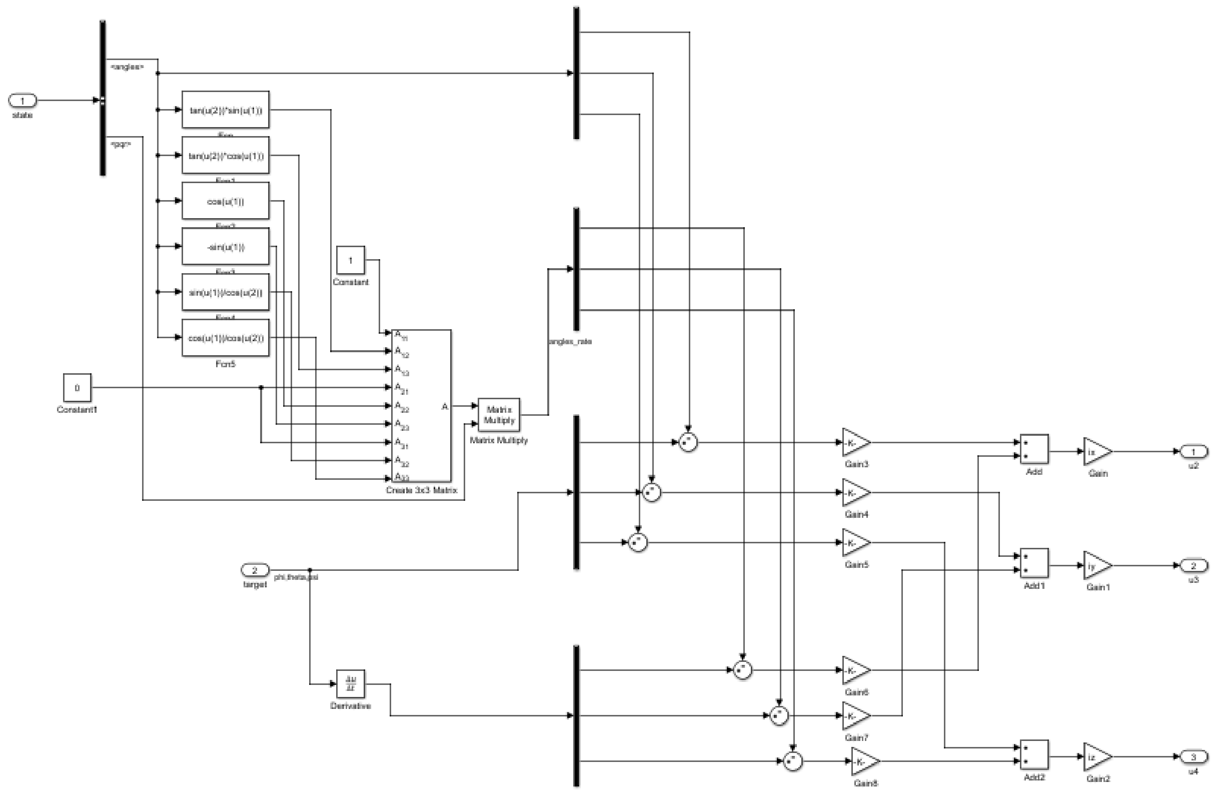


Figure II. 13: Altitude control system of the quadrotor (close lope system).

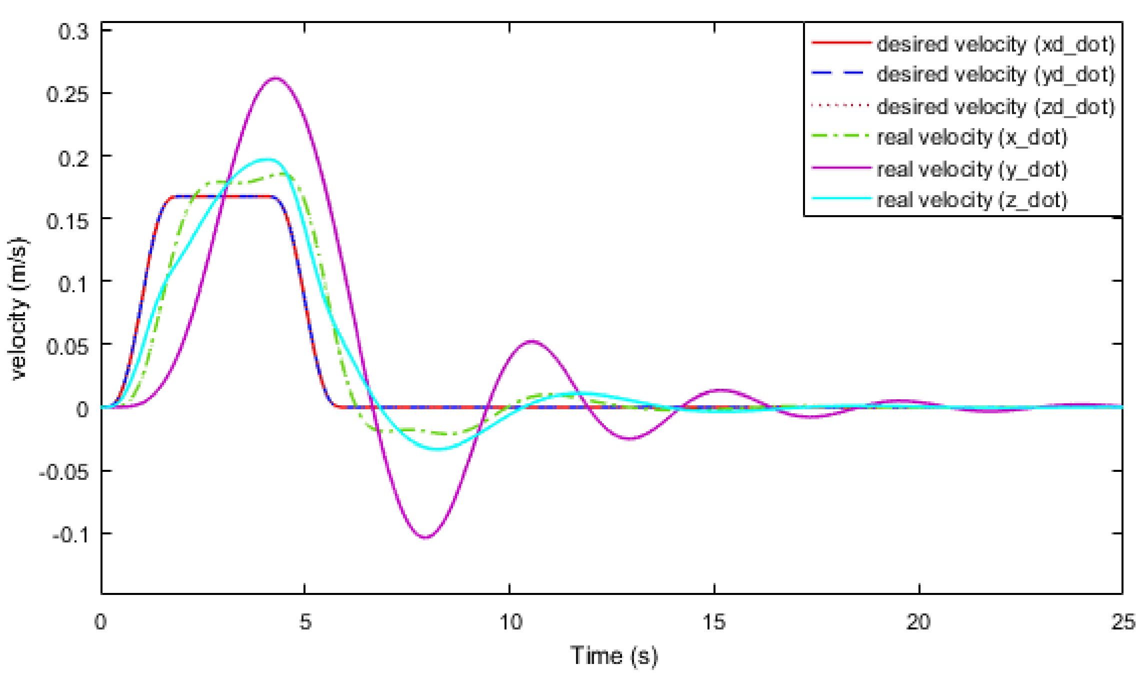


Figure II. 14: Targeted and actual Velocity.

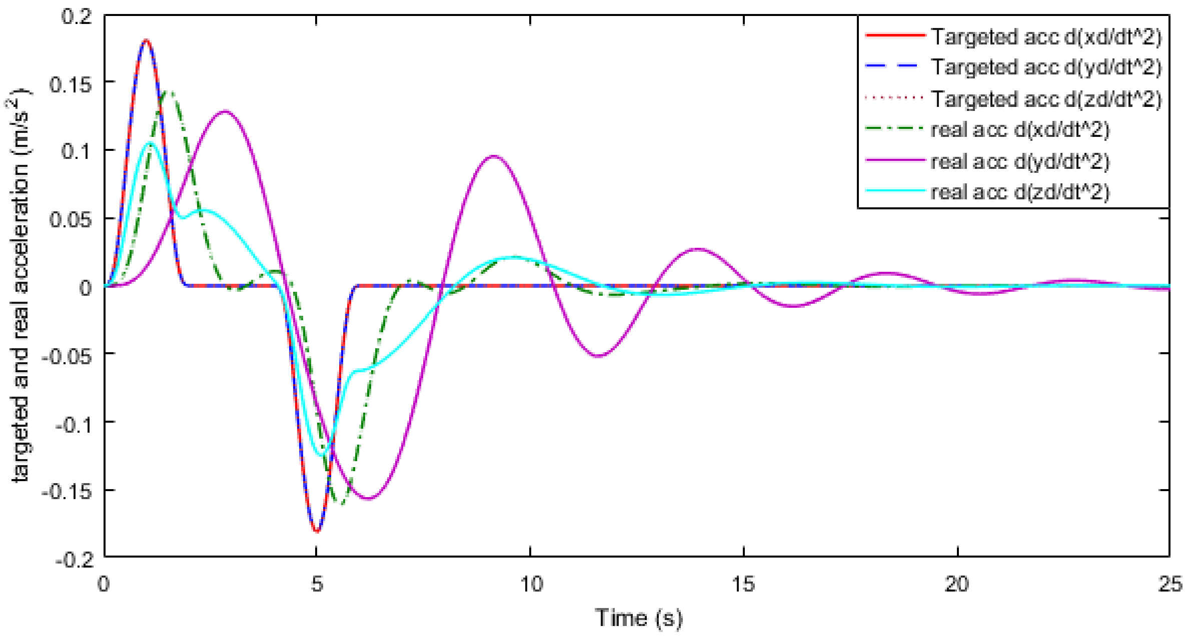
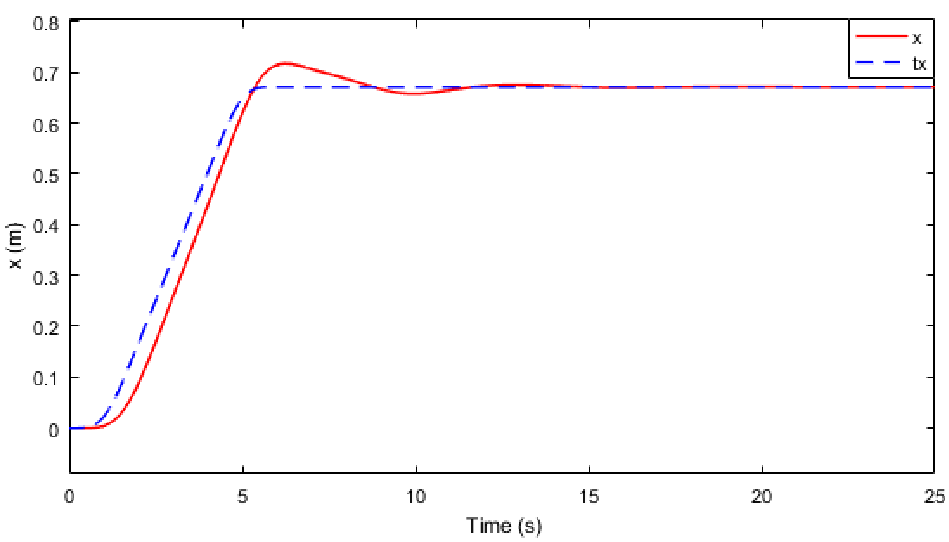
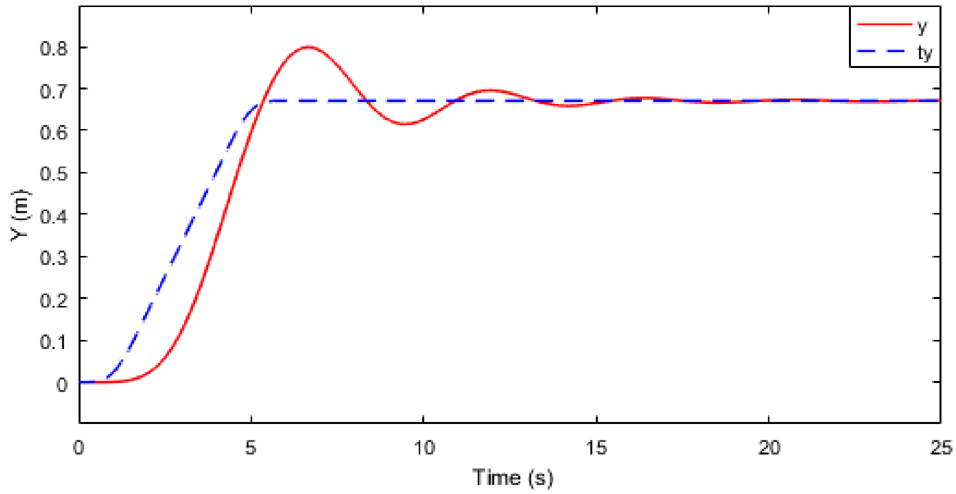


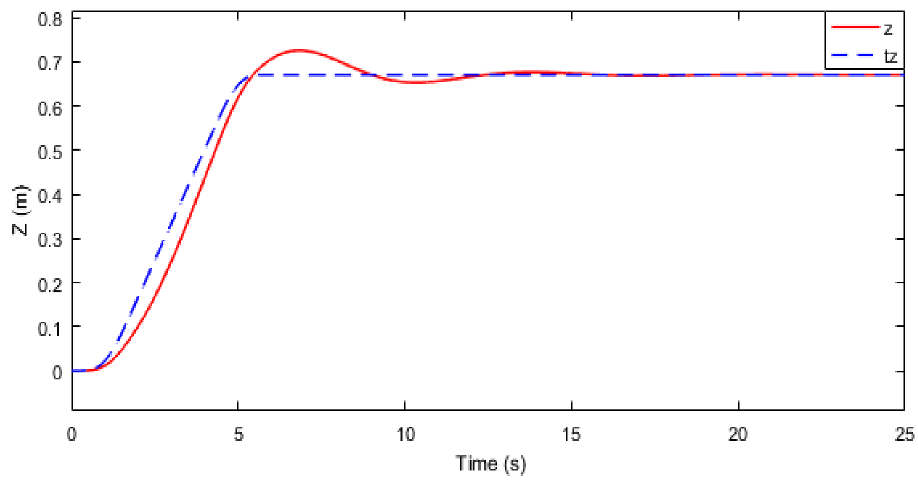
Figure II. 15: Targeted and actual acceleration.



A. X position



B. Y position



C. Z position

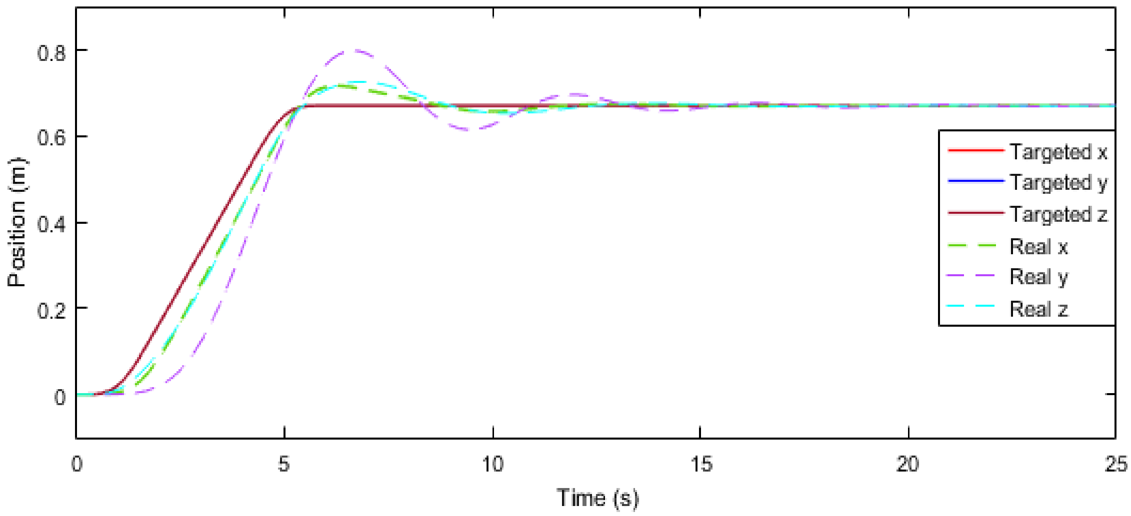
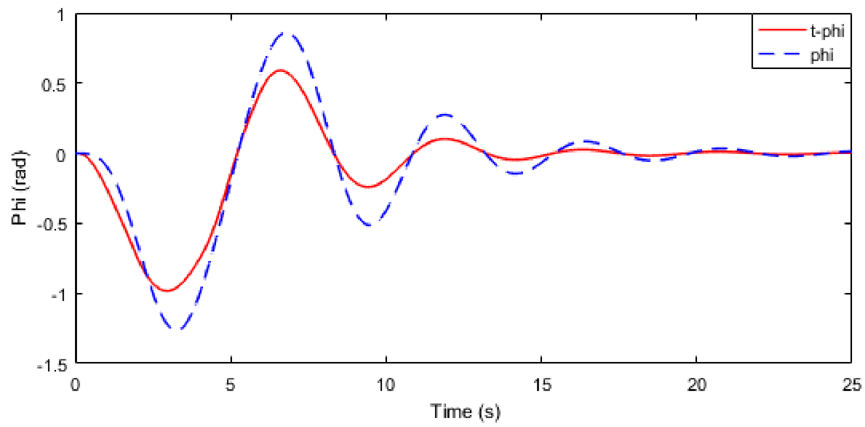
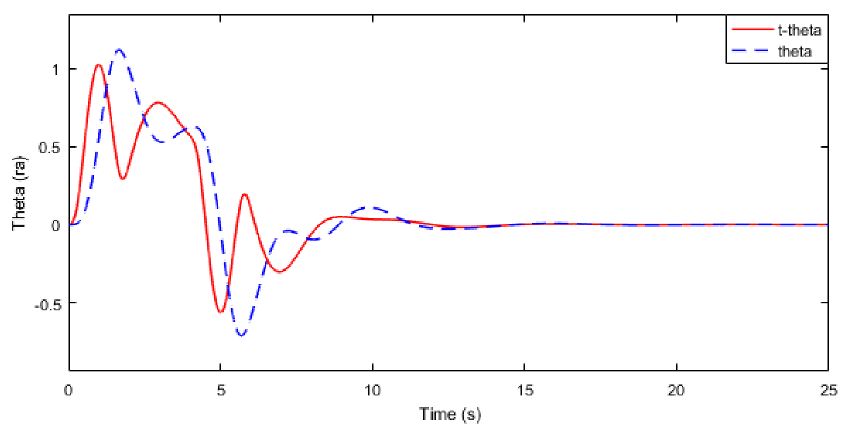


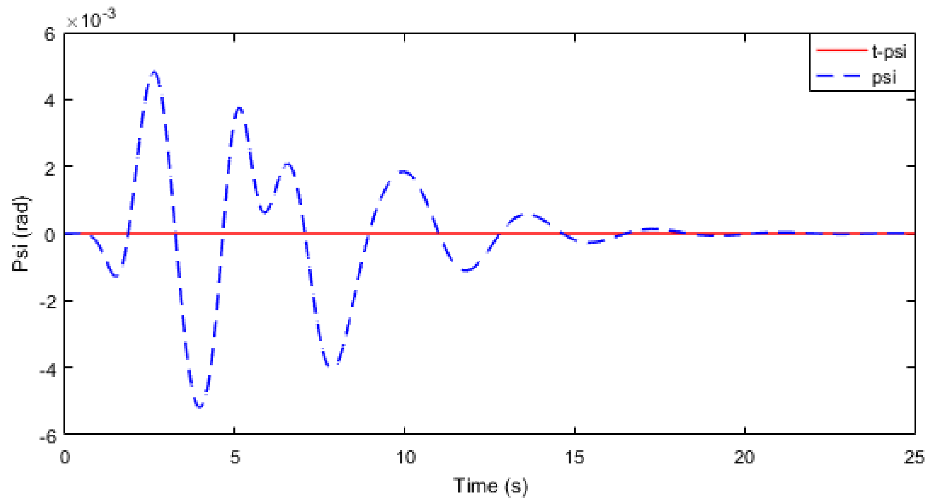
Figure II. 16: Targeted and actual position .



D. Phi Angle



E. Theta angle



F. Psi angle

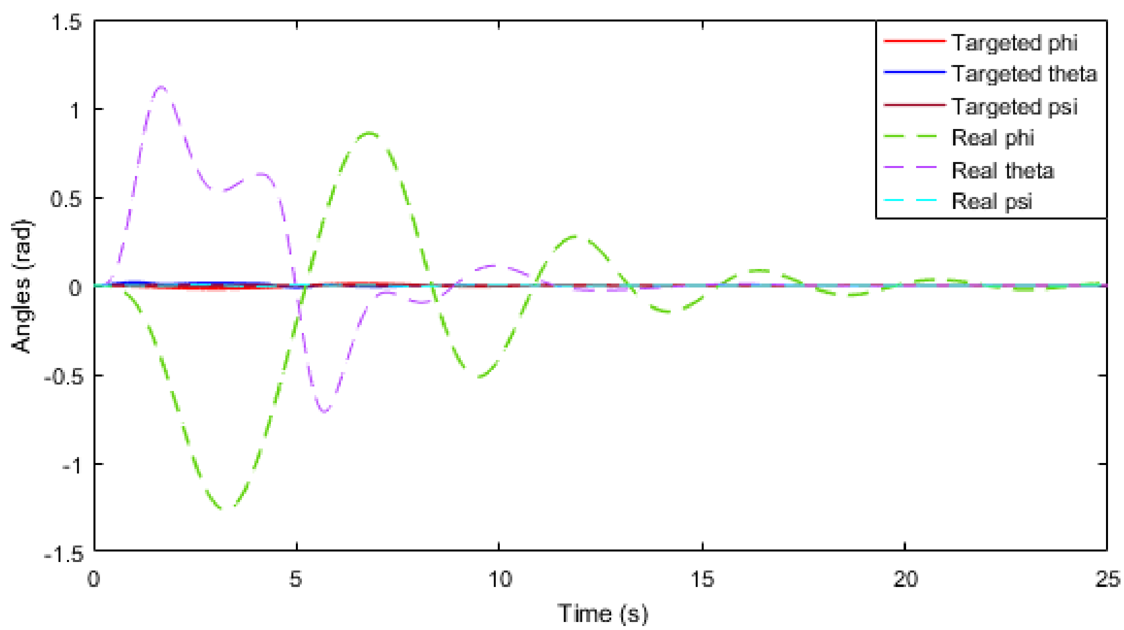


Figure II. 17: Targeted and actual Angles.

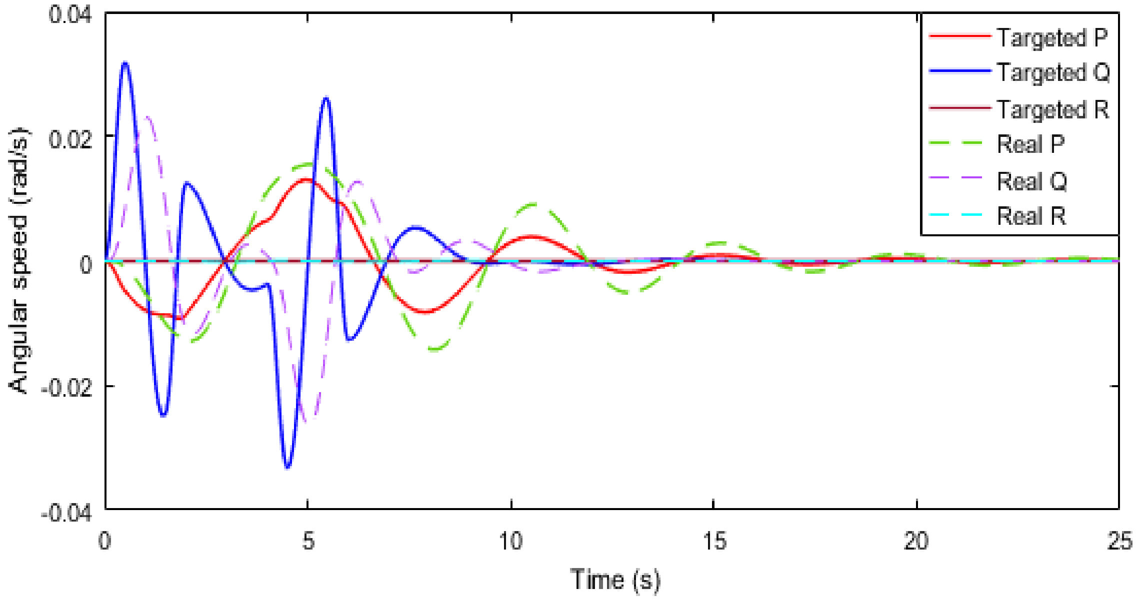


Figure II. 18: Targeted and actual Angular speeds PQR.

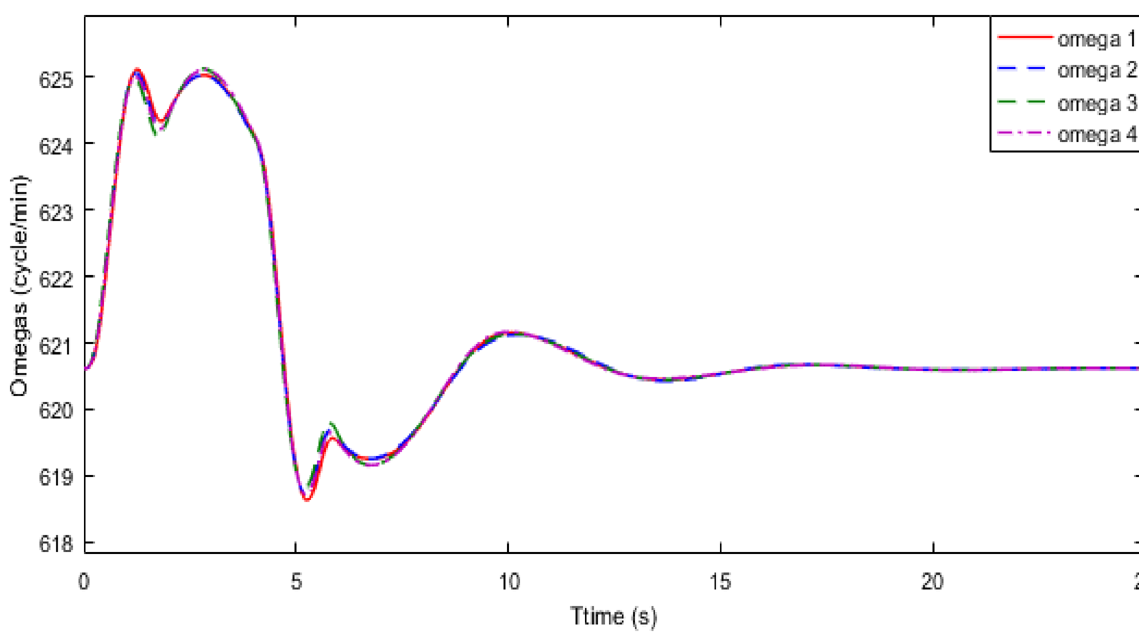


Figure II. 19: Close loop responses of the rotor speed.

The figures above shows the final results of the closed loop control system of a quadcopter, for this model we used a trajectory generator which allow us to generate the trajectory we desire for the quadretor to follow , we also used a tracking control system that tracks the quadretor's position in real-time , for the altitude control we designed an altitude control system which allows us to manipulate the altitude of the quaditor at all time , then we have our main model built with the use of the equations in chapter two .

starting with the positions as shown from figure(D) we can see precise results as the quadcopter follows greatly the desired point in all 3 axis , as well as the velocities shown in figure(e) , as for the angles and the rotors speeds we notice a small oscilation at the start because of the startup curent peak which is normal . The results from this approach has show great performances all aroud and that proves the reliability of the closed loop approach using the PID.

II.5. CONCLUSION

In this chapter we have seen the closed loop based control , we got to demonstrate what is a closed loop system.

A closed-loop controller is a common means of maintaining a desired motor speed under varying load conditions by changing the average voltage applied to the input from the controller.

We got to see both open loop and closed loop models and we have also seen the results of each It's clear that the closed loop approach is more precise and accurate than the open loop one from every angle.

After we explained this approach and seen it’s results on the positions velocities and angles which showed great performances in stability compared to the open loop based control , from this we can conclude that is the closed loop approach is the safest and the more suitable approach to control our system because of it's vulnerability to outside factors (environmental) which won't allow the system to operate in open loop .

Chapter III

Backstepping and Sliding Mode control

CHAPTER III: BACKSTEPPING AND SLIDING MODE CONTROL

III.1. INTRODUCTION

In this chapter, we will present the model of a quadrotor helicopter whose dynamical model is obtained via *Newton's* laws. A backstepping control then a sliding mode strategies are proposed. Starting with backstepping. having in mind that the quadrotor can be seen as the interconnected of three subsystems: under-actuated subsystem, fully-actuated subsystem and propeller subsystem. The output of the under-actuated subsystem are the x and y motion. In order to control them, tilt angles (pitch and roll) need to be controlled. It appeared judicious to us to use a backstepping technique to solve this problem. The idea is to stabilize the system in two phases. Firstly, the positions (x, y) are controlled by a virtual control based on the tilt angles. Secondly, the tilt angles are controlled by varying the rotor speeds, thereby changing the lift forces. The backstepping technique will be used to control the z motion and the yaw angle of the fully-actuated subsystem.

In the other hand we have the sliding mode appraoch. Sliding Mode Control (SMC) is a simple robust technique that permits designing controllers for linear and nonlinear processes [19-21]. SMC has become one of the selected solutions for many practical control designs such as electronics devices, robotics, chemical processes and so on [21-22]. SMC consists in two parts: firstly, a discontinuous control law to enforce the error vector toward a decision rule, called sliding surface, during the reaching phase. This part the control is switching on the different sides of the sliding surface equation and secondly, once the error vector is confined to the sliding surface, an equivalent part of the controller acts to follow the dynamics imposed by the equations describing the sliding surface [19].

There are some works of SMC applied to tracking and regulation tasks for quadrotors, let us make a briefly mention of them, for instances:[23], We present a controller approach where it is obtained by dividing the design in two steps: firstly, a high gain observer is considered and secondly, a dynamic sliding mode controller (DSMC) is proposed. [24].A direct adaptive sliding mode control is developed. The proposed controller is applied without considering disturbances and parameter uncertainties. The design of the adaptation laws is based on Lyapunov design principle [25]. in this chapter, sliding mode is presented and then compared with the backstepping method [26]. A controller based on high order sliding mode control (HOSMC) is designed for trajectory tracking [27]. Describes a feedback linearization-based controller with a high-order sliding mode observer. The observer works as an observer and estimator of the external disturbances effect [28] . The principal attention is based on the dynamic modeling of quadrotor taking into account the high-order non holonomic constraints. Using the Backstepping approach for the synthesis of tracking errors and Lyapunov functions, a sliding mode controller is obtained[29].An adaptive backstepping approach is utilized to design the sliding mode control.

The objective of chapter consists in designing a sliding mode control and applying it by simulation in a quadrotor. A PD sliding surface is considered for vertical take-off and landing aircraft, also changes in angles are done, and some disturbances are included. Therefore, the controller can be implemented using a PD controller as the sliding surface, and adding some algebra the complete controller algorithm is presented.

III.2. BACKSTEPPING

III.2.1. MODELLING OF THE QUADCOPTER

The dynamic model of the quadcopter is mostly derived either using Newton's laws or the Lagrange formula[15-17]. This paper considers the Newton-Euler equations for the dynamic model derivation, which are given as follows:

$$\vec{F} = m\dot{V} + \omega \times m\vec{V} \tag{1}$$

$$\vec{T} = I\dot{\omega} + \omega \times I\vec{\omega} \tag{2}$$

The state variables of the quadcopter are defined as, $\mathrm{x} = \begin{bmatrix} x & y & z & \phi & \theta & \psi & \dot{x} & \dot{y} & \dot{z} & \dot{\phi} & \dot{\theta} & \dot{\psi} \end{bmatrix}^T$ where, $\zeta = [\mathrm{x}, \mathrm{y}, \mathrm{z}]\mathrm{T}$ is the position described in the inertial coordinate frame B, $\mathrm{V} = [\dot{\mathrm{x}}, \dot{\mathrm{y}}, \dot{\mathrm{z}}]^{\mathrm{T}}$ is the translational velocity, $\eta = [\varphi, \theta, \psi]^{\mathrm{T}}$ are the roll-pitch-yaw angles describing the attitude of the quadcopter, $\dot{\eta} = [\dot{\phi}, \dot{\theta}, \dot{\psi}]^{\mathrm{T}}$ are the Euler angle rates and $\omega = [\mathrm{p}, \mathrm{q}, \mathrm{r}]^T$ the angular velocity of the quadcopter described in the body fixed frame A.

$$\text{With } \omega = R_r\dot{\eta} \text{ and } R_r = \begin{bmatrix} 1 & 0 & -s(\phi) \\ 0 & c(\phi) & c(\theta).s(\phi) \\ 0 & -s(\phi) & c(\phi) \end{bmatrix}$$

For small Euler angles, the angular velocity of the quadcopter is considered as $\omega = \dot{\eta}$. [15-18] Equation (1) can be formulated as given in (3), considering the term $\omega \times \mathrm{m}\vec{\mathrm{V}}$since the velocity of the quadcopter body is described in an inertial frame thus:

$$\mathrm{m}\dot{V} = -\mathrm{mG} + {}^{\mathrm{B}}\mathrm{R}_{\mathrm{A}}\mathrm{F} \tag{3}$$

where: ${}^{\mathrm{B}}\mathrm{R}_{\mathrm{A}}$ is the transformation matrix.

$${}^{\mathrm{B}}\mathrm{R}_{\mathrm{A}} = \begin{bmatrix} c(\theta)c(\psi) & s(\phi)s(\theta)c(\psi) - c(\phi)s(\psi) & c(\phi)s(\theta)c(\psi) + s(\phi)s(\psi) \\ c(\theta)s(\psi) & s(\phi)s(\theta)c(\psi) + c(\phi)s(\psi) & c(\phi)s(\theta)s(\psi) - s(\phi)c(\psi) \\ -s(\theta) & s(\phi)c(\theta) & c(\phi)c(\theta) \end{bmatrix} \tag{4}$$

m: the mass of the quadcopter in Kg. $F = [0,0,U_1]^T$ is the magnitude of the thrust force,

$G = [0, 0, g]^{\mathrm{T}}$ the gravity rate g in $\mathrm{m.s}^{-2}$. With,

$$U_1 = f_1 + f_2 + f_3 + f_4 \tag{5}$$

Where $f_i = b\omega^2$is the thrust force produced by propeller *i* with thrust coefficient *b* in $N.s^2.\omega_i$is the angular speed of motor i . Writing the Eq. (3) in matrix form, one gets:

$$m\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = -m\begin{bmatrix}0\\0\\g\end{bmatrix} + {}^{B}R_{A}\begin{bmatrix}0\\0\\U_1\end{bmatrix} \tag{6}$$

On the other hand, the equation of the angular motion (2) is formulated as follows:

$$\begin{aligned} I\dot{\omega} &= \tau + \tau_g - \omega \times I\vec{\omega} \\ \tau &= \left[\tau_\phi , \tau_\theta , \tau_\psi\right]^T = [U_2 , U_3 , U_4]^T \end{aligned} \tag{7}$$

$$\tau_g = \left[J_r \Omega_r \dot{\theta} \ \ -J_r \Omega_r \dot{\phi} \ \ 0\right]^T \tag{8}$$

$$\tau_\phi = l(f_4 - f_2) \tag{9}$$

$$\tau_\theta = l(f_3 - f_1) \tag{10}$$

$$\tau_\phi = T_1 - T_2 + T_3 - T_4 \tag{11}$$

$T_i = d\omega^2$ is the drag torque produced by propeller $i \ in \ N.m$ with corresponding drag coefficient ***d*** in $Nm^2.s.\tau_g$ represents the gyroscopic effect in the term of orientation change in the propellers plane. $\boldsymbol{l}$ is the distance between center of the quadcopter and center of propeller in $\omega \times I\vec{\omega}$ is the gyroscopic effect, I is the inertia matrix, where I_{xx}, I_{yy}, I_{zz} are moments of inertia about *x, y, z* axes, respectively in $Kg.m^2$.

$$\begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz}\end{bmatrix}\begin{bmatrix}\ddot{\phi}\\ \ddot{\theta}\\ \ddot{\psi}\end{bmatrix} = \begin{bmatrix}U_2\\U_3\\U_4\end{bmatrix} + \begin{bmatrix}J_r\Omega_r\dot{\theta}\\ -J_r\Omega_r\dot{\phi}\\ 0\end{bmatrix} - \begin{bmatrix}\dot{\varphi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} \times \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz}\end{bmatrix}\begin{bmatrix}\dot{\varphi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} \tag{12}$$

where J_r is the inertia of propeller and Ω_r is the sum of the four motors angular speed. Based on the above derivation and discussion, the equations of motion can be written as follows (13):

$$\begin{cases} \ddot{x} = -\frac{U_1}{m}[c(\phi)c(\psi)s(\theta) + s(\phi)s(\psi)] \\ \ddot{y} = \frac{U_1}{m}[c(\phi)s(\psi)s(\theta) - s(\psi)s(\phi)] \\ \ddot{z} = \frac{U_1}{m}[c(\phi)c(\theta)] - g \\ \ddot{\phi} = \frac{I_{zz}-I_{yy}}{I_{xx}}\dot{\theta}\dot{\psi} + \frac{J_r}{I_{xx}}\Omega_r\dot{\theta} + \frac{1}{I_{xx}}U_2 \\ \ddot{\theta} = \frac{I_{zz}-I_{xx}}{I_{yy}}\dot{\phi}\dot{\psi} + \frac{J_r}{I_{yy}}\Omega_r\dot{\phi} + \frac{1}{I_{yy}}U_3 \\ \ddot{\psi} = \frac{I_x-I_y}{I_z}\dot{\phi}\dot{\theta} + \frac{1}{I_{zz}}U_4 \end{cases} \quad (13)$$

$$\begin{cases} U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ U_2 = b(\Omega_3^2 - \Omega_1^2) \\ U_3 = b(\Omega_4^2 - \Omega_2^2) \\ U_4 = b(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) \end{cases} \quad (14)$$

III.2.2. QUADCOPTER CONTROL

This paper presents two nonlinear control methods derivations in order to control the quadcopter and achieve a high dynamic performance for trajectory tracking. This method is based on the integral backstepping technique.

III.2.2.1. INTEGRAL BACKSTEPPING

Control laws of the attitude and position of the quadcopter are derived using the integral backstepping approach. This approach depends on determining the error between the desired input and the system actual output, then outlining a Lyapunov function and determining virtual controls to make the derivative of the proposed Lyapunov function negative definite [18].

III.2.2.1.1. Altitude control

The control laws for the altitude of the quadcopter is derived as follows. The altitude error:

$$e_1 = z_d - z \quad (15)$$

and the Lyapunov function is chosen as

$$V_1 = \frac{1}{2}e_1^2 \quad (16)$$

With $\dot{V}_1 = e_1\dot{e}_1$, where

$$\dot{e}_1 = \dot{z}_d - \dot{z} = \dot{z}_d - v_z \quad (17)$$

if the term is $k_1 e_1$ added and subtracted to the $\dot{V}_1$ function, where $k_1 > 0$, then these yields

$$\dot{V}_1 = e_1\dot{e}_1 = e_1(\dot{z}_d - V_z + k_1e_1 - k_1e_1) \tag{18}$$

$$\dot{V}_1 = e_1\dot{e}_1 = -k_1e_1^2 + e_1(\dot{z}_d - V_z + k_1e_1) \tag{19}$$

The term $\dot{z}_d - v_z + k_1e_1$must vanish for a negative definite derivative of the Lyapunov function. That can be achieved by choosing the virtual control $v_z$such that

$$v_{zd} = \dot{z}_d + k_1e_1 + c_1\int e_1dt \tag{20}$$

Similar steps are repeated here to derive the control law,

$$e_2 = v_{zd} - v_z \tag{21}$$

The proposed Lyapunov function is

$$V_2 = \frac{1}{2}e_2^2 \tag{22}$$

The 1st order derivative of the proposed function is

$$\dot{V}_2 = e_2\dot{e}_2 \tag{23}$$

Using a similar strategy as for v_{zd}results

$$\dot{V}_2 = -k_2e_2^2 + e_1\left(\dot{v}_{zd} - \frac{c(\phi)c(\theta)}{m}U_1 + g + k_2e_2\right) \tag{24}$$

$$U_1 = \frac{m}{c(\phi)c(\theta)}[\ddot{z}_d + k_1\dot{e}_1 + g + k_2e_2] \tag{25}$$

III.1.2.2.2. Attitude control

The control laws of the attitude of the quadcopter were derived in this section depending on integral backstepping method as follows:

$$U_2 = \frac{1}{b_1}\left[\ddot{\phi}_d + k_3\dot{e}_3 - a_1\dot{\theta}\dot{\psi} - a_2\dot{\theta}\Omega_r + k_4e_4\right] \tag{26}$$

$$U_3 = \frac{1}{b_2}\left[\ddot{\theta}_d + k_5\dot{e}_5 - a_2\dot{\theta}\dot{\psi} + a_4\dot{\theta}\Omega_r\right] \tag{27}$$

$$U_4 = \frac{1}{b_3}\left[\ddot{\psi}_d + k_7\dot{e}_7 - a_5\dot{\theta}\dot{\phi}\right] \tag{28}$$

III.1.2.2.3. Position control

Since the motion of the quadcopter on the x axis depends on θ angle, whereas the motion on the y axis depends on φ angle, thus φ and θ angles have been considered as the outputs of x, y control laws, it is remarkable to mention that small Euler angles have not been considered in order to obtain the position control laws in this paper, which is significant to achieve high dynamic performance tracking control. The main contribution of this paper is deriving the position control laws from the quadcopter's model directly just by applying the procedure of the control approaches. Recalling Equation (13), for clarification, as follows

$$\ddot{x} = \frac{c(\phi)c(\psi)s(\theta)+s(\phi)s(\psi)}{m}U_1$$
$$\ddot{y} = \frac{c(\phi)s(\psi)s(\theta)-s(\psi)s(\phi)}{m}U_1 \qquad (29)$$

At first, the integral backstepping method is applied to derive the control law for the motion of the quadcopter on the x axis, and then the following control law is obtained:

$$\theta_d = \arcsin\left(\frac{m}{c(\phi)c(\theta)U_1}\left\{\dot{v}_{x_d} - \frac{s(\phi)s(\psi)}{m}U_1 + k_{10}e_{10}\right\}\right) \qquad (30)$$

The control law for the motion of the quadcopter on the y axis is derived in a similar way, and then the following law is obtained:

$$\phi_d = \arcsin\left(\frac{m}{c(\psi)U_1}\left\{\dot{v}_{y_d} - \frac{c(\phi)s(\theta)s(\psi)}{m}U_1 + k_{12}e_{12}\right\}\right) \qquad (31)$$

III.3. Sliding mode

III.3.1. Sliding mode basics

The quadrotor has twelve states, which are the following:

$$\mathrm{x} = \begin{bmatrix} x & y & z & \phi & \theta & \psi & \dot{x} & \dot{y} & \dot{z} & \dot{\phi} & \dot{\theta} & \dot{\psi} \end{bmatrix}^T \qquad \text{(III.32)}$$

Where, x, y and z are the position in the x, y, and z axes. $\dot{x}, \dot{y}$ and $\dot{z}$ are the speed in the axes.
ϕ, θ, ψ Are the roll, pitch, and yaw angles respectively, and the parameters are $\dot{\phi}, \dot{\theta}, \dot{\psi}$ the speed for roll, pitch and yaw.
The input signal U_1 is the total drag of the rotors. U_2, U_3 and U_4 Are the moments for pitch, roll and yaw respectively. m represents the mass of the quad rotor, J_r is the inertia of the rotor and I_x, I_y and I_z are the inertia of the quad rotor in 'x', 'y' and 'z' respectively.
The input signals are described from the equation (8) to (11).

$$\begin{cases} U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ U_2 = b(\Omega_3^2 - \Omega_1^2) \\ U_3 = b(\Omega_4^2 - \Omega_2^2) \\ U_4 = b(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) \end{cases} \qquad \text{(III..33)}$$

The angular speed for each rotor is noted as $\Omega_1, \Omega_2, \Omega_3, \Omega_4$. L is the distance between the rotor and the center of the quadrotor, and b is the drag factor. All of these parameters involved in equations (2) and(3) . are stated in [31].

And finally the value of Ω is:

$$\Omega = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4 \tag{III..34}$$

III.3.2. SLIDING MODE CONTROLLER DESIGN

Sliding Mode Control is a procedure that arises from Variable Structure Control (VSC)[32]. SMC objective involves two parts: in the first one, a control law to enforce the error vector toward a decision rule, called sliding surface, during the reaching phase is designed, this part the control is switching on the different sides of this sliding surface and secondly, once the error vector is restricted in the sliding surface, it tracks the dynamics imposed by the equations describing the sliding surface, this second part of the controller is called equivalent control [19].

Four control equations are used to keep the quadrotor on the reference value in spite of external disturbances. The signal *U1* is used to guarantee that the altitude follows the reference value, although the signals *U2*, *U3* and *U4* are used to control the roll, pitch and yaw of the system. Looking at the equations (13), we found four control input signals. The action of these in put signals makes the quadrotor moves forwards, backward, to the left, to the right, upwards or down.

As was mentioned before, the sliding surface or decision rule must be selected. This selection is made based on performance criteria, since the sliding surface equation determines the dynamics of the system. Therefore, it can be written as:

$$s = \dot{e} + \lambda e \tag{III.35}$$

Where λ is a tuning parameter design greater than zero.

The control law, $U(t)$, is composed by two parts: a continuous part, $u_{eq}(t)$, and a discontinuous part, $u_D(t)$. That is

$$U(t) = u_{eq}(t) + u_D(t) \tag{III.36}$$

The first part is a function of the controlled variable, and the reference value. The second part includes a nonlinear element that contains a switching element.

$$u_D(t) = k_D \Theta\big(\mathrm{s(t)}\big) \tag{III.37}$$

$k_D$is the tuning parameter responsible for the reaching phase, and Θ(s(t)) is a nonlinear function of $s(t)$. Then, Eq.(37) can be rewritten as follows:

$$u_D(t) = k_D \mathrm{sign}\big(\mathrm{s(t)}\big) \tag{III.38}$$

Chattering problem can be reduced if Eq. (39) is represented as:

$$u_D(t) = k_D \frac{\mathrm{s(t)}}{|\mathrm{s(t)}|+\delta} \tag{III.39}$$

δ is a tuning parameter used to reduce the chattering effect. In order to find $u_{eq}(t)$ the equivalent control procedure is followed:

The sliding surface is defined as in equation (35), for the altitude of the quadrotor the error would be.

$$e = z_d - z \tag{III.40}$$

z is measured state and z_d is the desired state. By replacing (40) in (35) the following result is obtained.

$$s = (z_d - z) + \lambda(z_d - z) \tag{III.41}$$

By applying the sliding condition, $\dot{s} = 0$, it is obtained:

$$\dot{s} = \ddot{e} + \lambda\dot{e} = 0 \tag{III.42}$$

And, it is obtained:

$$\dot{s} = (\ddot{z}_d - \ddot{z}) + \lambda(\dot{z}_d - \dot{z}) \tag{III.43}$$

Substituting $\ddot{z}$, into (43), it can be found:

$$\dot{s} = \left(\ddot{z}_d - \frac{U_1}{m}[c(\phi)c(\theta)] + g\right) + \lambda(\dot{z}_d - \dot{z}) \tag{III.44}$$

Considering$U_1 = u_{eq}$, since the system is in sliding condition, the above equation becomes.

$$\dot{s} = \left(\ddot{z}_d - \frac{U_{eq}}{m}[c(\phi)c(\theta)] + g\right) + \lambda(\dot{z}_d - \dot{z}) \tag{III.45}$$

Since, $\dot{s} = 0; u_{eq}$ becomes:

$$u_{eq} = [g + \lambda(\dot{z}_d - \dot{z}) + \ddot{z}_d]\ \frac{m}{c(\phi)c(\psi)} \tag{III.46}$$

And, the complete controller can be written as follows:

$$u_1 = [g + \lambda\dot{e} + \ddot{z}_d]\ \frac{m}{c(\phi)c(\psi)} + k_D sign(s) \tag{III.47}$$

To design u_D, a Lyapunov function V is defined. This function must be positive-definite.

$$V = \frac{1}{2}s^2 > 0 \tag{III.48}$$

The derivative of the function V must be negative-definite.

$$\dot{V} = s\dot{s} < 0 \tag{III.49}$$

In order to satisfy Eq.(51), u_D should be:

$$u_D = k_D sign(s) \tag{III.50}$$

Where s is defined by (37).

The reaching condition is given by the following inequality $\dot{V} > 0$. Therefore, to satisfy it:

$$k_D > 0 \; for \; all \; e < 0 \tag{III.51}$$

To avoid the chattering problem, u_D is re-defined as follows:

$$u_D(t) = k_D \frac{s}{|s|+\delta} \tag{III.52}$$

Thus, the complete controller equation is

$$u_1 = [g + \lambda\dot{e} + \ddot{z}_d] \frac{m}{c(\phi)c(\psi)} + k_D \frac{s}{|s|+\delta} \tag{III.53}$$

The same procedure is followed to get the controllers for, pitch, roll and yaw.

$$u_{eq\phi} = \left[\lambda\left(\dot{\phi}_d - \dot{\phi}\right) + \ddot{\phi}_d - \left(\frac{I_z - I_y}{I_x}\right)\dot{\theta}\dot{\psi} + \frac{J_r}{I_x}\,\Omega.\dot{\theta}\right]\frac{I_x}{l} \tag{III.54}$$

$$u_{eq\theta} = \left[\lambda(\dot{\theta}_d - \dot{\theta}) + \ddot{\theta}_d - \left(\frac{I_z - I_x}{I_y}\right)\dot{\phi}\dot{\psi} - \frac{J_r}{I_y}\,\Omega.\dot{\theta}\right]\frac{I_y}{l} \tag{III.55}$$

$$u_{eq\psi} = \left[\lambda\left(\dot{\psi}_d - \dot{\psi}\right) + \ddot{\psi}_d - \left(\frac{I_z - I_x}{I_y}\right)\dot{\phi}\dot{\theta}\right] I_z \tag{III.56}$$

III.4. CONCLUSIONS

This chapter presented the backstepping control approach, which dealt with the task of trajectory tracking for a quadcopter UAV. At first, the nonlinear dynamic model of the quadcopter has been presented using Newton laws. The control scheme has been based on two control loops. An inner loop for attitude control and an outer loop for altitude and position control. A nonlinear control method have been applied for both control loops.

After that we presented the SMC approach

The work of this study shows that the required objectives are accomplished to good extent. this includes PID, Backstepping and sliding mode. The limitations of the PID were taken into account, . The results revealed the limited capability of the PID control to deal with aggressive input signals especially in the presence of disturbance.

In backstepping approach, the control law is synthesized to force the system to follow the desired trajectory. Due to its complete independence from the other subsystem, firstly consider the control input for angular rotations subsystem and then the position control input is derived.

We have considered two nonlinear control techniques Backstepping Control, and Sliding Mode Control and a nonlinear unstable system, quadrotor which has several applications. The control equations have been derived for quadrotor dynamics..

This study has motivated to modify these schemes and simulate them for further improvement in performance which is under study.

GENERAL CONCLUSION

As it could be seen throughout this thesis, the proposed goals of this work were achieved, the fundamentals of flight dynamics of the quadrotor aircraft were investigated and a full mathematical non-linear model was derived using classical mechanics.

The purpose of the research is to design a quadcopter model and to stabilize it using a PID controller SMC and a BSC.

The Quadcopter dynamics show the design of quadcopter model and that it has been implemented with the help of Simulink library blocks.

Once the identification was complete, multiple control techniques were employed and their performances compared with each other, especially in terms of reference tracking. an open loop approach was conducted and following that a closed loop control as well followed by that we presented two leading theoretical approaches of two control techniques: backstepping control and sliding mode control.

Simulation results show the model is working fine and motion has been achieved. The quadcopter model is derived using Euler- equations. Later, a PID controller was integrated to the model to attain the stability which means controlling the quadcopter different motions. The simulation results show that the desired motions are attained by the controller which means that the controller is working efficiently. This model considers only the basic structure of a quadcopter. This model and the controller designed were tested using MATlab simulations only.

Altitude and position controller based on Backstepping and Sliding mode approach are presented. The performance and tracking ability of the controllers is examined and compared with traditional PID Controller through MATLAB simulation and stability is guaranteed using Lyapunov global stability theorem. The attitude and heading controller using SMC and BSC approach provide better control of orientation angles compare to PID and altitude controller also able to stabilize z motion with no overshoot and steady state error. Position controller introduced using the SMC approach provides average results compare to PID and BSC approach as chattering is associated with the SMC due to discontinuous control law. Each controller approves the ability to control the linear translations, although not quite as good. Therefore further research should be carried out on the robust control design and focus on extending the quadcopter dynamics and to implement the designed controller in real environment. the effect of disturbances and uncertainty is not taken in to account in this work so future work is to consider all these aspects and design a disturbance observer based control to estimate and reject the effect of disturbances and uncertainty.

Future works by applying BSC and SMC to continue our work on the visualization model design then a real life model. This will be built based on the work and results of this thesis so it would make it easier to built a safe well-functioning UAV .

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