



UNIVERSITY OF LAGHOUAT
FACULTY OF TECHNOLOGY



Course of Fluid Mechanics I

Intended for Second-Year Students in common Technology

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Chapter one: **Properties of fluids**



Learning Objectives :

- Determine the dimensions and units of physical quantities.
- Identify the key fluid properties used in the analysis of fluid behavior.
- Calculate common fluid properties given appropriate information.
- Explain effects of fluid compressibility.
- Use the concepts of viscosity, vapor pressure

1-1 Introduction and basic concepts

What's the Fluid Mechanics ?



- Mechanics is the oldest physical science that deals with both **stationary** and **moving** bodies under the influence of forces. The branch of mechanics that deals with bodies at rest is called **STATICS**, while the branch that deals with bodies in motion is called **DYNAMICS**.
- The subcategory fluid mechanics is defined as the science that deals with the **behavior** of fluids at rest (*fluid statics*) or in motion (*fluid dynamics*), and the interaction of fluids with solids or other fluids at the boundaries. Fluid mechanics is also referred to as fluid dynamics by considering fluids at rest as a special case of motion with zero velocity

- Fluid mechanics itself is also divided into several categories. The study of the motion of fluids that can be approximated as **INCOMPRESSIBLE** (such as liquids, especially water, and gases at **low speeds**) is usually referred to as **hydrodynamics**. A subcategory of hydrodynamics is **hydraulics**, which deals with liquid flows in pipes and open channels.

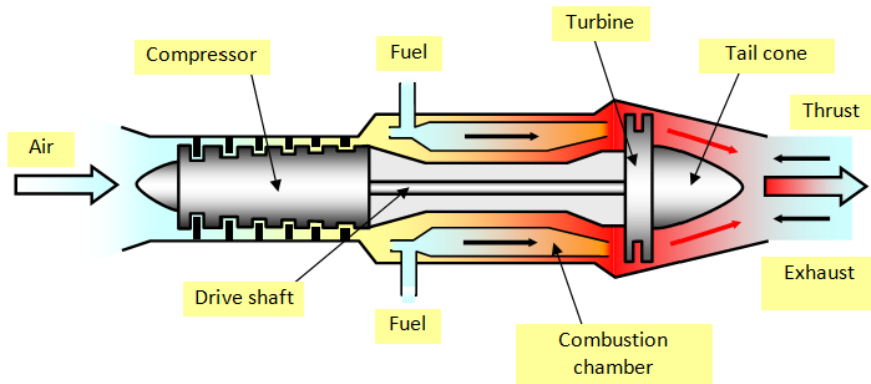


liquid flows in pipes



open channel flow

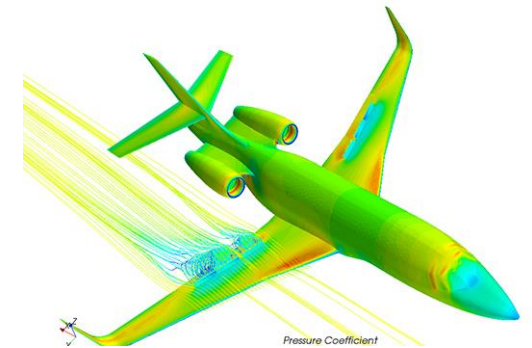
- **Gas dynamics** deals with the flow of fluids that undergo significant **density changes**, such as the flow of **gases** through nozzles at **high speeds** (exp : jet engine). The category aerodynamics deals with the flow of gases (especially air) over bodies such as aircraft, rockets, and automobiles at high or low speeds. Some other specialized categories such as meteorology, oceanography, and hydrology deal with naturally occurring flows

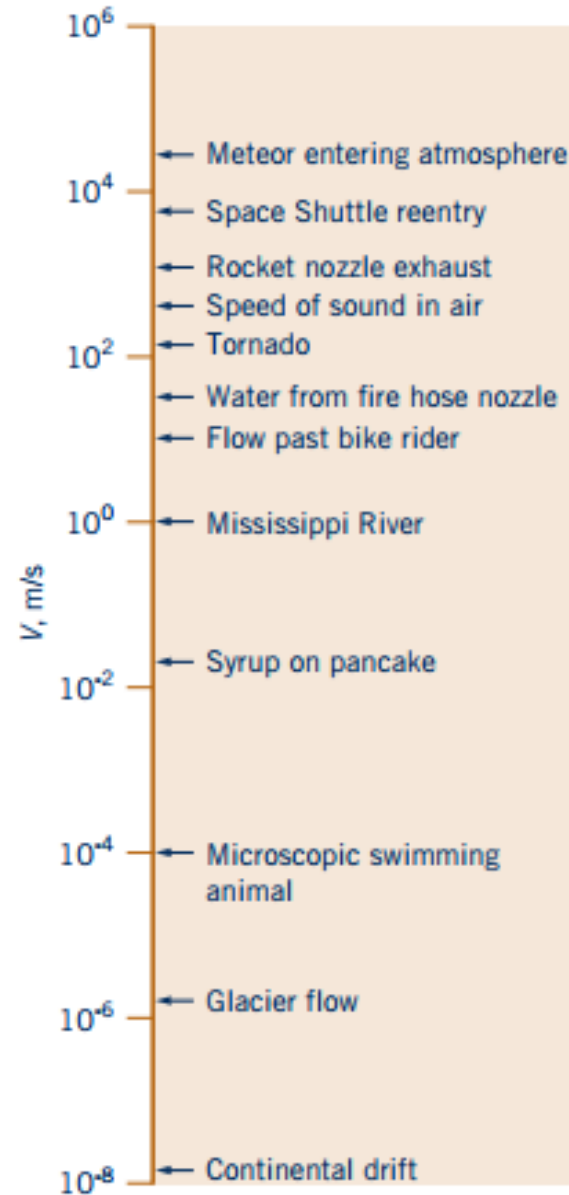
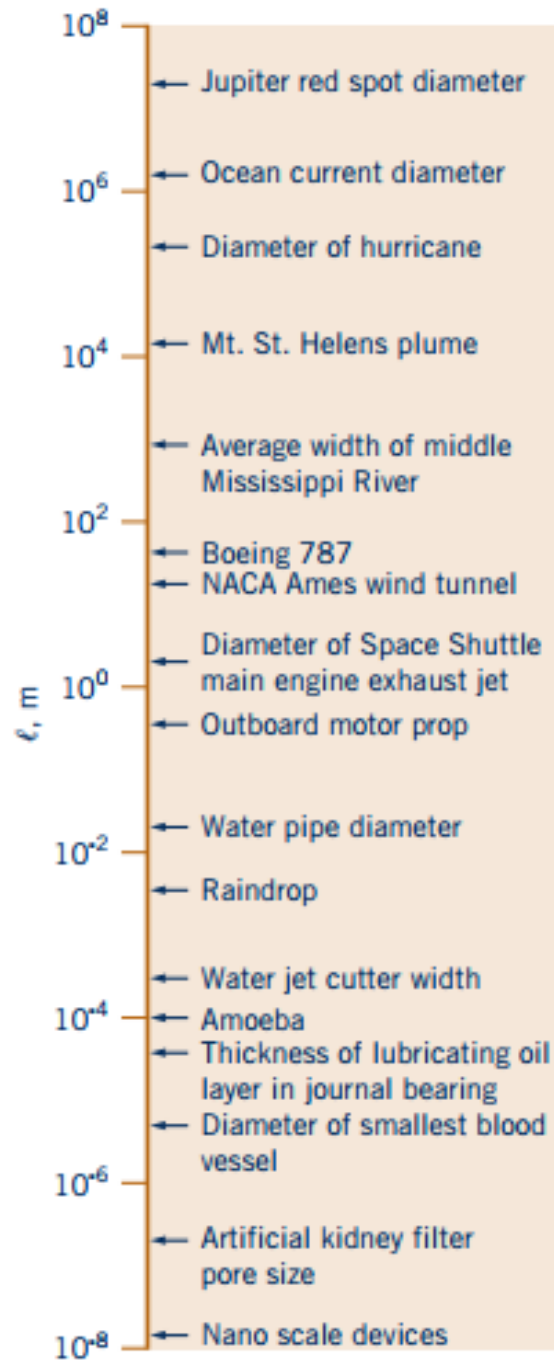


Jet engine

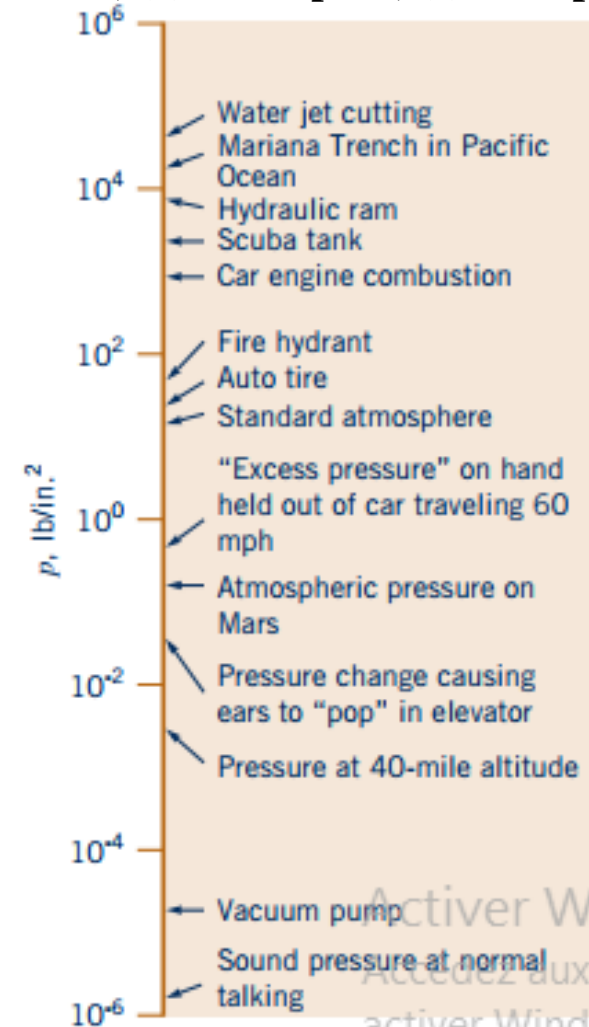


Air over aircraft





Characteristic values of some fluid flow parameters for a variety of flows: (a) object size, (b) fluid speed, (c) fluid pressure.



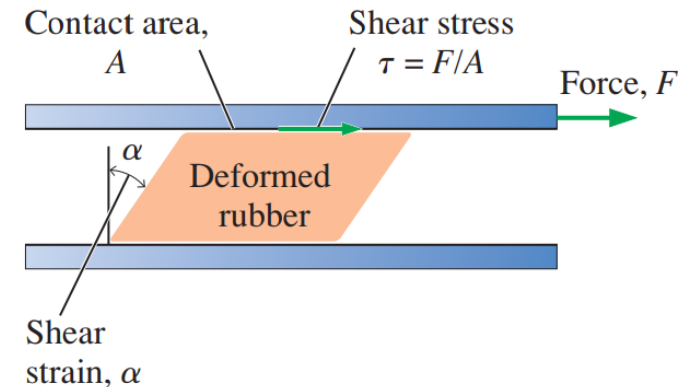
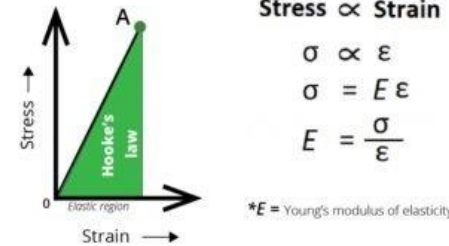
1-2 What is a Fluid?

We remember from physics that a substance exists in three primary phases: solid, liquid, and gas. (At very high temperatures, it also exists as plasma.) A substance in the liquid or gas phase is referred to as a **Fluid**. Distinction between a solid and a fluid is made on the basis of the substance's ability to resist an applied shear (or tangential) stress that tends to change its shape. A solid can resist an applied shear stress by deforming, whereas a fluid deforms continuously under the influence of a shear stress, no matter how small.

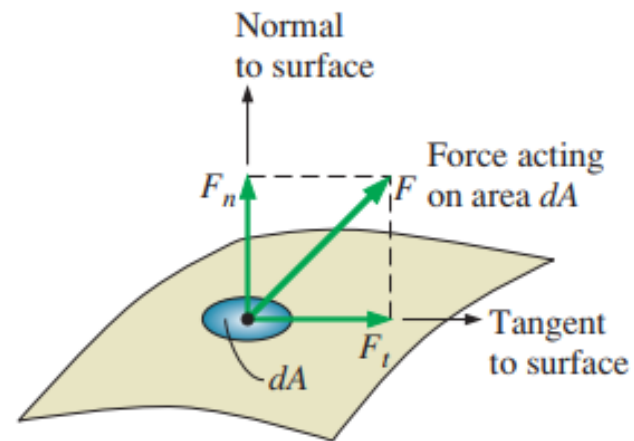
In solids, stress is proportional to strain, but in fluids,

stress is proportional to strain rate. When a constant shear force is applied,

a solid eventually stops deforming at some fixed strain angle, whereas a fluid never stops deforming and approaches a constant rate of strain.



We recall from statics that **stress** is defined as force per unit area and is determined by dividing the force by the area upon which it acts. The normal component of a force acting on a surface per unit area is called the **normal stress**, and the tangential component of a force acting on a surface per unit area is called **shear stress** (Fig). In a fluid at rest, the normal stress is called **pressure**. A fluid at rest is at a state of zero shear stress. When the walls are removed or a liquid container is tilted, a shear develops as the liquid moves to re-establish a horizontal free surface.



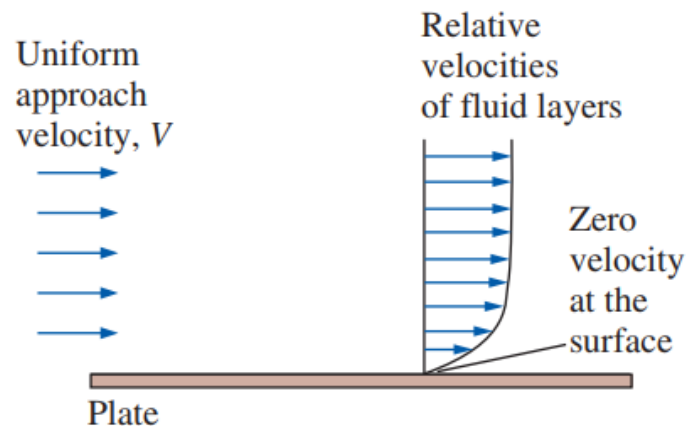
$$\text{Normal stress: } \sigma = \frac{F_n}{dA}$$

$$\text{Shear stress: } \tau = \frac{F_t}{dA}$$

1-3 The boundary layer

The layer that sticks to the surface slows the adjacent fluid layer because of viscous forces between the fluid layers, which slows the next layer, and so on. A consequence of the no-slip condition is that all velocity profiles must have zero values with respect to the surface at the points of contact between a fluid and a solid surface (Fig). Therefore, the no-slip condition is responsible for the development of the velocity profile. The flow region adjacent to the wall in which the viscous effects (and thus the velocity gradients) are significant is called the

boundary layer.



1-4 Dimensions and units

Some basic dimensions such as mass m , length L , time t , and temperature T are selected as **primary** or **fundamental dimensions**, while others such as velocity V , energy E , and volume V are expressed in terms of the primary dimensions and are called **secondary dimensions**, or **derived dimensions**. We accept as a fundamental premise that all equations describing physical phenomena must be **dimensionally homogeneous**

TABLE 1-1

The seven fundamental (or primary) dimensions and their units in SI

Dimension	Unit
Length	meter (m)
Mass	kilogram (kg)
Time	second (s)
Temperature	kelvin (K)
Electric current	ampere (A)
Amount of light	candela (cd)
Amount of matter	mole (mol)

Dimensions Associated with Common Physical Quantities

	<i>FLT</i> System	<i>MLT</i> System		<i>FLT</i> System	<i>MLT</i> System
Acceleration	LT^{-2}	LT^{-2}	Power	FLT^{-1}	ML^2T^{-3}
Angle	$F^0L^0T^0$	$M^0L^0T^0$	Pressure	FL^{-2}	$ML^{-1}T^{-2}$
Angular acceleration	T^{-2}	T^{-2}	Specific heat	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Angular velocity	T^{-1}	T^{-1}	Specific weight	FL^{-3}	$ML^{-2}T^{-2}$
Area	L^2	L^2	Strain	$F^0L^0T^0$	$M^0L^0T^0$
Density	$FL^{-4}T^2$	ML^{-3}	Stress	FL^{-2}	$ML^{-1}T^{-2}$
Energy	FL	ML^2T^{-2}	Surface tension	FL^{-1}	MT^{-2}
Force	F	MLT^{-2}	Temperature	Θ	Θ
Frequency	T^{-1}	T^{-1}	Time	T	T
Heat	FL	ML^2T^{-2}	Torque	FL	ML^2T^{-2}
Length	L	L	Velocity	LT^{-1}	LT^{-1}
Mass	$FL^{-1}T^2$	M	Viscosity (dynamic)	$FL^{-2}T$	$ML^{-1}T^{-1}$
Modulus of elasticity	FL^{-2}	$ML^{-1}T^{-2}$	Viscosity (kinematic)	L^2T^{-1}	L^2T^{-1}
Moment of a force	FL	ML^2T^{-2}	Volume	L^3	L^3
Moment of inertia (area)	L^4	L^4	Work	FL	ML^2T^{-2}
Moment of inertia (mass)	FLT^2	ML^2			
Momentum	FT	MLT^{-1}			

Where F is the basic dimensions of force. Since Newton's law states that force is equal to mass times acceleration, it follows that : $F=MLT^{-2}$ or $M=FL^{-1} T^2$

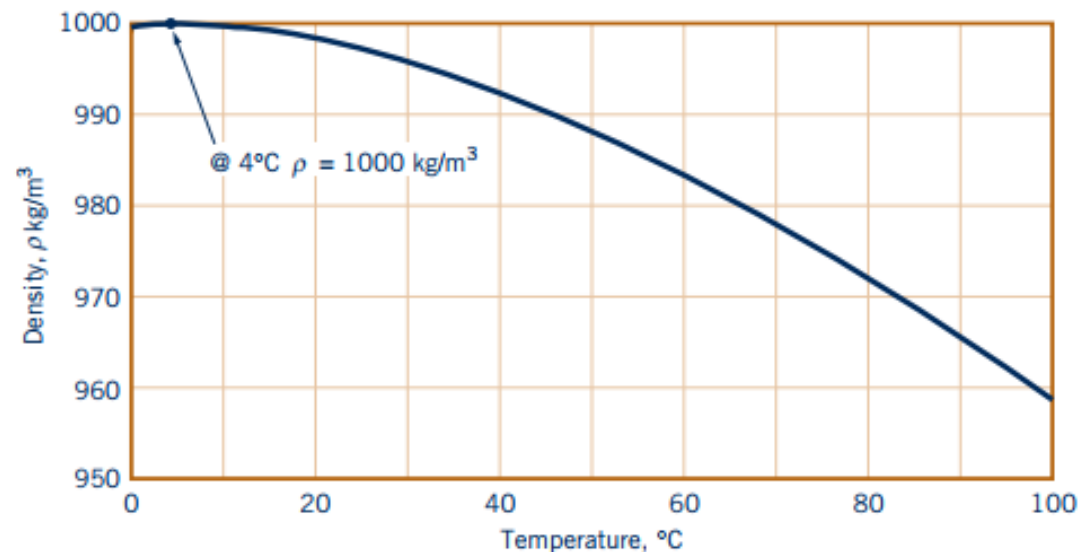
1-5 Analysis of fluid behavior

The study of fluid mechanics involves the same fundamental laws you have encountered in physics and other mechanics courses. These laws include:

- Newton's laws of motion
- Conservation of mass
- First and second laws of thermodynamics

1-6 Measures of fluid mass and weight

- **Density** : The *density* of a fluid, designated by (rho) ρ , is defined as its mass per unit volume. Density is typically used to characterize the **mass of a fluid system**. in SI the units are kg/m^3 , The value of density can vary widely between different fluids, but for liquids, variations in pressure and temperature generally have only a small effect on the value of ρ



■ **Figure 1.3** Density of water as a function of temperature.

Specific Weight :

The *specific weight* of a fluid, designated by γ (gamma), is defined as its *weight* per unit volume. Thus, specific weight is related to density through the equation

$$\gamma = \rho g$$

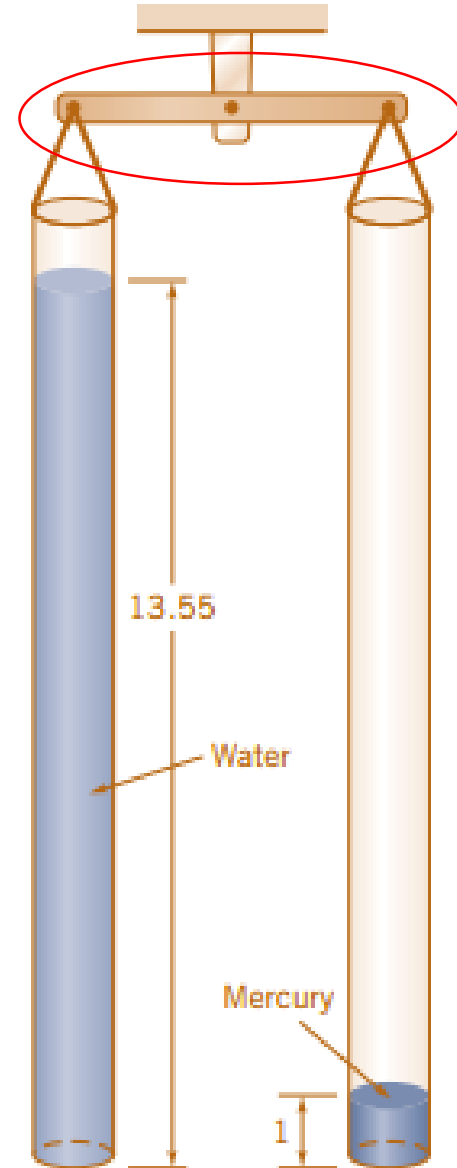
As the density is used to characterize the mass of a fluid system, the specific weight is used to characterize the **weight of the system**. In the SI the units are **N/m³**

Specific Gravity :

The *specific gravity* of a fluid, designated as SG , is defined as the ratio of the density of the fluid to the density of water at some specified temperature. Usually the specified temperature is taken as 4°C and at this temperature the density of water is 1000 kg/m^3 .

In equation form, specific gravity is expressed as:

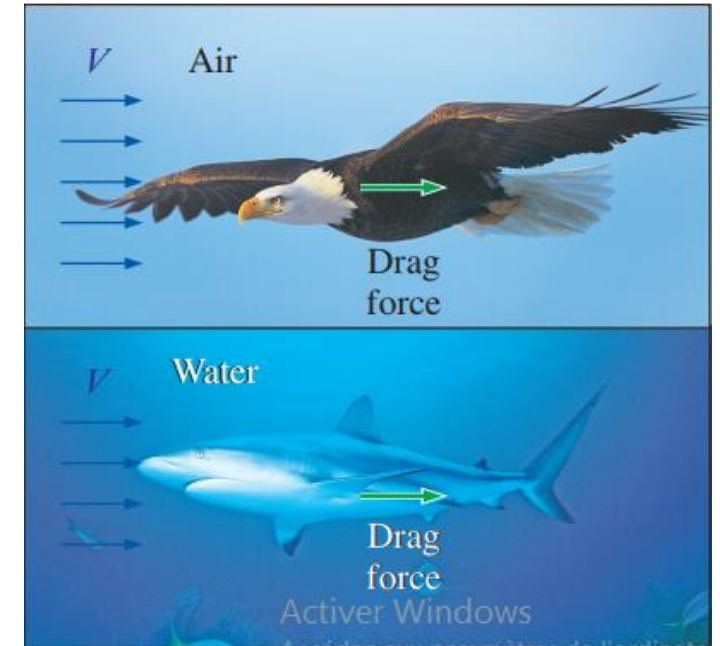
$$SG = \frac{\rho}{\rho_{\text{H}_2\text{O}@4^{\circ}\text{C}}}$$

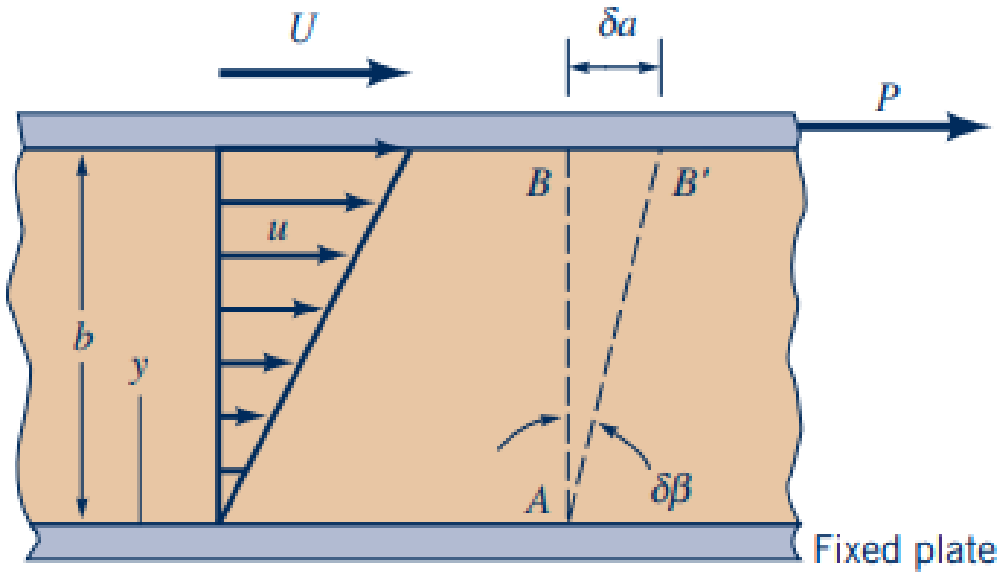


Viscosity:

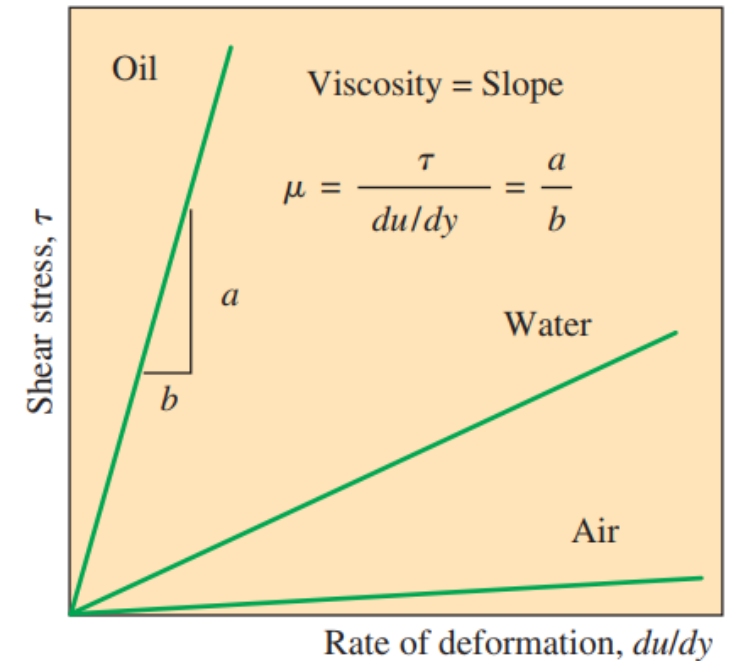
The properties of density and specific weight are measures of the “heaviness” of a fluid, some additional property is needed to describe the “fluidity” of the fluid. When a fluid moves relative to a solid or when two fluids move relative to each other; we move with relative ease in air, but not so in water.

It appears that there is a property that represents the **internal resistance** of a fluid to motion or the “fluidity,” and that property is the **VISCOSITY**. The force a flowing fluid exerts on a body in the flow direction is called the **DRAG FORCE**,



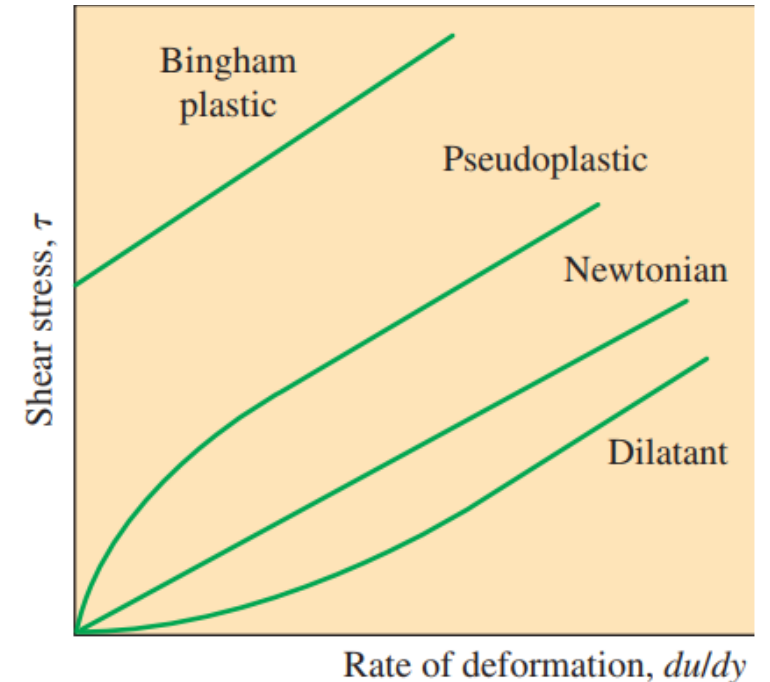


$$\tau = \mu \frac{du}{dy}$$



Where : $\delta\beta$ is the **shearing strain** and τ is the **shearing stress**

The constant of proportionality is designated by the Greek symbol μ (**mu**) and is called the **absolute viscosity, dynamic viscosity**, or simply the **viscosity of the fluid**, The **kinematic viscosity** is defined as the ratio of the absolute viscosity to the fluid density ρ .



1-7 Compressibility of Fluids

How easily can the volume of a given mass of the fluid be changed when there is a change in pressure ?

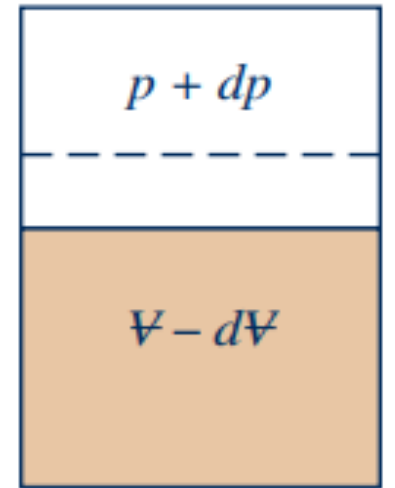
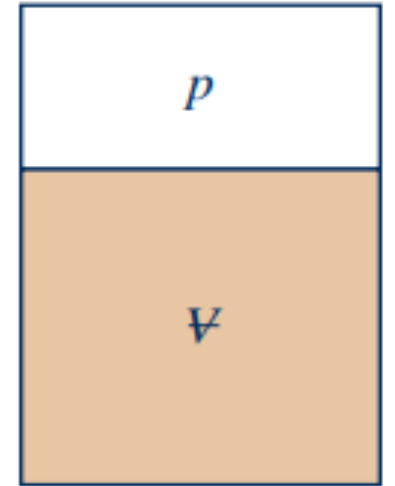
A property that is commonly used to characterize compressibility is the *bulk modulus of elasticity* or *coefficient of compressibility*

defined as :

$$E_v = -\frac{dp}{dV/V}$$

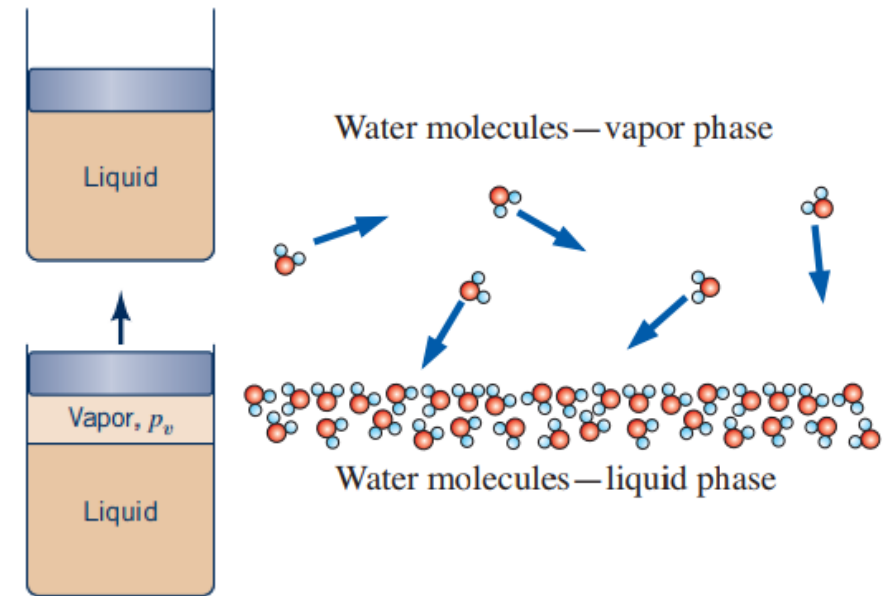
It can also be expressed as :

$$E_v = \frac{dp}{d\rho/\rho}$$



1-8 Vapor Pressure

At a given pressure, the temperature at which a pure substance changes phase is called the **saturation temperature** T_{sat} . Likewise, at a given temperature, the pressure at which a pure substance changes phase is called the **saturation pressure** P_{sat} . For example at an absolute pressure of 1 atm or 101.325 kPa, the saturation temperature of water is 100°C. The **vapor pressure** P_v of a pure substance is defined as *the pressure exerted by its vapor in phase equilibrium with its liquid at a given temperature*, P_v is a property of the pure substance, and turns out to be identical to the saturation pressure P_{sat} of the liquid ($P_v = P_{\text{sat}}$)



Saturation (or vapor) pressure of water at various temperatures

Temperature T , °C	Saturation Pressure P_{sat} , kPa
-10	0.260
-5	0.403
0	0.611
5	0.872
10	1.23
15	1.71
20	2.34
25	3.17
30	4.25
40	7.38
50	12.35
100	101.3 (1 atm)
150	475.8
200	1554
250	3973
300	8581

Approximate Physical Properties of Some Common Liquids (SI Units)

Liquid	Temperature (°C)	Density, ρ (kg/m ³)	Specific Weight, γ (kN/m ³)	Dynamic Viscosity, μ (N · s/m ²)	Kinematic Viscosity, ν (m ² /s)	Surface Tension, ^a σ (N/m)	Vapor Pressure, P_v [N/m ² (abs)]	Bulk Modulus, ^b E_v (N/m ²)
Carbon tetrachloride	20	1,590	15.6	9.58 E - 4	6.03 E - 7	2.69 E - 2	1.3 E + 4	1.31 E + 9
Ethyl alcohol	20	789	7.74	1.19 E - 3	1.51 E - 6	2.28 E - 2	5.9 E + 3	1.06 E + 9
Gasoline ^c	15.6	680	6.67	3.1 E - 4	4.6 E - 7	2.2 E - 2	5.5 E + 4	1.3 E + 9
Glycerin	20	1,260	12.4	1.50 E + 0	1.19 E - 3	6.33 E - 2	1.4 E - 2	4.52 E + 9
Mercury	20	13,600	133	1.57 E - 3	1.15 E - 7	4.66 E - 1	1.6 E - 1	2.85 E + 10
SAE 30 oil ^f	15.6	912	8.95	3.8 E - 1	4.2 E - 4	3.6 E - 2	—	1.5 E + 9
Seawater	15.6	1,030	10.1	1.20 E - 3	1.17 E - 6	7.34 E - 2	1.77 E + 3	2.34 E + 9
Water	15.6	999	9.80	1.12 E - 3	1.12 E - 6	7.34 E - 2	1.77 E + 3	2.15 E + 9

Approximate Physical Properties of Some Common Gases at Standard Atmospheric Pressure (SI Units)

Gas	Temperature (°C)	Density, ρ (kg/m ³)	Specific Weight, γ (N/m ³)	Dynamic Viscosity, μ (N · s/m ²)	Kinematic Viscosity, ν (m ² /s)	Gas Constant, ^a R (J/kg · K)	Specific Heat Ratio, ^b k
Air (standard)	15	1.23 E + 0	1.20 E + 1	1.79 E - 5	1.46 E - 5	2.869 E + 2	1.40
Carbon dioxide	20	1.83 E + 0	1.80 E + 1	1.47 E - 5	8.03 E - 6	1.889 E + 2	1.30
Helium	20	1.66 E - 1	1.63 E + 0	1.94 E - 5	1.15 E - 4	2.077 E + 3	1.66
Hydrogen	20	8.38 E - 2	8.22 E - 1	8.84 E - 6	1.05 E - 4	4.124 E + 3	1.41
Methane (natural gas)	20	6.67 E - 1	6.54 E + 0	1.10 E - 5	1.65 E - 5	5.183 E + 2	1.31
Nitrogen	20	1.16 E + 0	1.14 E + 1	1.76 E - 5	1.52 E - 5	2.968 E + 2	1.40
Oxygen	20	1.33 E + 0	1.30 E + 1	2.04 E - 5	1.53 E - 5	2.598 E + 2	1.40

Resume equations of chapter one

Specific weight

$$\gamma = \rho g$$

Specific gravity

$$SG = \frac{\rho}{\rho_{\text{H}_2\text{O}@4^\circ\text{C}}}$$

Ideal gas law

$$\rho = \frac{p}{RT}$$

Newtonian fluid shear stress

$$\tau = \mu \frac{du}{dy}$$

Bulk modulus

$$E_v = -\frac{dp}{dV/V}$$

1-9 Applications

Exercise 1:

The weight of a 500 ml volume of a liquid is 8N. Determine its specific gravity, density and specific gravity.

Exercise 2:

The kinematic viscosity of oxygen at 20°C and 150 kPa is 0.104 stokes. Determine the dynamic viscosity of oxygen under the same conditions.

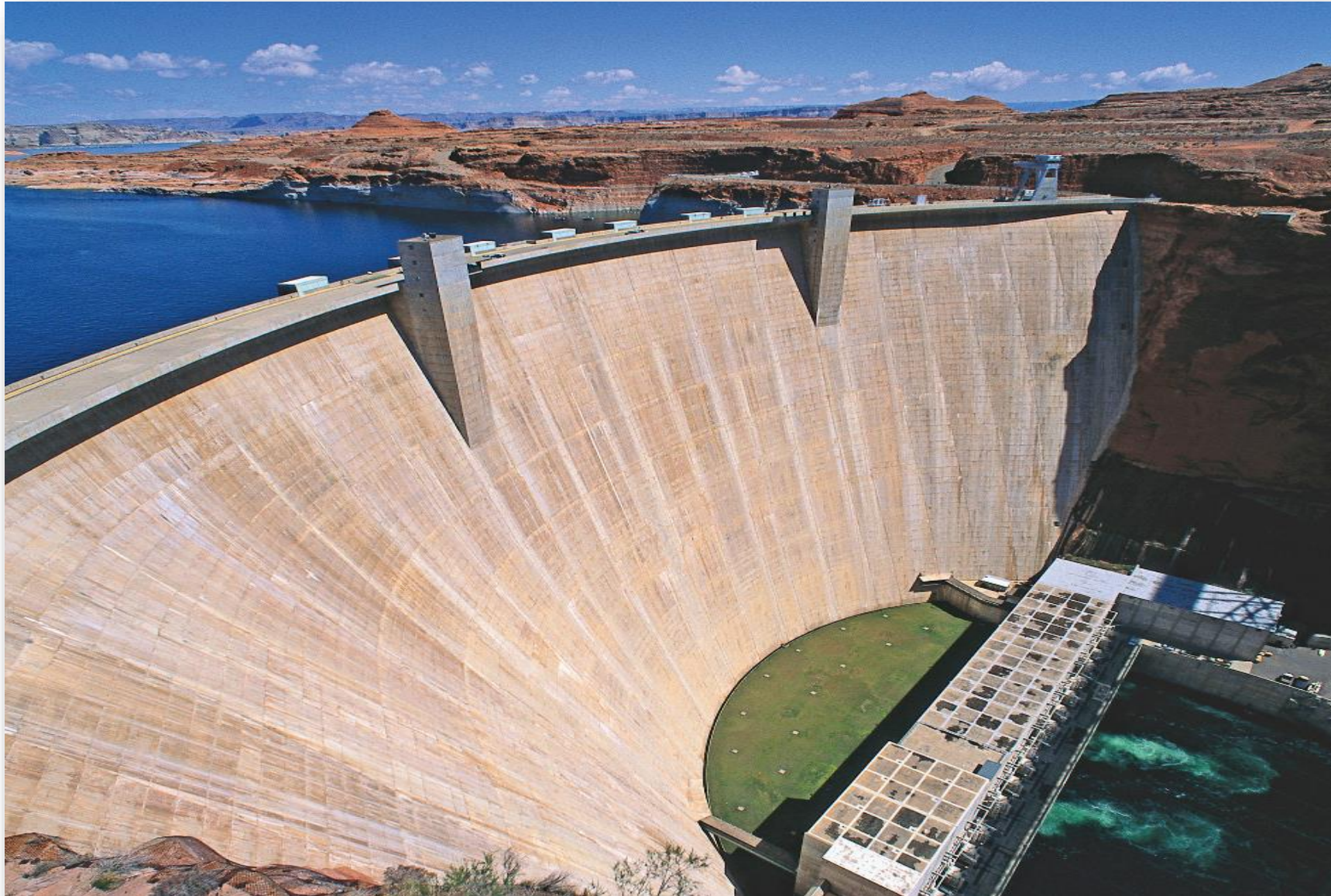
*We give the universal gas constant : $R = 259.8 \text{ j/Kg.K}$

Exercise 3 :

Pressures are sometimes determined by measuring the height of a column of liquid in a vertical tube. What diameter of clean glass tubing is required so that the rise of water at 20°C in a tube due to capillary action (as opposed to pressure in the tube) is less than 1.0 mm?

we give for water at 20°C : $s = 0.0728 \text{ N/m}$, $g = 9789 \text{ N/m}^3$.

Chapter two : Pressure and fluid statics

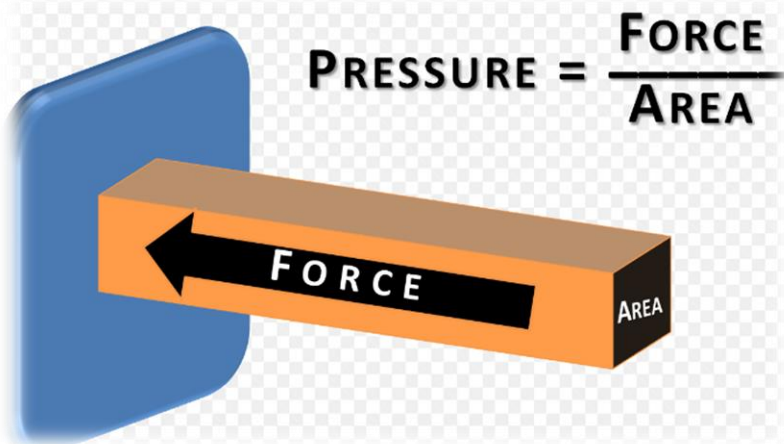


Learning Objectives:

- Determine the pressure at various locations in a fluid at rest.
- Explain the concept of manometers and apply appropriate equations to determine pressures.
- Calculate the hydrostatic pressure force on a plane or curved submerged surface.
- Calculate the buoyant force and discuss the stability of floating or submerged objects.

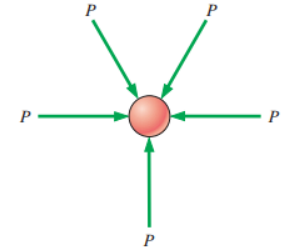
2-1 Pressure

Pressure is defined as a normal force exerted by a fluid per unit area. We speak of pressure only when we deal with a gas or a liquid. The counterpart of pressure in solids is *normal stress*. Since pressure is defined as force per unit area, it has the unit of newtons per square meter (N/m^2), which is called a **Pascal (Pa)**. That is : $1 \text{ Pa} = 1 \text{ N/m}^2$



Pressure at a Point :

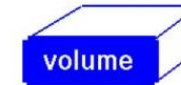
Pressure is a **scalar quantity, or a vector !!** the pressure at a point in a fluid is the same in all directions ?! It can be demonstrated by considering a small wedge-shaped fluid element of unit length ($\Delta y = 1$) in equilibrium. The mean pressures at the three surfaces are P_y , P_z , and P_s , and the force acting on a surface is the product of mean pressure and the surface area. From Newton's second law, a force balance in the y- and z-directions gives:



A scalar quantity has only **magnitude**.
A vector quantity has both **magnitude** and **direction**.

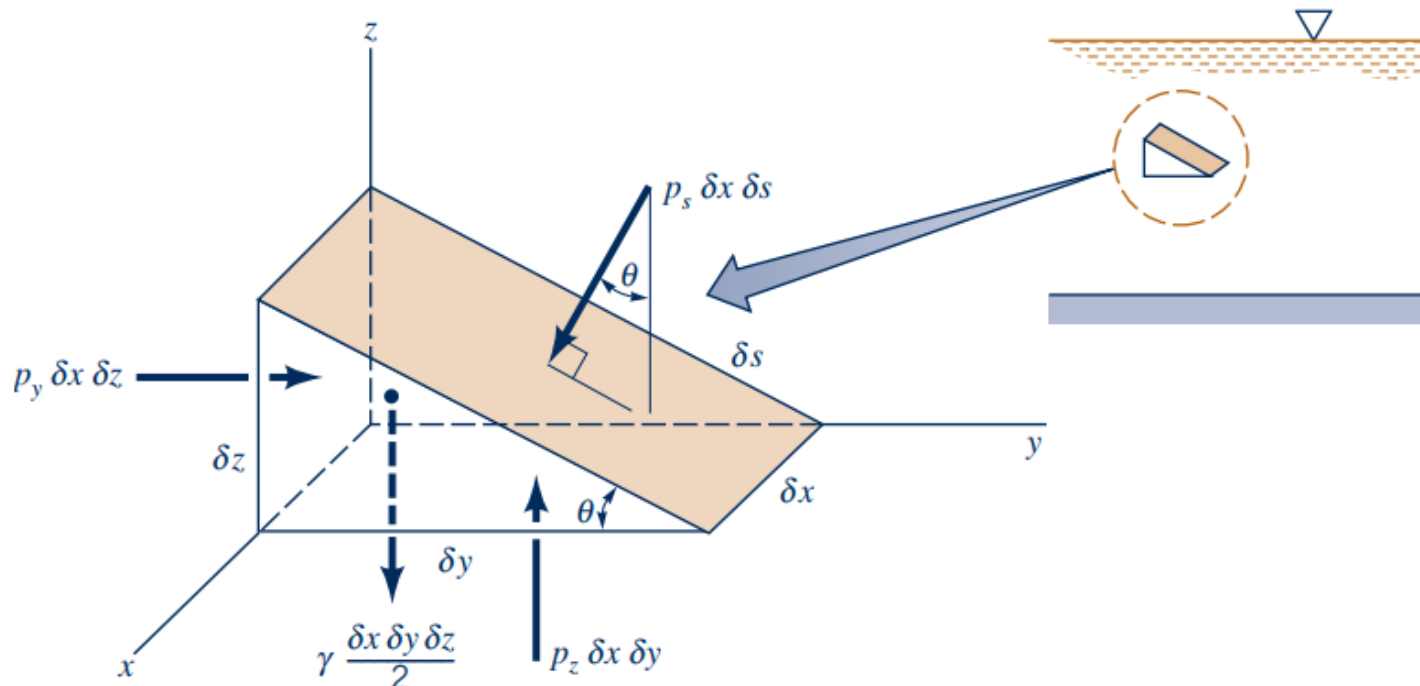
Scalar Quantities

length, area, volume
speed
mass, density
pressure
temperature
energy, entropy
work, power



Vector Quantities

displacement
velocity
acceleration
momentum
force
lift, drag, thrust
weight



$$\sum F_y = p_y \delta x \delta z - p_s \delta x \delta s \sin \theta = \rho \frac{\delta x \delta y \delta z}{2} a_y$$

$$\sum F_z = p_z \delta x \delta y - p_s \delta x \delta s \cos \theta - \gamma \frac{\delta x \delta y \delta z}{2} = \rho \frac{\delta x \delta y \delta z}{2} a_z$$

It follows from the geometry that : $\delta y = \delta s \cos \theta$ $\delta z = \delta s \sin \theta$

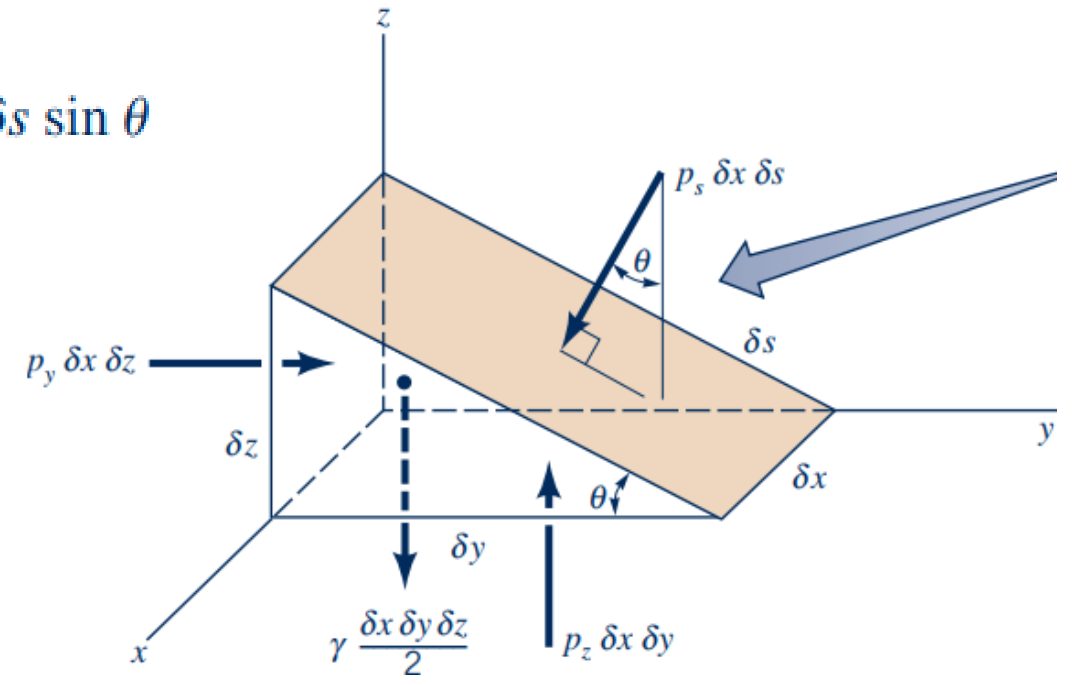
$$p_y - p_s = \rho a_y \frac{\delta y}{2}$$

So :

$$p_z - p_s = (\rho a_z + \gamma) \frac{\delta z}{2}$$

Since we are really interested in what is happening at a point, we take the limit as δx , δy and δz approach zero , it follows that :

$$p_y = p_s \quad p_z = p_s$$



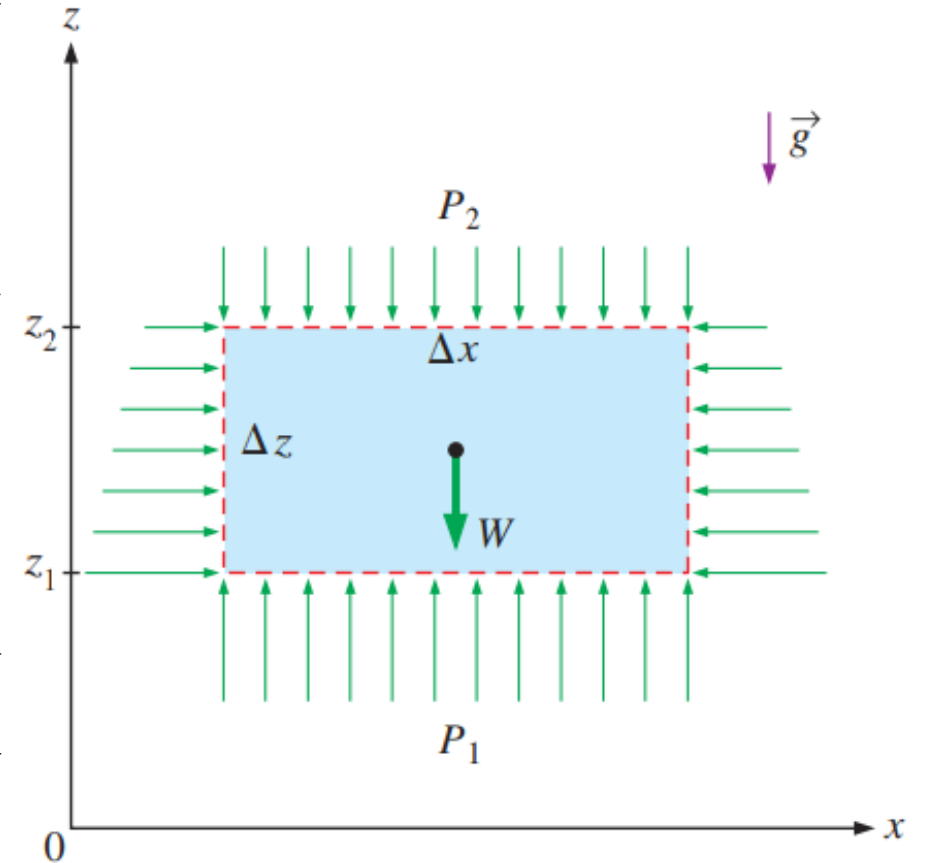
Thus we conclude that the pressure at a point in a fluid has the same magnitude in all directions. This result is applicable to fluids in motion as well as fluids at rest since pressure is a scalar, not a vector..

2-2 Variation of Pressure with Depth

To obtain a relation for the variation of pressure with depth, consider a rectangular fluid element of height Δz , length Δx , and unit depth $\Delta y = 1$ in equilibrium, as shown in Fig. Assuming the density of the fluid ρ to be constant, a force balance in the vertical z -direction gives :

$$\sum F_z = ma_z = 0: \quad P_1 \Delta x \Delta y - P_2 \Delta x \Delta y - \rho g \Delta x \Delta y \Delta z = 0$$

where $W = mg = \rho g \Delta x \Delta y \Delta z$ is the weight of the fluid element and $\Delta z = z_2 - z_1$. Dividing by $\Delta x \Delta y$ and rearranging gives :

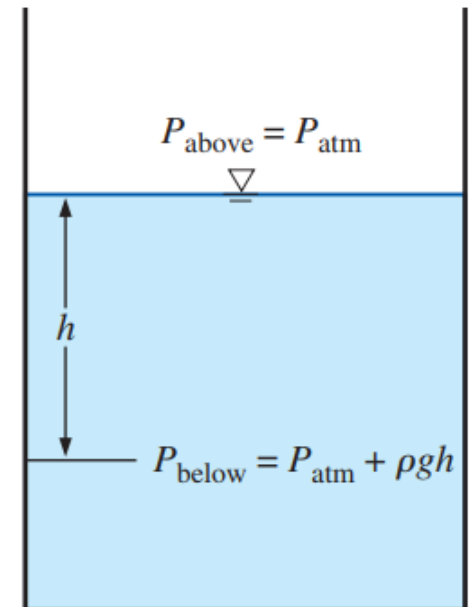


$$\Delta P = P_2 - P_1 = -\rho g \Delta z = -\gamma_s \Delta z$$

where $\gamma_s = \rho g$ is the *specific weight* of the fluid. Thus, we conclude that the pressure difference between two points in a constant density fluid is proportional to the vertical distance Δz between the points and the density ρ of the fluid. Noting the negative sign, *pressure in a static fluid increases linearly with depth*

An easier equation to remember and apply between any two points in the **same** fluid under hydrostatic conditions is :

$$P_{\text{below}} = P_{\text{above}} + \rho g |\Delta z| = P_{\text{above}} + \gamma_s |\Delta z|$$



For fluids whose density changes significantly with elevation, a relation for the variation of pressure with elevation can be obtained by dividing the equation :

$$\Delta P = P_2 - P_1 = -\rho g \Delta z = -\gamma_s \Delta z$$

by Δz , and taking the limit as $\Delta z \rightarrow 0$. This yields :

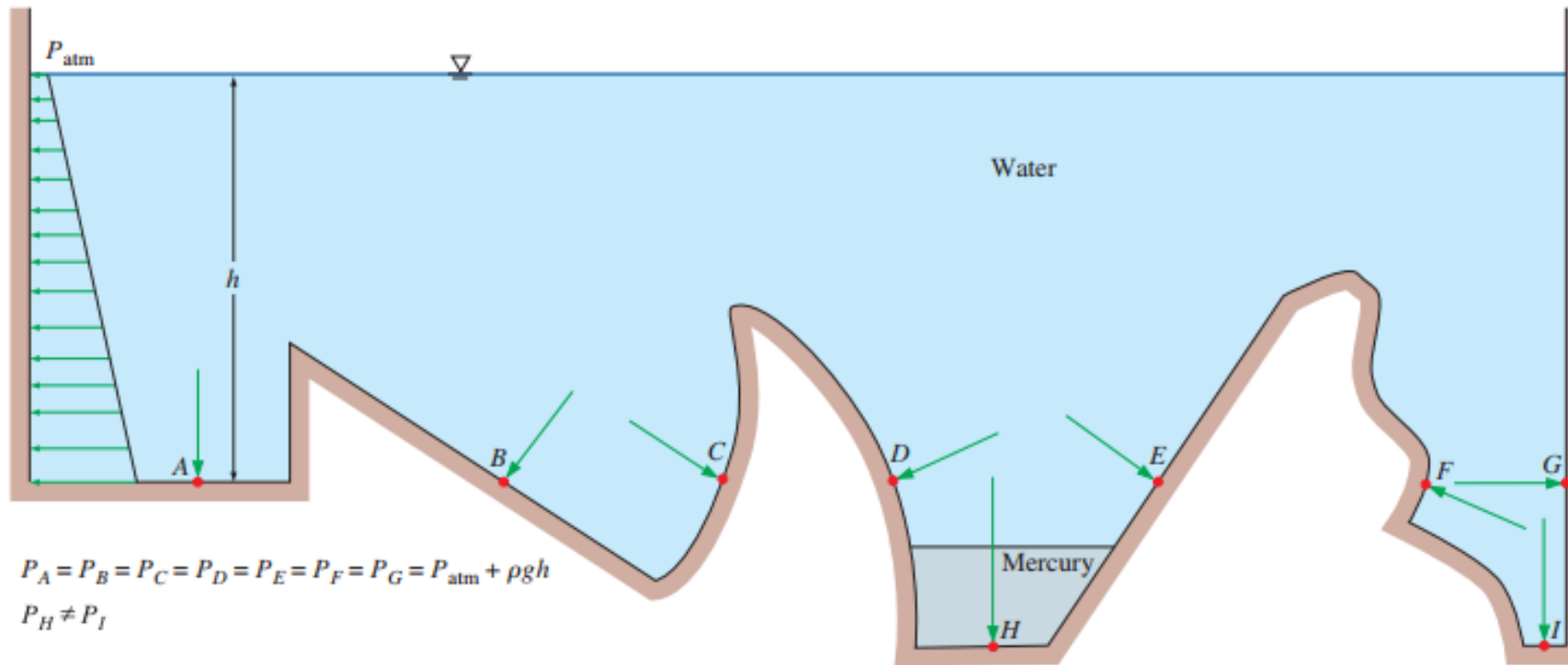
$$\frac{dP}{dz} = -\rho g$$

When the variation of density with elevation is known, the pressure difference between any two points 1 and 2 can be determined by integration to be :

$$\Delta P = P_2 - P_1 = -\int_1^2 \rho g dz$$

2-3 Pascal's law

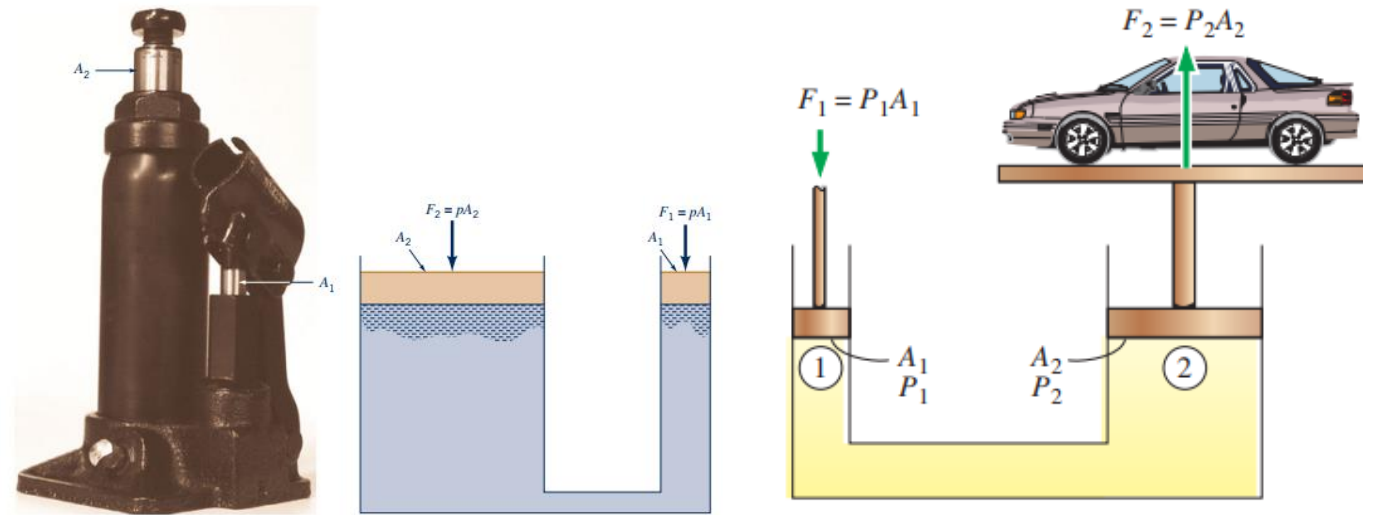
Pressure in a fluid at rest is independent of the shape or cross section of the container. It changes with the vertical distance, but remains constant in other directions, A consequence of the pressure in a fluid remaining constant in the horizontal direction is that *the pressure applied to a confined fluid increases the pressure throughout by the same amount.* This is called **Pascal's law**



Application of the Pascal's law:

Pascal's machine : It has been the source of many inventions that are a part of our daily lives such as hydraulic brakes and lifts. This is what enables us to lift a car easily by one arm, the ratio of output force to input force is determined to be

$$P_1 = P_2 \quad \rightarrow \quad \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \rightarrow \quad \frac{F_2}{F_1} = \frac{A_2}{A_1}$$



For the compressible Fluids :

$$\Delta P = P_2 - P_1 = - \int_1^2 \rho g dz$$

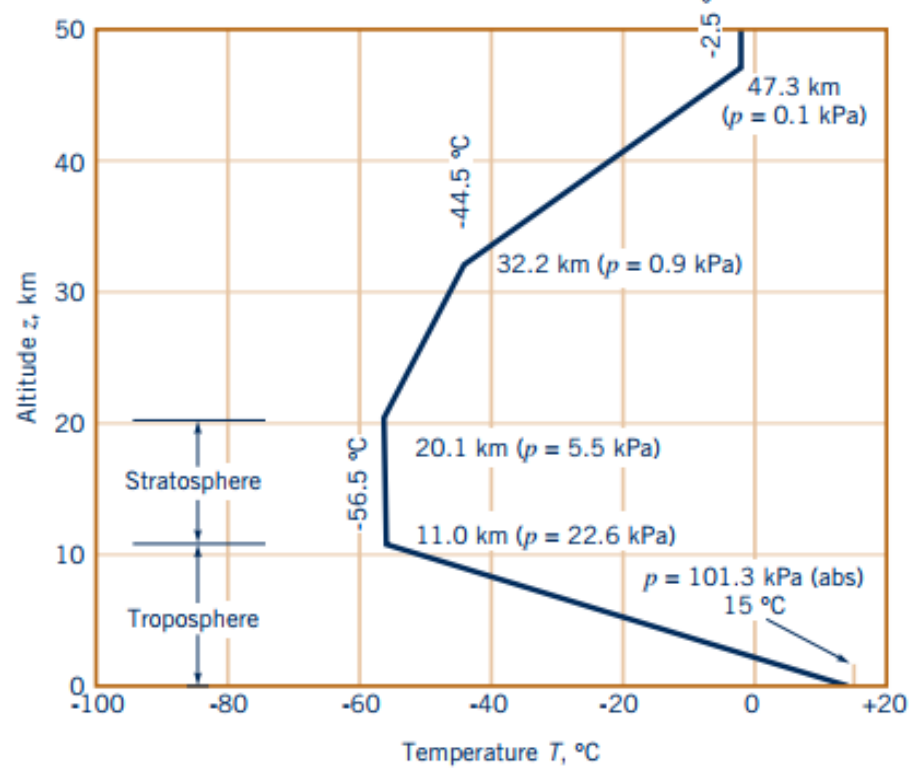
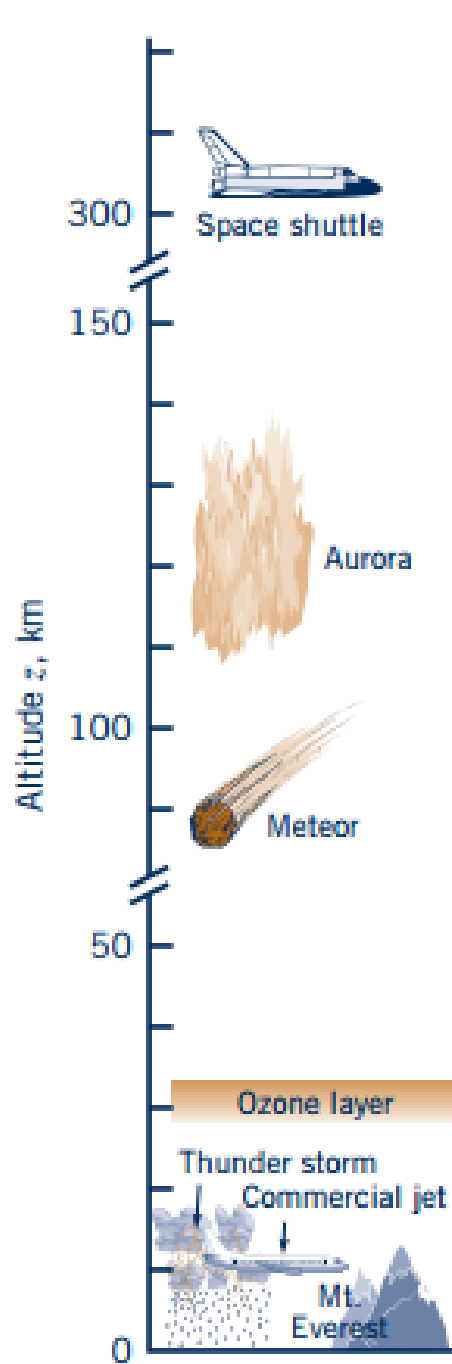
$$\rho = \frac{p}{RT}$$

$$\frac{dp}{dz} = - \frac{gp}{RT}$$

$$\int_{p_1}^{p_2} \frac{dp}{p} = \ln \frac{p_2}{p_1} = - \frac{g}{R} \int_{z_1}^{z_2} \frac{dz}{T}$$

$$p_2 = p_1 \exp \left[- \frac{g(z_2 - z_1)}{RT_0} \right]$$

$$T = T_a - \beta z$$



Properties of U.S. Standard Atmosphere at Sea Level*

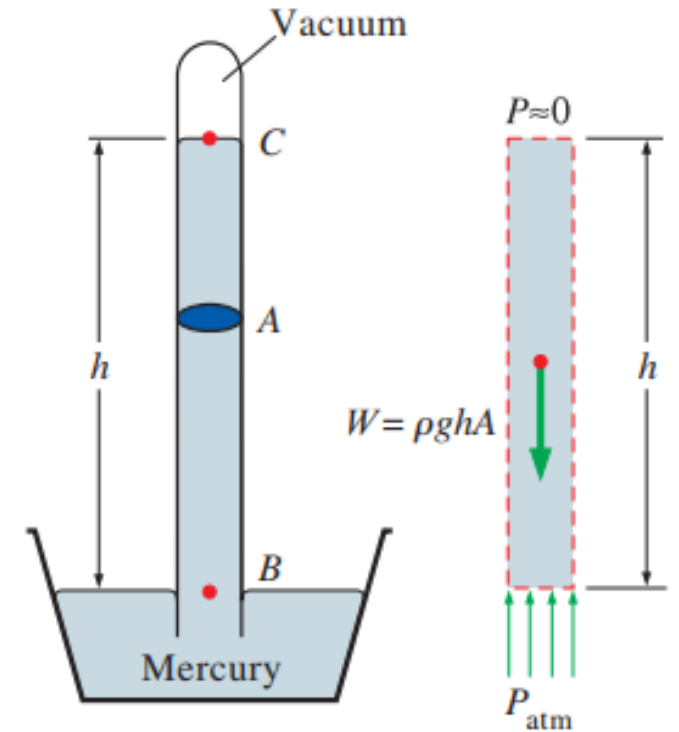
Property	SI Units	BG Units
Temperature, T	288.15 K (15 °C)	518.67 °R (59.00 °F)
Pressure, p	101.33 kPa (abs)	2116.2 lb/ft ² (abs) [14.696 lb/in. ² (abs)]
Density, ρ	1.225 kg/m ³	0.002377 slugs/ft ³
Specific weight, γ	12.014 N/m ³	0.07647 lb/ft ³
Viscosity, μ	1.789×10^{-5} N · s/m ²	3.737×10^{-7} lb · s/ft ²

*Acceleration of gravity at sea level = $9.807 \text{ m/s}^2 = 32.174 \text{ ft/s}^2$.

2-4 Pressure Measurement

The Barometers: Atmospheric pressure is measured by a device called a **barometer**; thus, the atmospheric pressure is often referred to as the *barometric pressure*. The Italian Torricelli (1608–1647) was the first to conclusively prove that the atmospheric pressure can be measured by inverting a mercury-filled tube into a mercury container that is open to the atmosphere. The pressure at point *B* is equal to the atmospheric pressure, and the pressure at point *C* can be taken to be **zero** since there is only mercury vapor above point *C* and the pressure is very low relative to P_{atm} and can be neglected to an excellent approximation. Writing a force balance in the vertical direction gives :

$$P_{\text{atm}} = \rho gh$$



The standard atmosphere, is defined as the pressure produced by a column of mercury 760 mm in height at 0°C

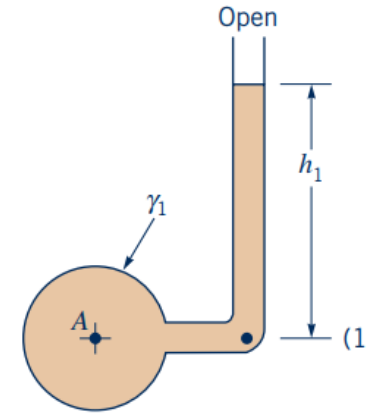
The Manometers :

A standard technique for measuring pressure involves the use of liquid columns in vertical or inclined tubes. Pressure-measuring devices based on this technique are called *manometers*, there are many other configurations possible depending on the particular application. Three common types of manometers include the **piezometer tube**, the **U-tube manometer**, and the **inclined-tube manometer**.

1) Piezometer Tube:

$$p_A = \gamma_1 h_1$$

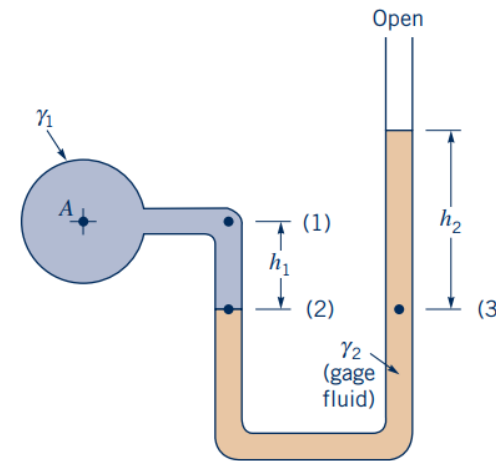
To measure **low values of pressures**



2) U-Tube Manometer:

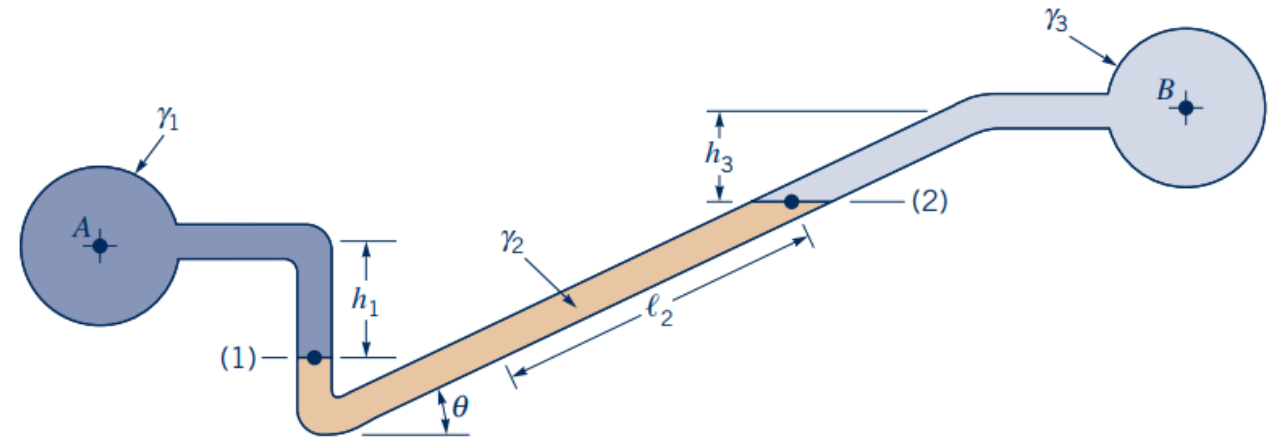
$$p_A + \gamma_1 h_1 - \gamma_2 h_2 = 0$$

To measure **high values of pressures**

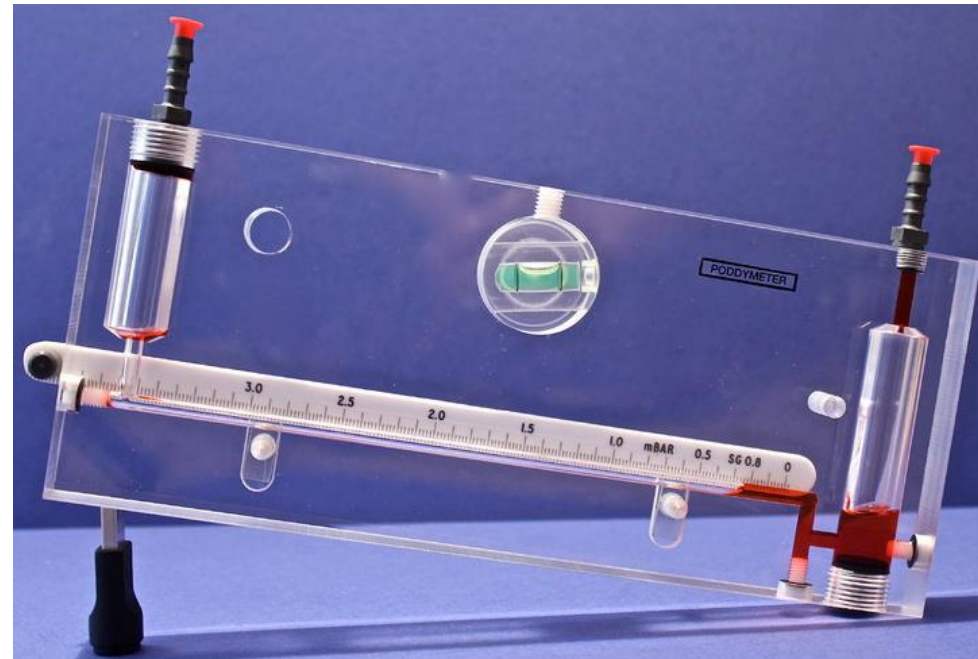


3) Inclined-Tube Manometer

To measure **small pressure changes**



$$p_A - p_B = \gamma_2 \ell_2 \sin \theta + \gamma_3 h_3 - \gamma_1 h_1$$



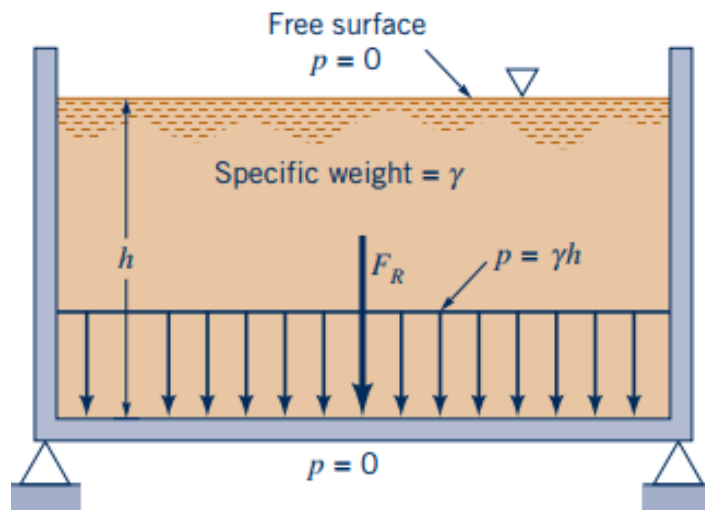
2-5 Hydrostatic Force (FR)

When a surface is submerged in a fluid, forces develop on the surface due to the fluid. The determination of these forces is important in the design of storage tanks, ships, dams, and other hydraulic structures,

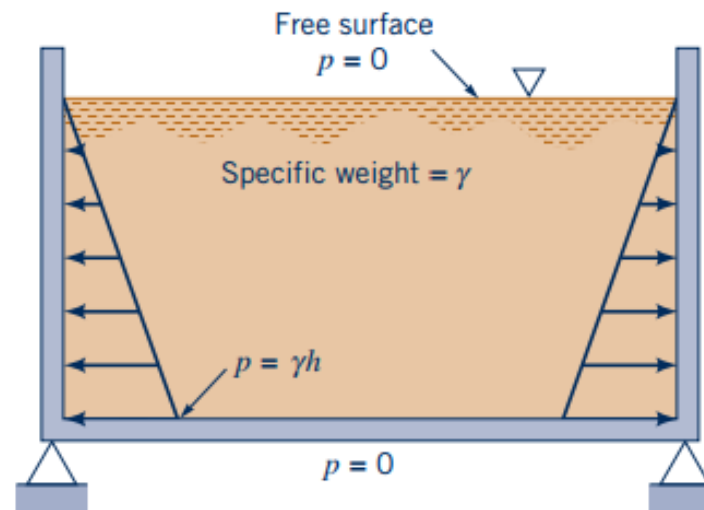


* Hydrostatic Force on a Plane Surface :

In the incompressible fluids, we know that the pressure will vary linearly with depth as shown in Fig(b). For a horizontal surface, such as the bottom of a liquid filled tank in Fig(a), the magnitude of the **resultant force** $F_R = p / A$, where : $p = \gamma h$ is the uniform pressure on the bottom and A is the area of the bottom. If the pressure on the ends of the tank is not uniformly distributed Fig (b),determination of the resultant force for situations is important .



(a) Pressure on tank bottom



(b) Pressure on tank ends



For the more general case in which a submerged plane surface is inclined

$$F_R = \int_A \gamma h \, dA = \int_A \gamma y \sin \theta \, dA$$

where $h = y \sin \theta$. For constant γ and θ

$$F_R = \gamma \sin \theta \int_A y \, dA$$

The integral appearing in Eq. 2.17 is the *first moment of the area* with respect to the x axis, so we can write

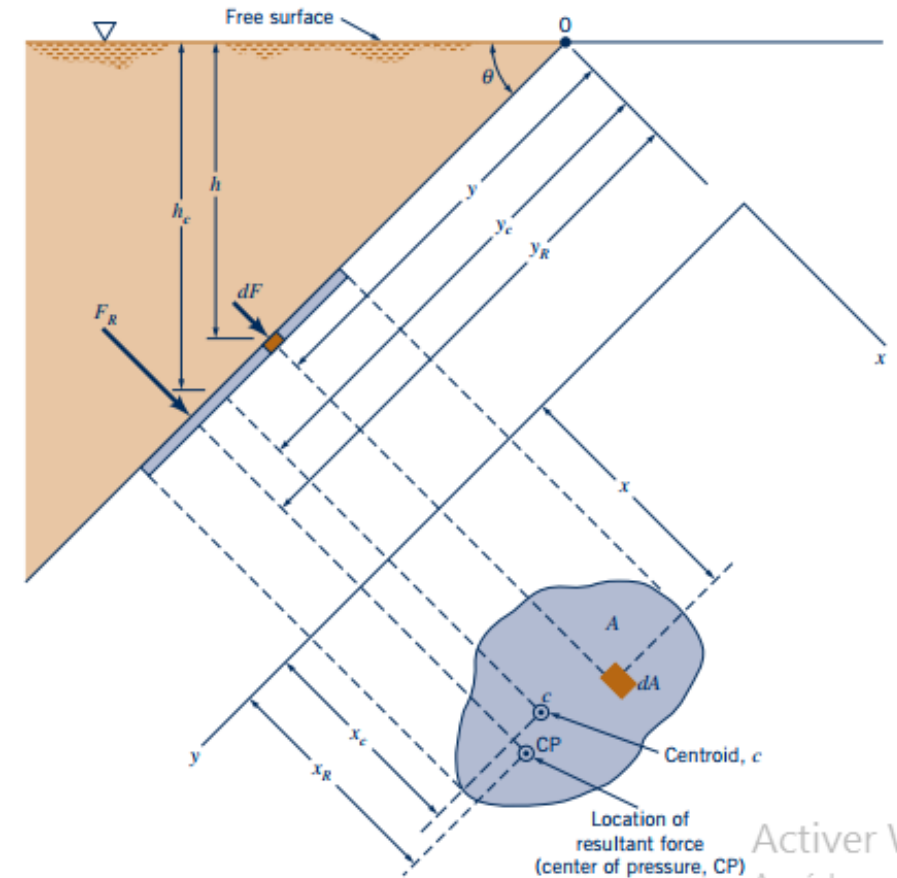
$$\int_A y \, dA = y_c A$$

where y_c is the y coordinate of the centroid of area A measured from the x axis which passes through 0. Equation 2.17 can thus be written as

$$F_R = \gamma A y_c \sin \theta$$

or more simply as

$$F_R = \gamma h_c A$$



Note that the magnitude of the force is independent of the angle, it depends only on the **specific weight** of the fluid, **the total area**, and the **depth of the centroid** of the area below the surface

The y coordinate (y_R) of the **resultant force** can be determined by summation of moments around the x axis. That is, the moment of the resultant force must equal the moment of the distributed pressure force, or :

$$F_R y_R = \int_A y dF = \int_A \gamma \sin \theta y^2 dA$$

and, therefore, since $F_R = \gamma A y_c \sin \theta$

$$y_R = \frac{\int_A y^2 dA}{y_c A}$$

The integral in the numerator is the *second moment of the area (moment of inertia)*, I_x , with respect to an axis formed by the intersection of the plane containing the surface and the free surface (x axis). Thus, we can write

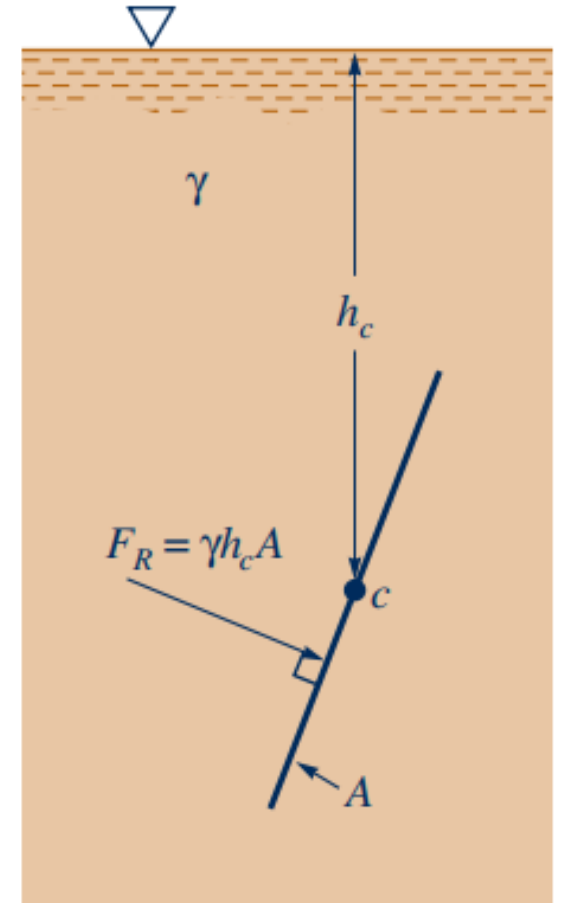
$$y_R = \frac{I_x}{y_c A}$$

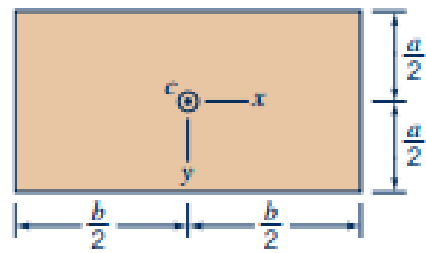
Use can now be made of the parallel axis theorem to express I_x as

$$I_x = I_{xc} + A y_c^2$$

where I_{xc} is the second moment of the area with respect to an axis passing through its *centroid* and parallel to the x axis. Thus,

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$





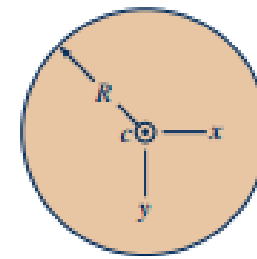
$$A = ba$$

$$I_{xc} = \frac{1}{12} ba^3$$

$$I_{yc} = \frac{1}{12} ab^3$$

$$I_{xyc} = 0$$

(a) Rectangle

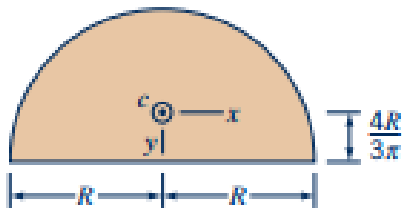


$$A = \pi R^2$$

$$I_{xc} = I_{yc} = \frac{\pi R^4}{4}$$

$$I_{xyc} = 0$$

(b) Circle



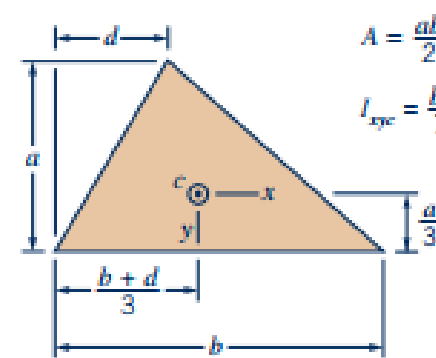
$$A = \frac{\pi R^2}{2}$$

$$I_{xc} = 0.1098R^4$$

$$I_{yc} = 0.3927R^4$$

$$I_{xyc} = 0$$

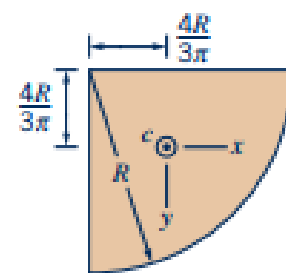
(c) Semicircle



$$A = \frac{ab}{2} \quad I_{xc} = \frac{ba^3}{36}$$

$$I_{xyc} = \frac{ba^2}{72}(b - 2d)$$

(d) Triangle



$$A = \frac{\pi R^2}{4}$$

$$I_{xc} = I_{yc} = 0.05488R^4$$

$$I_{xyc} = -0.01647R^4$$

(e) Quarter circle

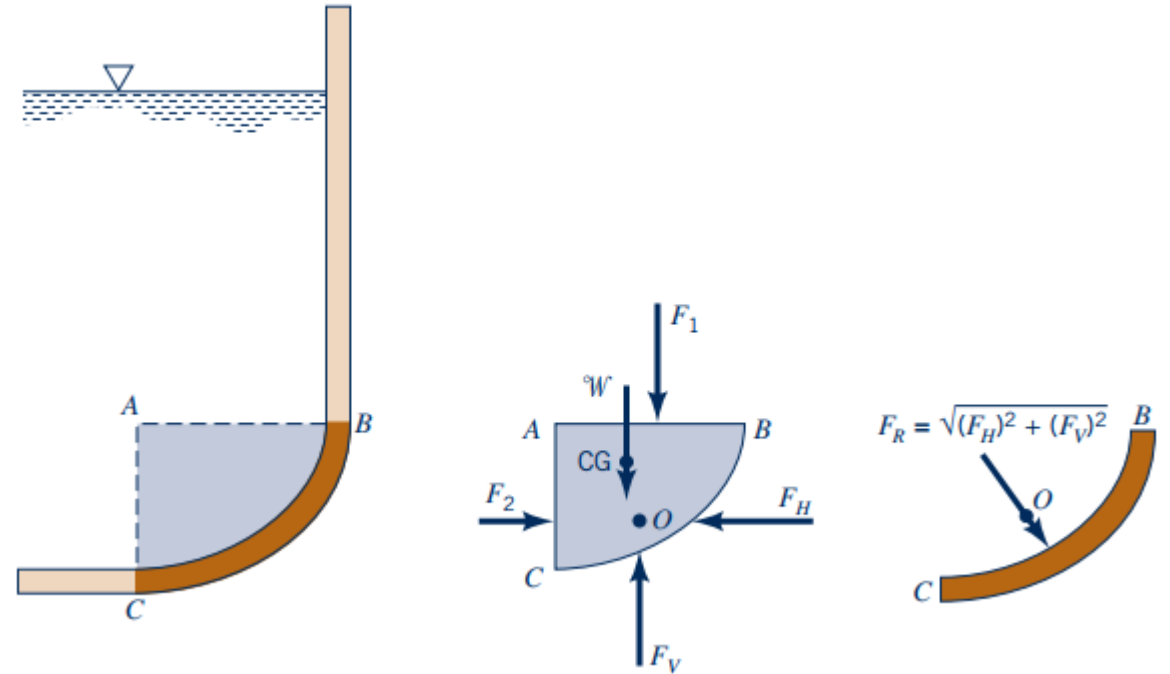
■ **Figure 2.18** Geometric properties of some common shapes.

* Hydrostatic Force on a Curved Surface

$$F_H = F_2$$
$$F_V = F_1 + W$$

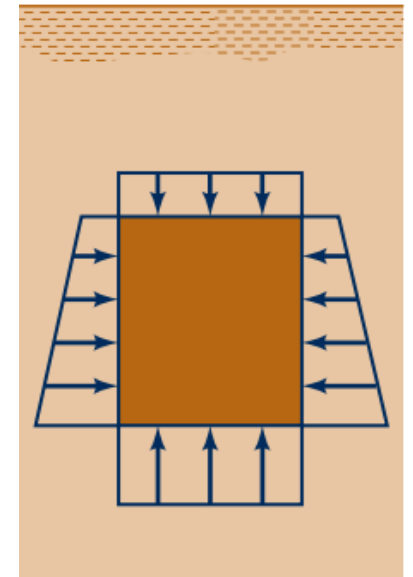
and the magnitude of the resultant is obtained from the equation

$$F_R = \sqrt{(F_H)^2 + (F_V)^2}$$



2-6 Buoyancy and Flotation

Archimede's Principle :When a stationary body is completely submerged in a fluid such as the hot air balloon, or floating so that it is only partially submerged, the resultant fluid force acting on the body is called the *buoyant force*. A net upward vertical force results because pressure increases with depth and the pressure forces acting from below are larger than the pressure forces acting from above



$$F_B = F_2 - F_1 - W$$

If the specific weight of the fluid is constant, then

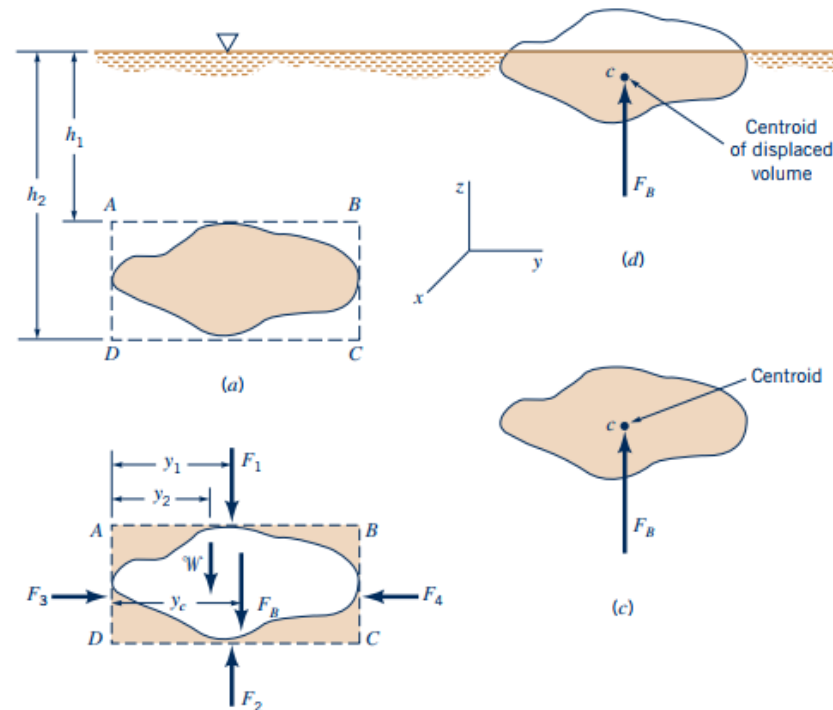
$$F_2 - F_1 = \gamma(h_2 - h_1)A$$

where A is the horizontal area of the upper (or lower) surface of the parallelepiped, and Eq. 2.21 can be written as

$$F_B = \gamma(h_2 - h_1)A - \gamma[(h_2 - h_1)A - V]$$

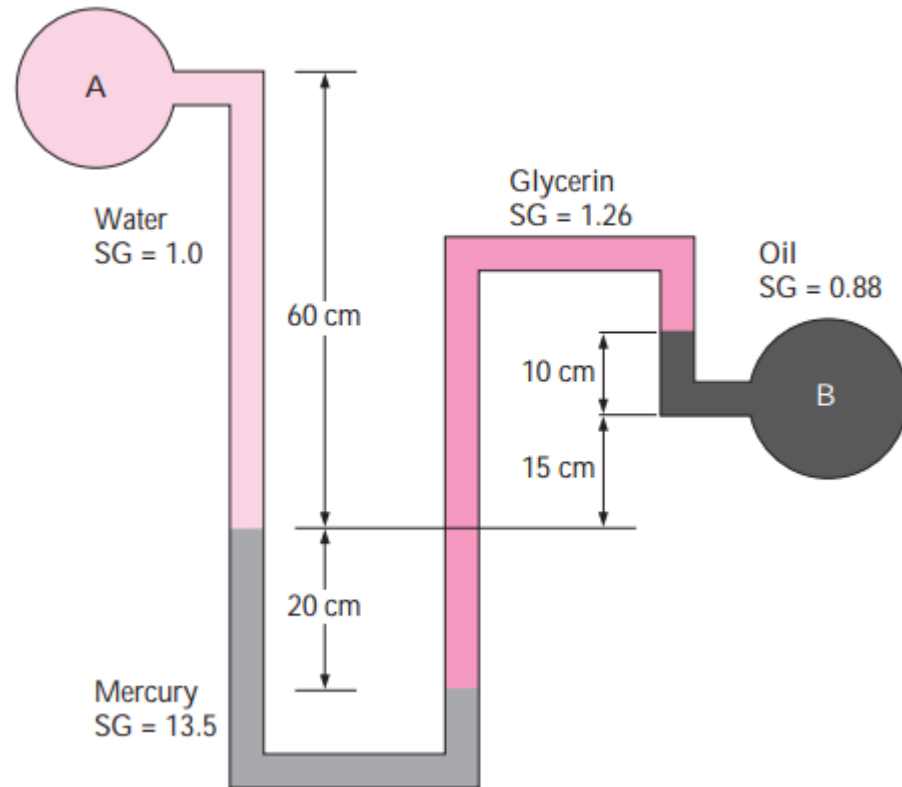
Simplifying, we arrive at the desired expression for the buoyant force

$$F_B = \gamma V$$

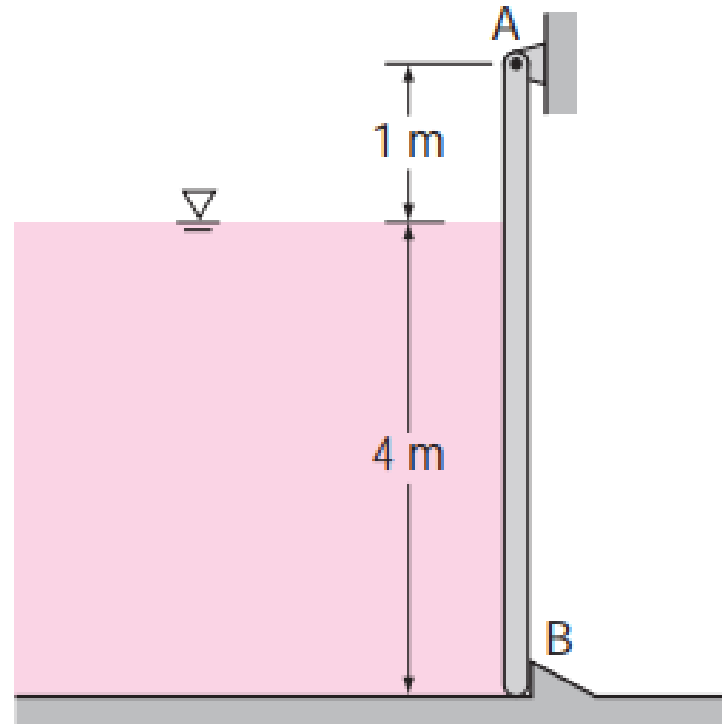


2-7 Applications

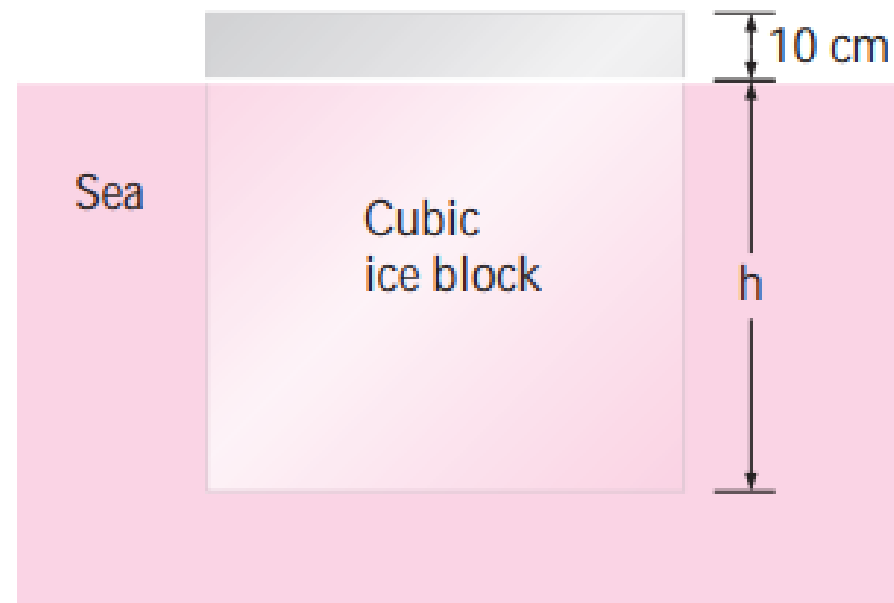
Exercise 1 The pressure difference between an oil pipe and water pipe is measured by a double-fluid manometer, as shown in Fig. P3–49. For the given fluid heights and specific gravities, calculate the pressure difference $\Delta P = P_B - P_A$.



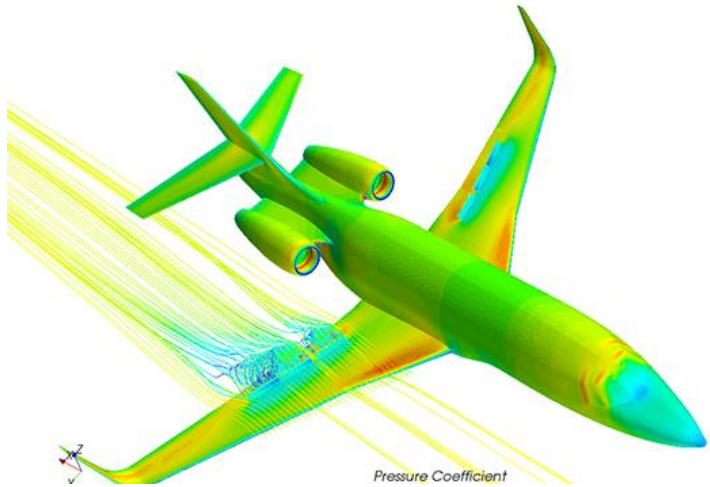
Exercise 2 A 4-m-high, 5-m-wide rectangular plate blocks the end of a 4-m-deep freshwater channel, as shown in Fig. P3–66. The plate is hinged about a horizontal axis along its upper edge through a point A and is restrained from opening by a fixed ridge at point B. Determine the force exerted on the plate by the ridge.



Exercise 3 Consider a large cubic ice block floating in seawater. The specific gravities of ice and seawater are 0.92 and 1.025, respectively. If a 10-cm-high portion of the ice block extends above the surface of the water, determine the height of the ice block below the surface. *Answer: 87.6 cm*

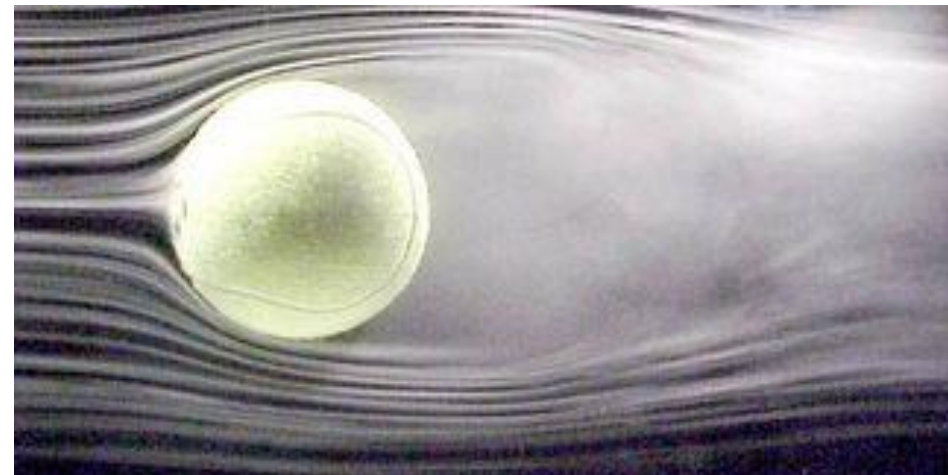


Chapter three : Fluid dynamics , Bernoulli and energy equations



*** Conservation of Energy Principle**

*** Conservation of Mass Principle**



Learning Objectives

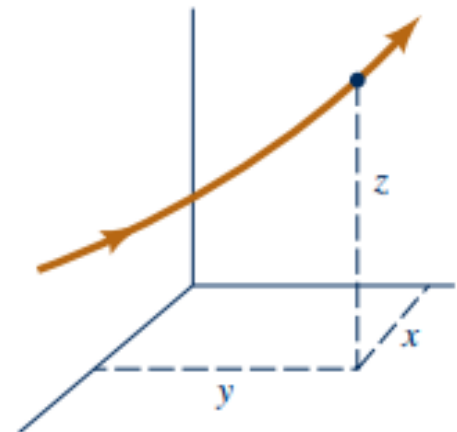
- Discuss the application of Newton's second law to fluid flows.
- Explain the development, uses, and limitations of the Bernoulli equation.
- Use the Bernoulli equation (stand-alone or in combination with the continuity equation) to solve simple flow problems.
- Apply the concepts of static, stagnation, dynamic, and total pressures.

Newton's Second Law :

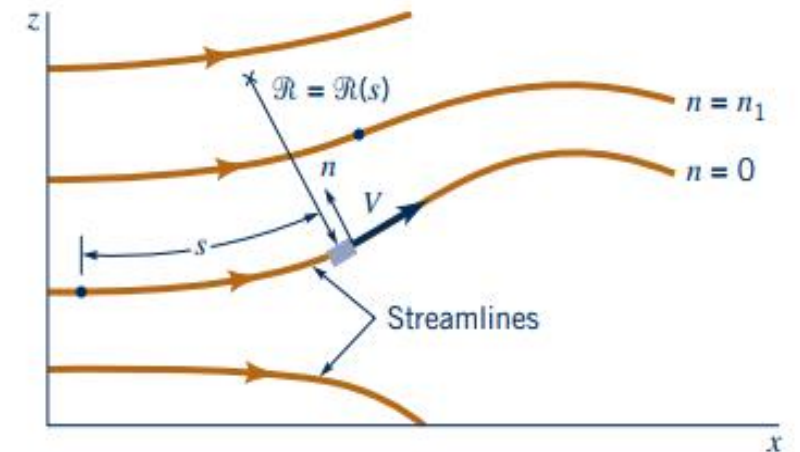
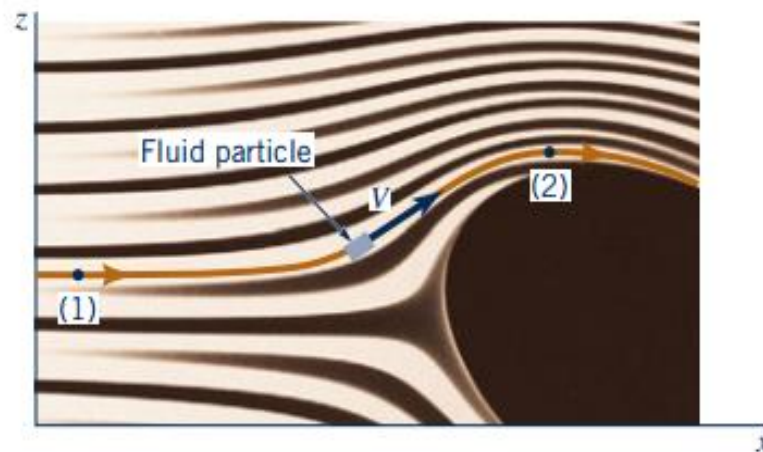
- We assume that the fluid motion is governed by pressure and gravity forces only and examine Newton's second law as it applies to a fluid particle in the form:

$$\text{(Net pressure force on particle)} + \text{(net gravity force on particle)} = \text{(particle mass)} \times \text{(particle acceleration)}$$

Inviscid fluid flow is governed by pressure and gravity forces.



- For **steady flows** (nothing changes with time at a given location in the flow field) each particle slides along its path, and its velocity vector is everywhere tangent to the path. The lines that are tangent to the velocity vectors throughout the flow field are called **STREAMLINES**. In two-dimensional flow, the acceleration can be decomposed into two components: streamwise acceleration: $a_s = \frac{dV}{dt}$ along the streamline and normal acceleration a_n in the direction normal to the streamline, which is given as: $a_n = V^2/R$



3-1 Conservation of Energy Principle

Force Balance along Streamlines (Bernoulli Equation)

$$\sum F_s = ma_s \quad a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s} \frac{ds}{dt} = \frac{\partial V}{\partial s} V = V \frac{dV}{ds}$$

$$P dA - (P + dP) dA - W \sin \theta = mV \frac{dV}{ds}$$

$$-dP dA - \rho g dA ds \frac{dz}{ds} = \rho dA ds V \frac{dV}{ds}$$

Canceling dA from each term and simplifying,

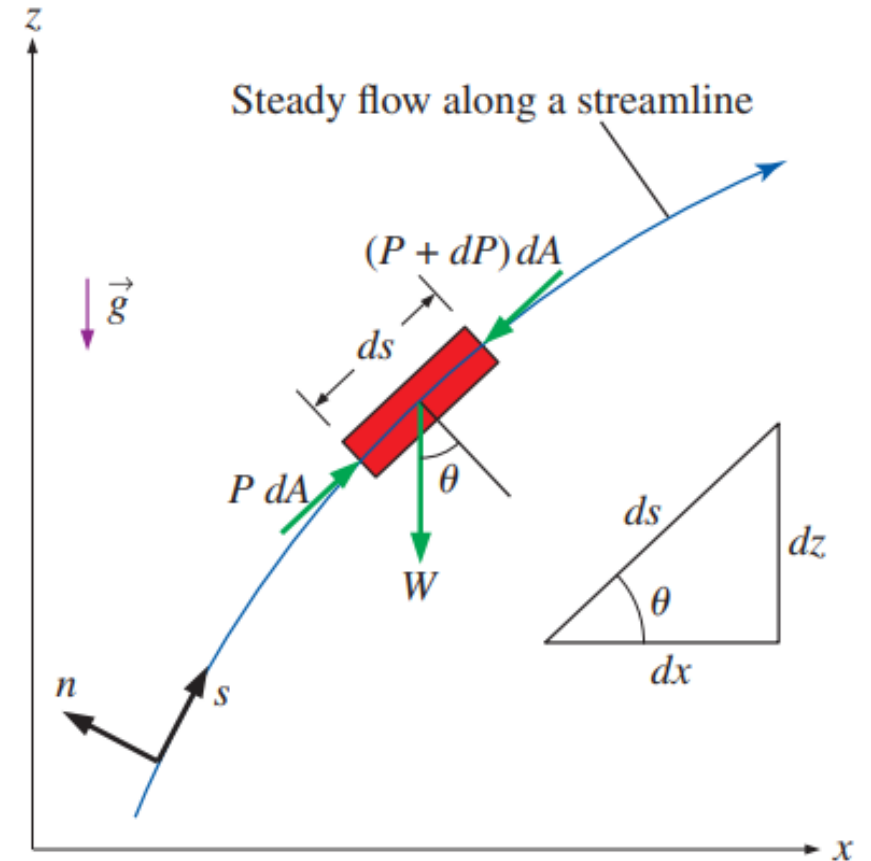
$$-dP - \rho g dz = \rho V dV$$

Noting that $V dV = \frac{1}{2} d(V^2)$ and dividing each term by ρ gives

$$\frac{dP}{\rho} + \frac{1}{2} d(V^2) + g dz = 0$$

Integrating,

Steady flow: $\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)}$



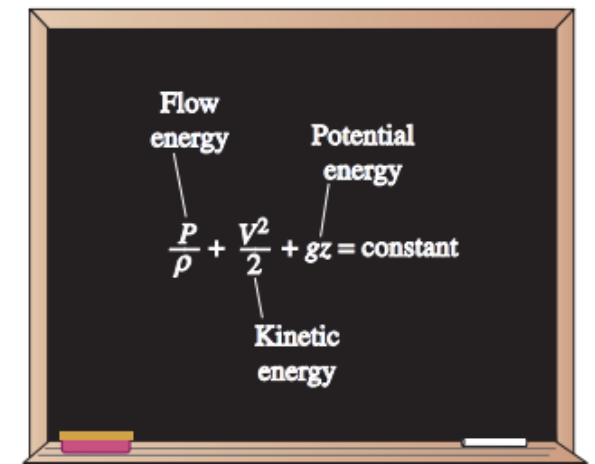
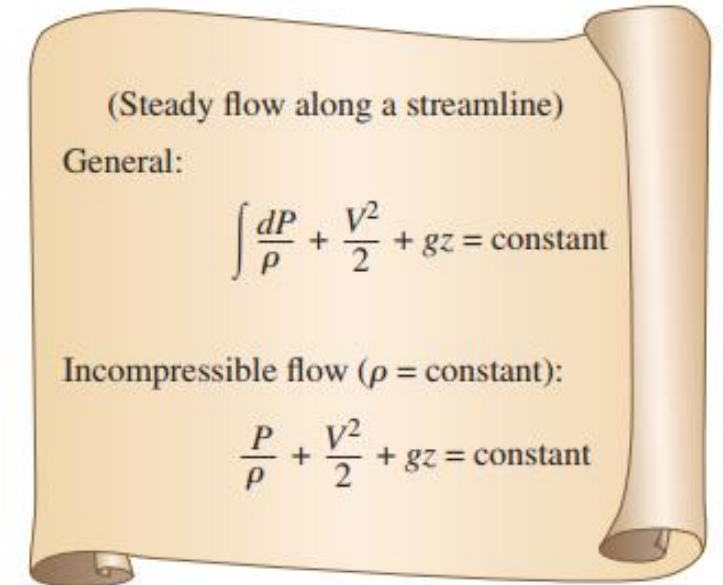
- since the last two terms are exact differentials. In the case of incompressible flow, the first term also becomes an exact differential, and integration gives

Steady, incompressible flow: $\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)}$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

This is the famous **Bernoulli equation** which is commonly used in fluid mechanics for steady, incompressible flow along a streamline in inviscid regions of flow. The Bernoulli equation was first stated in words by the Swiss mathematician Daniel Bernoulli (1700–1782) :

The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when compressibility and frictional effects are negligible.

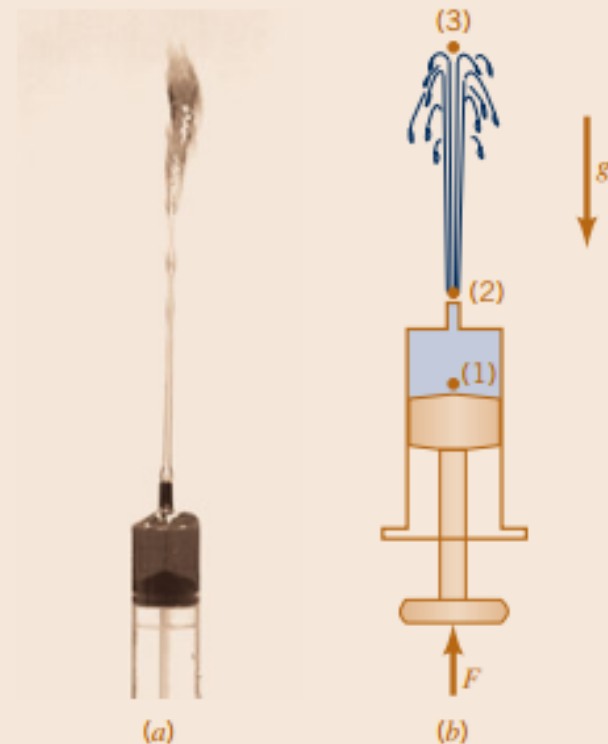


EXAMPLE 3.4 Kinetic, Potential, and Pressure Energy

GIVEN Consider the flow of water from the syringe shown in Fig. E3.4a. As indicated in Fig. E3.4b, a force, F , applied to the

plunger will produce a pressure greater than atmospheric at point (1) within the syringe. The water flows from the needle, point (2), with relatively high velocity and coasts up to point (3) at the top of its trajectory.

FIND Discuss the energy of the fluid at points (1), (2), and (3) by using the Bernoulli equation.



Point	Energy Type		
	Kinetic $\rho V^2/2$	Potential γz	Pressure p
1	Small	Zero	Large
2	Large	Small	Zero
3	Zero	Large	Zero

3-2 Force Balance across Streamlines

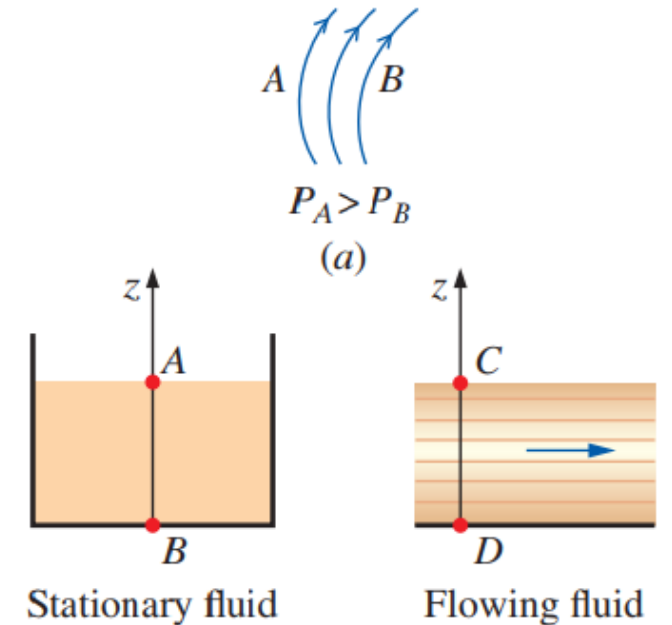
Note that streamwise acceleration is due to a change in speed along a streamline, and normal acceleration is due to a change in direction. For particles that move along a *straight path*, $a_n = 0$ since the radius of curvature is infinity and thus there is no change in direction,

$$\frac{P}{\rho} + \int \frac{V^2}{R} dn + gz = \text{constant} \quad (\text{across streamlines})$$

For flow along a straight line, $R \rightarrow \infty$ and the equation reduces to

$$P/\rho + V^2/2 + gz = C \quad \rightarrow \quad P_1/\rho + gz_1 = P_2/\rho + gz_2$$

$$a_n = \frac{dV}{dt} \quad \rightarrow \quad \Delta P = -\rho g \Delta z$$



$$P_B - P_A = P_D - P_C$$

(b)

Limitations on the Use of the Bernoulli Equation :

- **Steady flow**
- **Negligible viscous effects**
- **No shaft work** (is not applicable in a flow section that involves a pump, turbine, fan, or any other machine)
- **Incompressible flow** ($\rho = \text{constant}$)
- **Negligible heat transfer** (The density of a gas is inversely proportional to temperature)

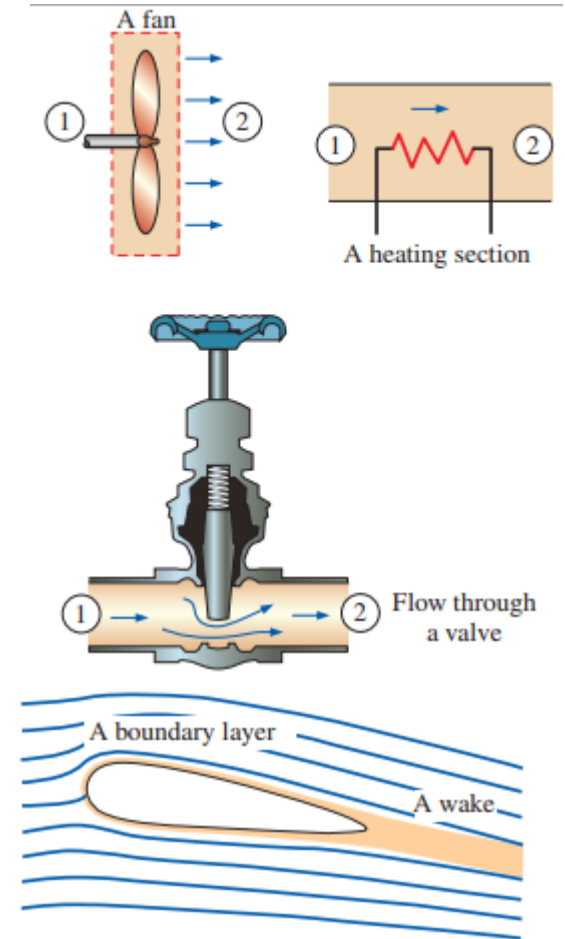
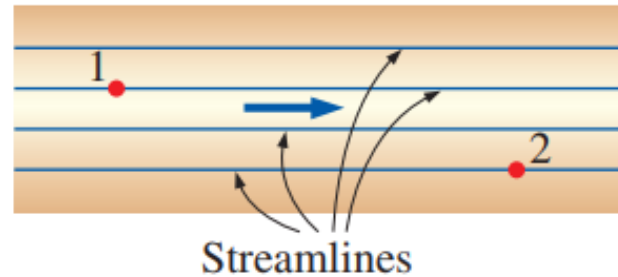


FIGURE 5-32

Frictional effects, heat transfer, and components that disturb the streamlined structure of flow make the Bernoulli equation invalid. It should *not* be used in any of the flows shown here.

3-3 Flow along a streamline

The Bernoulli equation $P/\rho + V^2/2 + gz = C$ is applicable along a streamline, and the value of the constant C is generally different for different streamlines. However, when a region of the flow is *irrotational* and there is *no vorticity* in the flow field, the value of the constant C remains the same for all streamlines)



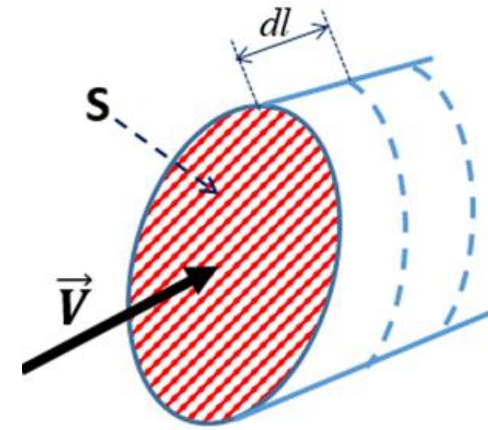
$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

When the flow is irrotational, the Bernoulli equation becomes applicable between any two points along the flow (not just on the same streamline).

3-4 Conservation of Mass Principle

- **Mass flow rate** : The amount of mass passing through a section per unit time is called mass flow rate and is denoted by :

$$\dot{m} = \frac{dm}{dt} = \rho S \frac{dl}{dt} = \rho SV \quad [\text{kg/s}]$$



- **Volume flow rate** : The amount of volume passing through a section per unit time is called volume flow rate and is denoted by :

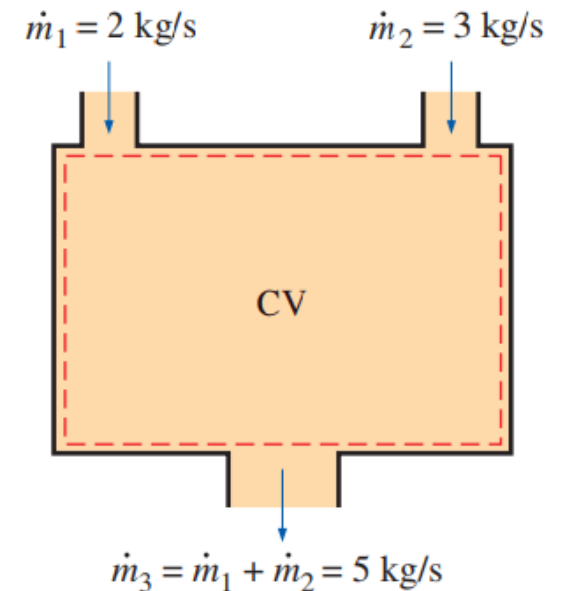
$$q_v = \frac{dV}{dt} = \frac{dm / \rho}{dt} = SV \quad [m^3 / s]$$

Mass Balance for Steady-Flow Processes

During a steady-flow process, the total amount of mass contained within a control volume does not change with time ($m_{CV} = \text{constant}$). Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.

Steady flow:

$$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m} \quad (\text{kg/s})$$

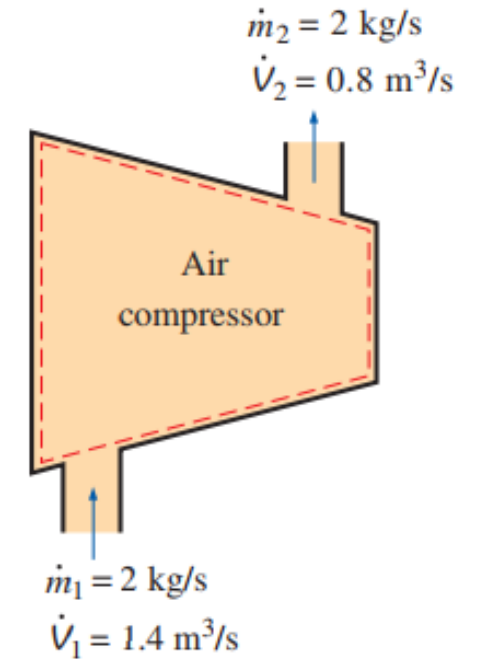


- Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet).

Steady flow (single stream): $\dot{m}_1 = \dot{m}_2 \rightarrow \rho_1 V_1 A_1 = \rho_2 V_2 A_2$

- **Incompressible Flow**: The conservation of mass relations can be simplified even further when the fluid is incompressible, which is usually the case for liquids. Canceling the density from both sides of the general steady-flow relation gives

Steady, incompressible flow (single stream): $\dot{V}_1 = \dot{V}_2 \rightarrow V_1 A_1 = V_2 A_2$



During a steady-flow process, volume flow rates are not necessarily conserved although mass flow rates are.

3-5 Examples of Use of the Bernoulli Equation

• Static, Dynamic, and Stagnation Pressures (Pitot Tubes)

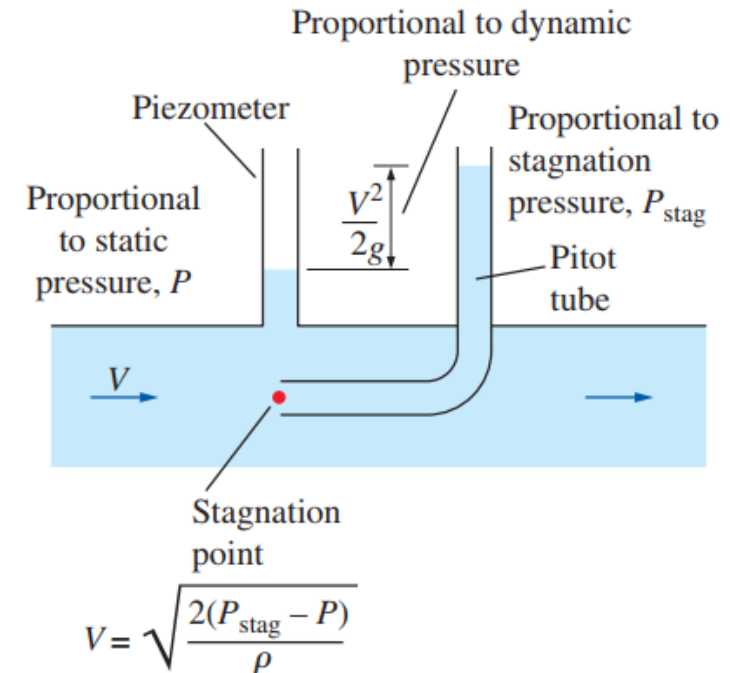
$$P + \rho \frac{V^2}{2} + \rho g z = \text{constant (along a streamline)}$$

Each term in this equation has pressure units, and thus each term represents some kind of pressure:

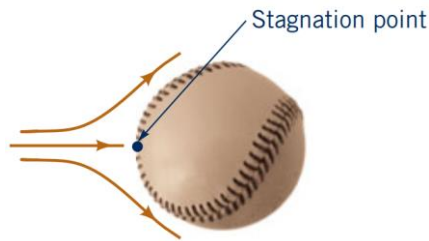
P is the **STATIC PRESSURE** (it does not incorporate any dynamic effects); it represents the actual thermodynamic pressure of the fluid.

$\rho V^2/2$ is the **DYNAMIC PRESSURE**; it represents the pressure rise when the fluid in motion is brought to a stop isentropically.

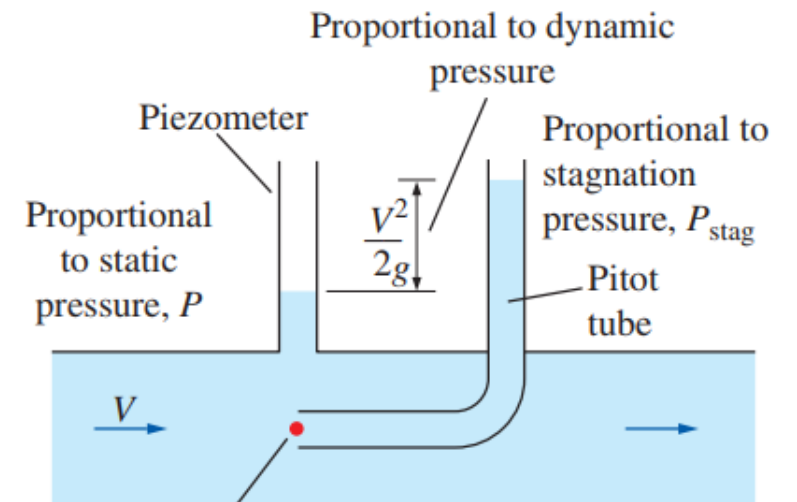
$\rho g z$ is the **HYDROSTATIC PRESSURE** term, which is not pressure in a real sense since its value depends on the reference level selected; it accounts for the elevation effects.



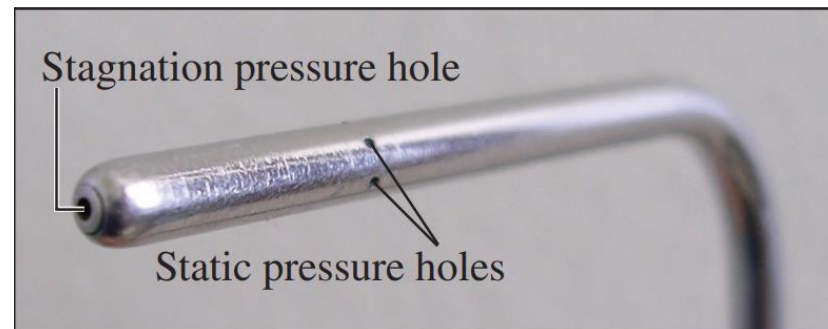
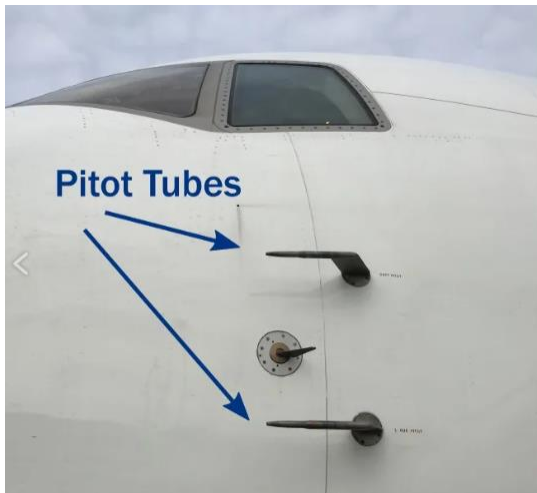
- The sum of the static, dynamic, and hydrostatic pressures is called the **total pressure**. Therefore, the Bernoulli equation states that *the total pressure along a streamline is constant*. The sum of the **static** and **dynamic** pressures is called the **Stagnation Pressure (measured by Pitot tubes)**, it is expressed as :



$$P_{\text{stag}} = P + \rho \frac{V^2}{2} \quad (\text{kPa})$$

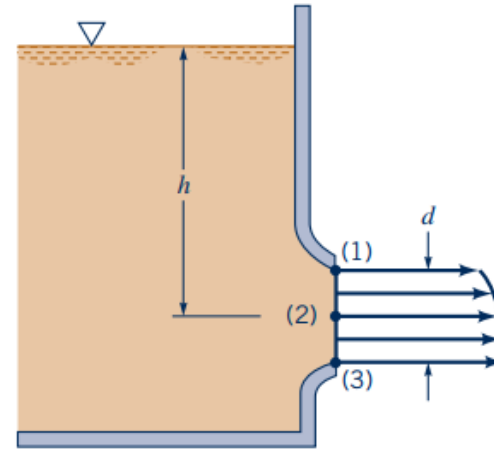


$$V = \sqrt{\frac{2(P_{\text{stag}} - P)}{\rho}}$$



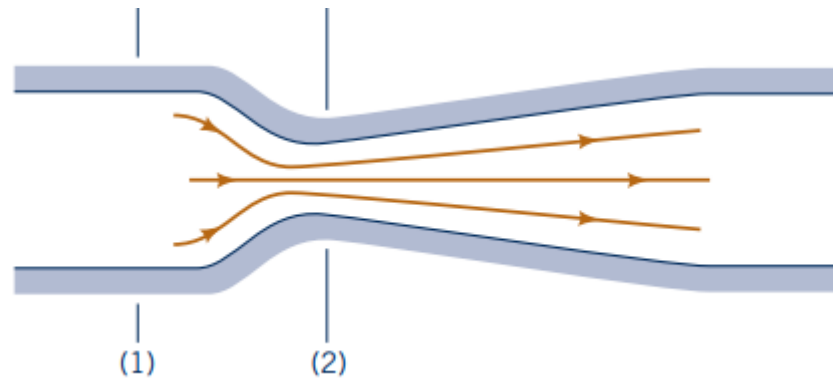
• Free Jets

$$V = \sqrt{2 \frac{\gamma h}{\rho}} = \sqrt{2gh}$$



• Flowrate Measurement

$$Q = A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (A_2/A_1)^2]}}$$

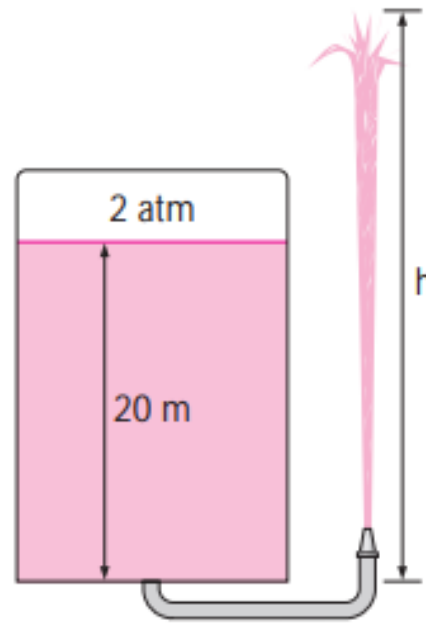


Venturi

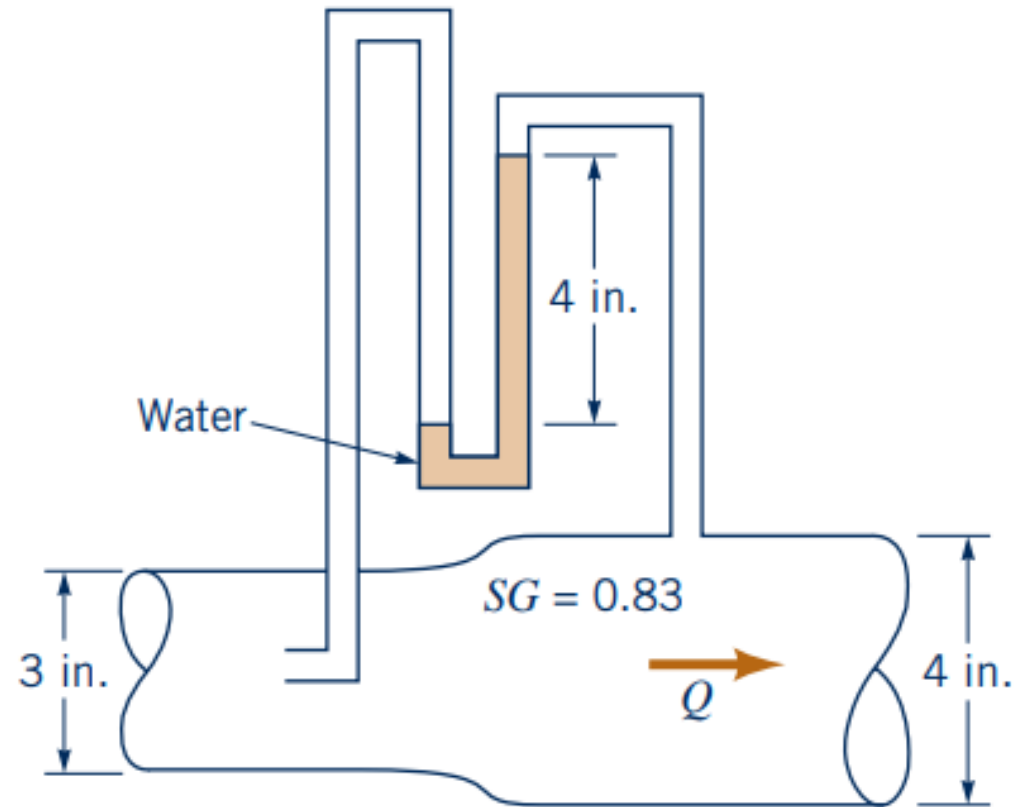


3-6 Applications

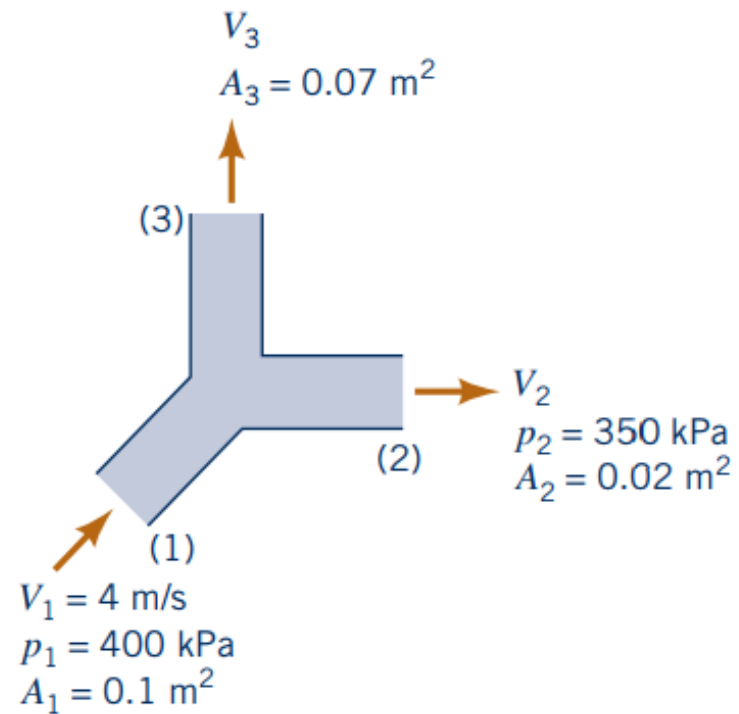
Exercise 1 The water level in a tank is 20 m above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The tank cover is airtight, and the air pressure above the water surface is 2 atm gage. The system is at sea level. Determine the maximum height to which the water stream could rise. *Answer: 40.7 m*



Exercise 2 Oil of specific gravity 0.83 flows in the pipe shown in Fig. P3.67. If viscous effects are neglected, what is the flowrate?



Exercise 3 Water flows through a horizontal branching pipe as shown in Fig. P3.100. Determine the pressure at section (3).



References

- Fluid Mechanics - Fundamentals and Applications 3rd Edition Cengel and Cimbala(2014)
- Fundamentals of Fluid Mechanics 7th Munson (2013)
- Fluid Mechanics 4th ed - F. White (1997)
- Engineering Fluid Mechanics 9th ED Book – Crowe(2009)
- Google photos